X-RAY BURST SOURCES - A SIMPLE TWO ZONE MODEL

J. Robert Buchler and Manuel Barranco
Department of Physics, University of Florida and
Mario Livio
Department of Physics and Astronomy, University of Florida
and Tel-Aviv University, Ramat-Aviv

ABSTRACT

With the help of a very simple two zone model, we demonstrate the possibility of periodic thermal relaxation (limit cycle) oscillations in the helium burning envelope of accreting neutron stars. Physically reasonable model parameters can be chosen which yield agreement with the observed features of x-ray bursts and we suggest that this limit cycle is operative in neutron stars which have an accretion rate in a specific range. For hydrogen burning a similar cycle is possible, but it operates at such high temperatures that an unrealistically large accretion rate would be required.

1. INTRODUCTION

X-ray burst sources are known to exhibit a luminosity behavior which is very reminiscent of relaxation oscillations (although not strictly periodic). The nature of the burst has been studied extensively (see e.g., Lewin and Joss 1977 for references). Thermonuclear flashes on accreting neutron stars have been suggested as a possible cause of the bursts (Woosley and Taam 1976, Maraschi and Cavaliere 1977, Joss 1977, Lamb and Lamb 1978). The complete calculation of model envelopes on neutron stars is quite involved and requires intricate evolutionary programs (Joss 1978, Joss and Li 1980). It seems, however, that the major features of these calculations can be reproduced by means of a very simple two zone model. It includes the essential physical processes and it exhibits a limit cycle oscillation as the solution to two nonlinear differential equations.

2. THE TWO ZONE MODEL

The neutron star and its accreted envelope are divided into the following zones:

Space Science Reviews 27 (1980) 585–590. 0038–6308/80/0274–585 $00.90.

https://doi.org/10.1017/S0252921100082099 Published online by Cambridge University Press

1. The core which is assumed to be inert and of fixed mass M_{core}, radius R_{core} and temperature T_{core}.

2. The burning shell (denoted by subscript 1) which is assumed to be very narrow and of small mass.

3. A buffer zone which consists of the envelope above the burning shell (denoted by subscript 2). This zone can become entirely or partially convective during the flash.

4. The outer atmosphere, treated also as inert and at constant temperature T_{ph}.

We incorporate the hydrostatic adjustment of the zones as a result of heat input by the method of Henyey and Ulrich (1972). We write (Barranco, Buchler and Livio 1980)

$$\delta T_i = \left(\frac{1}{c_{pi}} - G_{ii}\right) T_i \delta S_i - T_i G_{ij} \Delta_{mj}\ T_j\ \delta S_j \qquad (1)$$

$$i,j = 1,2$$

where $G_{ij} = G(q_i,q_j)$ is a Green's function of the mass ratios q_i (Henyey and Ulrich 1972).

The following simplifying assumptions are made in writing the energy balance equations for the two time dependent zones:

1. We assume the energy transfer between zones 1 and 2 to be radiative. This in a way follows from the definition of zone 1 as being very narrow. The transfer between zone 2 and the photosphere is clearly radiative. Conduction is taken into account between zone 1 and the core.

2. We neglect terms proportional to Δm_1, which is assumed to be very small compared to Δm_2.

3. The cross terms which contain G_{12}, G_{21} are neglected. This is justified by the expectation that the Green functions are sharply peaked (Henyey and Ulrich 1972).

The energy balance equations then assume the form (Barranco, Buchler and Livio 1980):

$$T_1 \frac{ds_1}{dt} = \varepsilon_1(T_1) - \frac{1}{\Delta m_1} \left[K_c(T_1^4 - T_c^4) + K_1(T_1^4 - T_2^4)\right] \qquad (2)$$

https://doi.org/10.1017/S0252921100082099 Published online by Cambridge University Press

$$T_2 \frac{ds_2}{dt} = \frac{1}{\Delta m_2} [K_1(T_1^4 - T_2^4) - K_2(T_2^4 - T_{ph}^4)] \tag{3}$$

where

$$K_i = \frac{(4\pi R_i^2)^2 \; ac}{3\kappa_i \Delta m_i^*} \tag{4}$$

$$T_{h2} = (c_{p2} \; \Delta m_2 \; G_{22})^{-1} \tag{5}$$

here K_i is the effective opacity, Δm_i^* are the edge centered masses, ε_1 is the energy generation rate and s is the entropy (both per unit mass). Since the buffer zone is nondegenerate from application of the virial theorem, we expect it to have a negative effective heat capacity, namely, we assume that $T_2 > T_{h2}$ (Buchler and Perdang 1979).

Introducing the parameters θ and τ with dimensions of temperature and time respectively, we obtain the dimensionless equations:

$$\frac{dT_1}{dt} = \alpha\{\frac{\varepsilon_1(T_1)}{\varepsilon^*} - \gamma_{c1}(T_1^4 - T_c^4) - (T_1^4 - T_2^4)\} \tag{6}$$

$$\frac{dT_2}{dt} = \alpha\chi(1 - \frac{T_2}{T_{h2}})\{T_1^4 - T_2^4) - \gamma_{12}(T_2^4 - T_{ph}^4)\} \tag{7}$$

where

$$\varepsilon^* = \frac{K_1 \theta^4}{\Delta m_1} \tag{8}$$

$$\alpha = \frac{K_1 \theta^3 \tau}{C_{p1} \Delta m_1} = \frac{\varepsilon^* \tau}{C_{p1} \theta}$$

$$\gamma_{c1} = \frac{K_c}{K_1} \cong \frac{\Delta m_1^*}{\Delta m_c} \cong \frac{\kappa_1}{\kappa_c}$$

$$\gamma_{12} = \frac{K_2}{K_1} \cong \frac{\Delta m_1^*}{\Delta m_2^*}$$

$$\chi = \frac{C_{p1} \Delta m_1}{C_{p2} \Delta m_2} \cong \frac{\Delta m_1}{\Delta m_2} << 1$$

Here α, χ, ε^*, γ_{12} and γ_{C1} are assumed to be constant except for temperature dependence of the opacities.

3. RESULTS

Detailed calculations have shown that the x-ray bursts can be best reproduced by helium burning (Joss 1977, 1978, Taam and Picklum 1979, after original work by Hansen and Van Horn 1975). We first assume that energy is generated via the triple alpha reaction, the reaction rate of which is taken from Clayton (1968). Neutrino losses have been included following Beaudet, Petrosian and Salpeter (1967) but are found to be negligible. The dominant radiative opacity is electron scattering (Joss 1978) and for the conductive opacity fitting formulae of Bodenheimer et al (1965) have been used.

In order to study the existence of limit cycle oscillations one has to investigate the shape of the equilibrium curves (EC) $F_1(T_1,T_2) = 0$ and $F_2(T_1,T_2) = 0$, in the (T_1,T_2) "phase plane". With the appropriate choice of the parameters the first EC has an S-shape while the second is almost a straight line. These curves are represented in Barranco et al (1980). Necessary and sufficient conditions for the existence of a limit cycle are (Buchler and Perdang 1979):

a. Both functions F_1 and F_2 are negative to the left of their respective EC.

b. The EC of F_2 must intersect the first EC on the middle (unstable) branch of the S curve. These two conditions ensure that the intersection point is an unstable nodal point. In the limit $\chi \to 0$ relaxation oscillations will be obtained. The parameters which determine most sensitively whether limit cycle oscillations ensue are ε^*, on which the existence of the S-shape depends and γ_{12} which governs the location of the intersection point; ε^* also determines the lowest value of T_2 during the cycle. The parameters used by Barranco et al (1980) were:

$$\varepsilon^* = 0.957 \times 10^{16}, \quad \gamma_{12} = 2, \quad \gamma_{C1} = 0.01, \quad T_{core} = 2.52,$$

$$\theta = 10^8 \text{ °K}.$$

With

$$T_{h2} = 0.5, \; \rho_2 \, \Delta R_{12} = 2.6 \times 10^8 \text{ g/cm}^2$$

(ΔR_{12} is the extent of the buffer zone) a burst period of ~2 hours was obtained which is well in the range of x-ray burst source. The temporal behavior of the temperatures is shown in Barranco et al (1980). In this model values of $1 \lesssim \gamma_{12} \lesssim 10$ ensured the existence of a limit cycle.

An implicit assumption made in the calculation is that of continuous replenishment of the helium that has been burnt. This in turn implies a specific (constant) accretion rate. It is interesting to note that the model predicts that the thermal oscillations (and therefore the bursts) can be obtained only for a relatively limited range of accretion rates. Other accretion rates result in burning temperatures which are either too high or too low compared to the average limit cycle temperature.

4. SUMMARY

A simple two zone model is found to exhibit limit cycle oscillations. We suggest that this may be the underlying mechanism for the burst behavior found in more complex thermonuclear calculations. Nonperiodic changes in parameters assumed constant in our simplified picture would cause departure from the precise periodicity of the thermal relaxation oscillations as would the interaction between more than two zones as would be required in a more detailed model. The inclusion of more complete burning networks is also now in progress, in particular it is interesting to see whether hydrogen burning via the now equilibrium hot CNO cycle can produce an oscillation both on neutron star and on white dwarf surfaces.

This work has been supported in part by the National Science Foundation (AST 79-20024).

REFERENCES

Barranco, M., Buchler, J. R., and Livio, M.: 1980, Ap. J. in press.
Beaudet, G., Petrosian, V., and Salpeter, E. E.: 1967, Ap. J. 150, p. 979.
Bodenheimer, P., Forbes, J. E., Gould, N. L., and Henyey, L. G.: 1965, Ap. J. 141, p. 1019.
Buchler, J. R. and Perdang, J.: 1979, Ap. J. 231, p. 524.
Clayton, D. D.: 1968, "Principles of Stellar Evolution and Nucleosynthesis," New York: McGraw-Hill.
Hansen, C. J. and Van Horn, H. M.: 1975, Ap. J. 195, p. 735.
Henyey, L. G. and Ulrich, R. K.: 1972, Ap. J. 173, p. 109.
Joss, P. C.: 1977, Nature 270, p. 310.
Joss, P. C.: 1978, Ap. J. Lett. 225, p. L123.
Joss, P. C. and Li, F. K.: 1980, Ap. J. 238, p.
Lamb, D. A. and Lamb, F. K.: 1978, Ap. J. 220, p. 291.
Lewin, W. H. G. and Joss, P. C.: 1977, Nature 270, p. 211.
Maraschi, L. and Cavaliere, A.: 1977, Highlights of Astronomy, Vol. 4, ed. E. A. Müller, (Dordrecht: Reidel), Part 1, p. 127.
Taam, R. E. and Picklum, R. E.: 1979, Ap. J. 233, p.327.
Woosley, S. E. and Taam, R. E.: 1976, Nature 263, p. 101.

DISCUSSION

J. COX: Can you apply the same model to the oscillatory behavior of the secular stability problem?

BUCHLER: We have applied this to the thin helium shell burning instability, the Schwarzschild-Härm instability. It is on a sounder basis there because the Greens functions are known. The same approximations are valid there, and it reproduces very nicely the pulses.

JOSS: Hydrogen burning has a fundamental problem. At relevant temperatures the CNO cycle is completely saturated because one has to wait for the positron decays. There is, therefore, no temperature dependence of the hydrogen burning, and there is no limit cycle for pure hydrogen burning.

BUCHLER: We assumed here CNO equilibrium.

COLGATE: What is the behavior if helium is diluted with hydrogen?

JOSS: Then life gets much more complicated. Triple alpha burning makes carbon and oxygen on which protons capture to make heavier elements. The energetics are more complicated, but the fundamental instability is the triple alpha reaction.

BUCHLER: That may change the oscillation period as these two shells interact. Rose, studying pure helium stars, got strict periodicity. Schwarzschild and Härm considered the hydrogen shell and found two periods - the oscillations decayed and then reappeared. Maybe that will happen here too.