Radial and Nonradial Pulsations as Probes of Stellar Physics
ASP Conference Series, Vol. 259, 2002
C. Aerts, T.R. Bedding, & J. Christensen-Dalsgaard, eds.

Convective Overshooting: Physical Properties and Seismic Evidence

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Abstract. We review briefly the different prescriptions which have been proposed to predict the extent of convective penetration (or overshoot) in stellar interiors, and we confront them with the results of numerical simulations and with helioseismic data. It appears that the penetrative motions are structured in plumes, and that thermal diffusion plays an important role in controlling the temperature stratification in the stable domain. The most recent high-resolution simulations suggest that these plumes are less space-filling than thought before, and that they are therefore less efficient in establishing an adiabatic temperature profile. This property is compatible with the solar profiles obtained through acoustic sounding.

1. Introduction

Thirty years ago, convective overshoot has been introduced in stellar models to achieve better agreement with observational data. By adjusting the amount of overshoot from convective cores, thus increasing the size of the well mixed inner region, it became easier to match theoretical and observed isochrones in the temperature/luminosity diagram (Castellani et al., 1971; Prather & Demarque, 1974; Cogan, 1975; Maeder, 1975).

There were also good physical reasons for proceeding in that way. When a fluid element crosses the border of a convectively unstable region, it continues to move into the stable adjacent domain, until the adverse buoyancy force brings it to a halt. Thus the question is not whether such overshoot occurs – there are many examples of it in geophysical fluids and in the laboratory – but rather how much the motions overshoot beyond the unstable region.

It was soon realized that this question has no easy answer. By comparing the steep subadiabatic temperature gradient in the radiation zone with that, superadiabatic and rather low, in the convection zone, Saslaw & Schwarzschild (1965) concluded that the amount of overshoot would be very small. But Shaviv & Salpeter (1973) pointed out that the overshooting eddies, by depositing their heat content in the radiation zone, would lower there the entropy gradient, thereby reducing its stability and easing the penetration of motions. This idea prevailed, and in most subsequent work it was assumed that the temperature gradient was close to adiabatic in the overshoot region.

https://doi.org/10.1017/S0252921100015542 Published online by Cambridge University Press

2. Roxburgh's integral constraint

A significant step forward was made by Roxburgh (1978), who gave an upper limit to the overshoot from a convective core. Assuming that the temperature gradient is almost adiabatic in the whole region occupied by the convective motions, he writes the horizontally averaged heat equation in the form

$$T\frac{d}{dr}\left(4\pi r^2\overline{\rho w s'}\right) = \frac{d}{dr}\left(L - L_{\mathrm{rad}}\right), \tag{1}$$

with the usual notations, w and s' being respectively the vertical velocity and the fluctuation of specific entropy from its horizontal mean. He then divides this equation by T and integrates it over the entire convective core, up to the edge of the overshoot region ($r = r_c$). He thus obtains the following relation

$$\int_0^{r_c} (L - L_{\mathrm{rad}})\, d\left(\frac{1}{T}\right) = 0, \tag{2}$$

which yields the value of r_c, with the contributions of the unstable region ($L > L_{\mathrm{rad}}$) and of the stable region ($L < L_{\mathrm{rad}}$) compensating each other.

This constraint has the merit of being very simple and apparently rigorous. However it predicts a substantial overshoot, much more than allowed by the observations. The reason is that it neglects viscous dissipation. When this effect is included,

$$\int_0^{r_c} (L - L_{\mathrm{rad}})\, d\left(\frac{1}{T}\right) = \int_0^{r_c} \left\{\nu\left(\frac{\partial v_i}{\partial x_j}\right)^2\right\}\frac{dm}{T} > 0, \tag{3}$$

with $dm = 4\pi r^2 \rho\, dr$. The term in brackets is the viscous dissipation rate, whose contribution is far from negligible. To illustrate this, it may be expressed in terms of the largest convective eddies (of velocity V_i), which initiate the turbulent cascade:

$$\int_0^{r_c} \left\{\nu\left(\frac{\partial v_i}{\partial x_j}\right)^2\right\}\frac{dm}{T} = \int_0^{r_c} \left\{\nu_t\left(\frac{\partial V_i}{\partial x_j}\right)^2\right\}\frac{dm}{T}, \tag{4}$$

where we have introduced a suitably defined eddy-viscosity ν_t. By standard mixing-length arguments one can verify that this viscous dissipation integral may be large enough to cancel most of the negative part of the integral on the left-hand side of (3).

Aware of that problem, Roxburgh & Simmons (1993) have performed numerical simulations of 3-dimensional penetrative convection, in order to estimate the effect of viscous dissipation. However the spatial resolution they used did not allow them to reach the regime where the result would not depend any more from mesh size.

3. Early numerical simulations

Such numerical simulations have been performed well before, with much less computer resources, because it was clear that the non-linear terms would play an essential role in convective penetration, and these could not be captured easily by the analytical approach, as was pointed out by Veronis (1963). The first calculations have been carried out in the Boussinesq approximation, thus neglecting the density stratification; they were either 2-dimensional, or they postulated a given horizontal structure (one may call them 1.5-dimensional), a technique called modal expansion. Because of the lack of spatial resolution, they remained confined to rather low values of the Rayleigh number (typically 10 times critical); as a result the solutions were laminar, and most often stationary. They all showed a substantial amount of penetration into the stable layers (Moore & Weiss, 1973), which seemed to correlate with the flux of kinetic energy (Zahn et al., 1982).

The effect of stratification has been explored using the anelastic approximation, whose role is to filter out the acoustic waves, while keeping the other properties of compressibility. As expected, the solutions showed a pronounced up/down asymmetry, but there was no significant change in the amount of penetration (Massaguer et al., 1984). The first realistic case examined was that the upper layers of an A-type star, which has two superposed unstable zones, one due to the ionization of hydrogen and the other to the second ionization of helium. In the mixing-length treatment, these are two separated convection zones. However the modal simulations showed that motions connect the two unstable zones (Latour et al., 1981), a result which was later confirmed by more sophisticated 2-D simulations (Freytag et al., 1996).

Similar 2-D simulations had been carried out earlier by Hurlburt et al. (1986) in a simplified atmosphere (superposed polytropic layers). They showed that penetration was achieved by slim, strong downdrafts which were long-lived, but highly time-dependent. However it was not clear whether those structures were not an artifact of reducing the problem to 2 dimensions, where large eddies may build up through inverse turbulent cascade.

4. A scaling law

The idea that the up/down asymmetry could play an important role in convective penetration was carried further by Schmitt et al. (1984). They postulated that penetration below the solar convection zone was achieved by descending plumes, which they modeled as is done in atmospheric sciences. In addition, they assumed that the (negative) convective flux in the stable region was transported exclusively by these plumes. With this simplified model, they were able to explore a vast parameter space, and they found that the penetration depth scales as $d_{\rm pen} \propto f^{1/2} W_i^{3/2}$, where W_i is the initial velocity of the plumes at the edge of the unstable zone and f their filling factor (i.e. the fraction of horizontal area occupied by them).

It is not difficult to derive the scaling law which was established empirically by Bohn et al. For simplicity, let us work in plane-parallel geometry. We linearize the radiative flux around its value at the base of the unstable zone, located at $z = z_i$ (z is directed downwards, but fluxes are counted positive when upwards):

$$F_{\rm rad} = \chi \left(\frac{dT}{dz}\right)_{\rm ad} = F_{\rm total} \left[1 + \left(\frac{d\ln\chi}{dz}\right)_i (z - z_i)\right], \tag{5}$$

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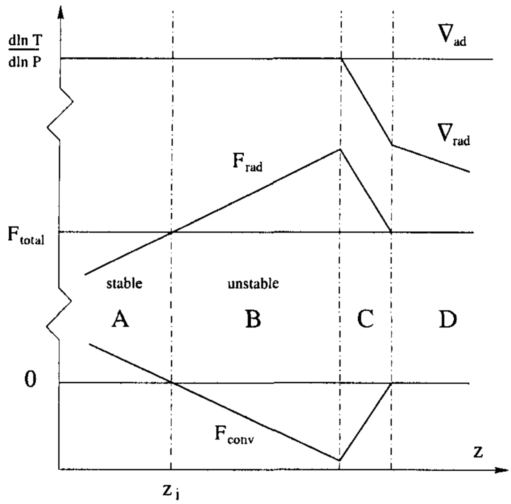

Figure 1. Convective penetration at the base of a convection zone (from Zahn, 1991). (A) designates the unstable region, which ends at $z = z_i$, where the radiative flux, which increases with the conductivity χ, matches the total flux. The motions penetrate into the stable region (B), where they are slowed down by the buoyancy force, and where they establish an almost adiabatic temperature gradient, due to their high Peclet number (see text). When the Peclet number becomes less than unity, the temperature gradient relaxes to the radiative gradient, in the thermal adjustment layer (C). (D) is the radiative interior.

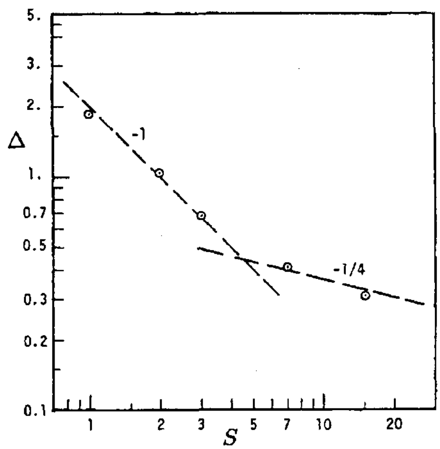

Figure 2. Extent of convective penetration as a function of the stability parameter S characterizing the stratification in the stable layer (Hurlburt et al., 1994, courtesy ApJ).

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where χ is the radiative conductivity. Neglecting the kinetic energy flux, the convective flux is given by $F_{\rm conv} = F_{\rm total} - F_{\rm rad}$, or

$$F_{\rm conv} = -F_{\rm total} \left(\frac{d\ln\chi}{dz}\right)_i (z - z_i). \tag{6}$$

But $F_{\rm conv}$ is also the flux of enthalpy:

$$F_{\rm conv} = -f\rho C_p W \Delta T, \tag{7}$$

where W is the vertical velocity, ΔT the temperature fluctuation from its horizontal mean, and f the filling factor. The variation of kinetic energy obeys

$$\frac{1}{2}\frac{dW^2}{dz} = g\frac{\Delta\rho}{\rho} = -g\frac{\Delta T}{T}, \tag{8}$$

assuming the perfect gas law. After some straightforward eliminations, we integrate Eq. (8) from $W = W_i$ to $W = 0$, which yields the penetration depth (Zahn, 1991)

$$d_{\rm pen}^2 = \frac{3}{5} H_p H_\chi \left\{ f\frac{\rho W_i^3}{F_{\rm total}} \right\}, \tag{9}$$

with H_p and H_χ being respectively the scale-height of pressure and conductivity. This is the relation found by Bohn et al.

One may expect that the term in brackets does not depend much on the size of the unstable zone, provided it is thick enough, and this was confirmed by subsequent numerical simulations, which gave $\{...\} \approx 1/5$. Thus the extent of penetration predicted by this scaling is $0.2 - 0.3H_p$.

So far we have assumed that the motions proceed adiabatically, which allows them to establish an almost adiabatic stratification in the stable zone. But this is true only as long as the Peclet number characterizing the flow, $Pec = Wd/K$, remains larger than unity (d is the size of the plume and $K = \chi/\rho C_p$ the thermal diffusivity). When $Pec < 1$, radiative diffusion must be taken into account, and one finds that the temperature gradient changes from nearly adiabatic to radiative in a thermal adjustment layer, labelled (C) in Fig. 1, whose thickness is of order

$$d_{\rm th}^2 \approx K t_d, \quad \text{where } t_d = \left(\frac{H_p}{g}\right)^{1/2} \tag{10}$$

is the local dynamical time. In the Sun, this is merely a boundary layer, since $d_{\rm th}$ is of order of 1 km.

5. Penetration versus overshoot

We thus see that we have to distinguish between two regimes, depending on the value of the Peclet number. Since we have two names which are loosely employed to designate what seems at first sight a single phenomenon, we may use them in a way to insist on that difference. When $Pec \gg 1$, the fluid motions retain their heat content, and they tend to establish an adiabatic stratification beyond the unstable zone, where they are slowed down by the buoyancy force;

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we propose to call this *convective penetration.* On the contrary, when $Pec < 1$, the motions are unable to keep their temperature and density contrasts, and since they do not feel the buoyancy force, they are able to proceed much deeper into the stable zone, which remains in nearly radiative stratification; we suggest to call that *convective overshoot.* For instance, the link between the two unstable zones in the atmosphere of an A-type star, which we mentioned above, is due to overshoot: the convective motions have a Peclet number less than unity, and they do not disturb the radiative stratification. On the contrary, at the base of the solar convection zone $Pec \approx 10^6$, and we are in presence of convective penetration.

In numerical simulations, where the Peclet number is not much larger than unity, it is possible to observe the transition from one regime to the other. Take for instance the 2-D calculations made by Hurlburt et al. (1994), in a piecewise polytropic envelope. The unstable layer, with polytropic index $m_{\rm unst}$, is on the top of the stable domain with polytropic index $m_{\rm st}$. The relative stability of this stable stratification may be characterized by the parameter

$$S = \frac{m_{\rm st} - m_{\rm ad}}{m_{\rm ad} - m_{\rm unst}}, \tag{11}$$

where $m_{\rm ad}$ is the adiabatic index (= 1.5 for a perfect gas). The simulations show that the properties of convection in the unstable layer are not much affected by the degree of stability outside and by the extent of penetration. Thus the Peclet number at the top of the stable region varies as the inverse of the conductivity, which means roughly as $1/S$. For small S, the penetration is adiabatic, and based on the arguments developed above, its extent should scale as S^{-1}; for large S, thermal diffusion dominates, and the extent of overshoot should scale as $S^{-1/4}$. (This shows that one cannot extrapolate results of calculations done at low Pec number to the high Pec regime – to penetration in the Sun, for instance.) The results of the simulations agree rather well with these predictions, as can be verified on Fig. 2.

One is thus tempted to take this agreement as a proof that penetration is due to such plumes, and that it extends over a fraction of the pressure scale-height (or of the size of the unstable domain). How does this compare with the observations ?

6. The seismic evidence

Convective penetration has two signatures which may be detected in stars. Firstly, it leads to a larger well-mixed zone than predicted by the classical mixing-length treatment, and this is visible in the evolutionary tracks in the temperature/luminosity diagram. And secondly it modifies the temperature stratification, which can be detected through acoustic sounding, at least in the Sun, where the seismic diagnostic reaches a precision of order 10^{-4}.

If there were no penetration at the base of the solar convection zone, the temperature gradient would be continuous there and its derivative would present a discontinuity; thus the second derivative of the sound speed would be discontinuous. With penetration, there will be a jump in the temperature gradient, since the thermal adjustment layer (which is labeled C in Fig. 1) is extremely

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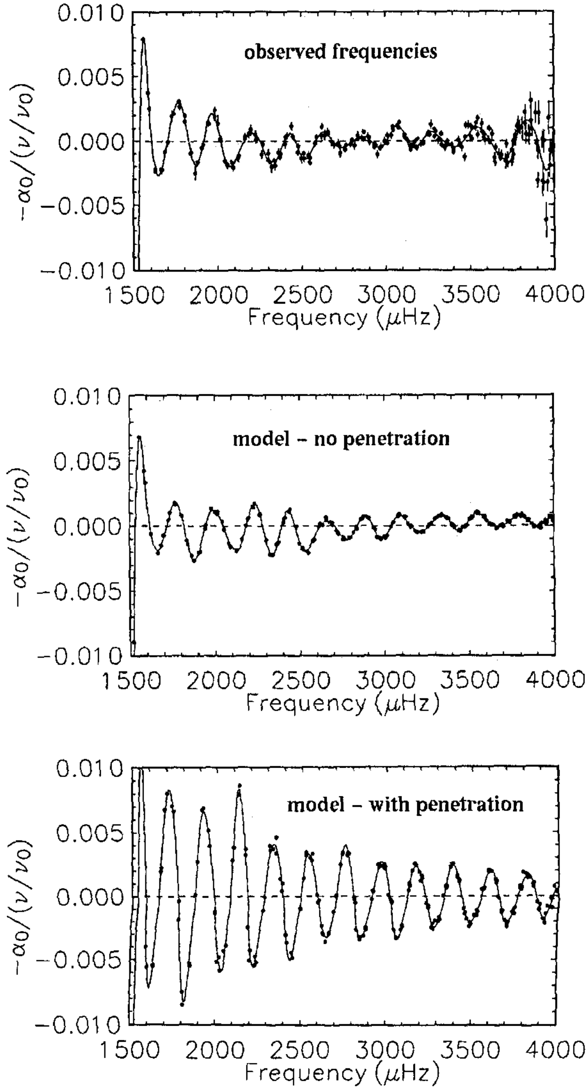

Figure 3. Phase shift α in the asymptotic relation (14) versus frequency ν. The upper panel shows the observed mode frequencies, the mid panel the frequencies predicted from a solar model without penetration below the convection zone, and the lower panel shows the same but using a model with a penetration of $0.56H_p$ (Roxburgh & Vorontsov, 1994, courtesy MNRAS).

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thin, and it is the first derivative of the sound speed which will be discontinuous. These discontinuities can be detected by helioseismology.

To show this, we follow the treatment by Roxburgh & Vorontsov (1994). They cast the wave equation in the form of a Schrödinger equation:

$$\frac{d^2\zeta}{d\tau^2} + \left[\omega^2 - V(\tau)\right]\zeta = 0, \tag{12}$$

where τ is the acoustic depth ($d\tau = dr/c$), ζ^2 is the kinetic energy density of the wave, and V the acoustic potential

$$\begin{aligned} V &= N^2 - \frac{c}{2}\frac{d}{dr}\left[c\left(\frac{2}{r} + \frac{N^2}{g} - \frac{g}{c^2} - \frac{1}{2c^2}\frac{dc^2}{dr}\right)\right] \\ &+ \frac{c^2}{4}\left(\frac{2}{r} + \frac{N^2}{g} - \frac{g}{c^2} - \frac{1}{2c^2}\frac{dc^2}{dr}\right)^2 - 4\pi G\rho. \end{aligned} \tag{13}$$

The oscillation frequency ω obeys an asymptotic relation, first described by Duvall (1982), which may be written

$$\pi\frac{n+\alpha}{\omega} = F\left(\frac{\ell+1/2}{\omega}\right) + \text{higher order terms}, \tag{14}$$

where n is the radial order and ℓ the spherical order of the mode. When the acoustic potential V is a smooth function of τ, the phase shift does not vary much with frequency ω. But a sharp variation or a discontinuity in the derivatives of the sound speed c imprints an oscillation on α, whose frequency indicates the location of the discontinuity, and whose amplitude is proportional to its strength.

Figure 3 displays this phase shift with the observed solar frequencies, and compares it with that obtained with two solar models, one without penetration and the other with a penetration of $0.56 H_p$. By visual inspection it seems that there is a small amount of penetration in the Sun.

This is confirmed by a more quantitative treatment. Roxburgh & Vorontsov have calculated the amplitude of the phase shift oscillation as a function of penetration, and concluded that the observed amplitude corresponds to an extent of penetration of about $0.20 H_p$

One assumption made in predicting the extent of penetration in the described in Sect. 4 is that all plumes cross the top of the radiation zone with the same velocity W. This is of course a very crude picture. One would rather expect a distribution of initial velocities, and that would smoothen the transition from adiabatic to radiative temperature gradient. Christensen-Dalsgaard et al. (1995) have calculated the effect of such temperature profiles on the amplitude of the phase shift amplitude. Their result is shown in Fig. 4: the observed amplitude is compatible with profiles that connect smoothly the adiabatic and radiative temperature gradients.

7. Recent work

Another assumption made in establishing the scaling law (9) is that, for high Peclet number, the penetration establishes an almost adiabatic temperature gra-

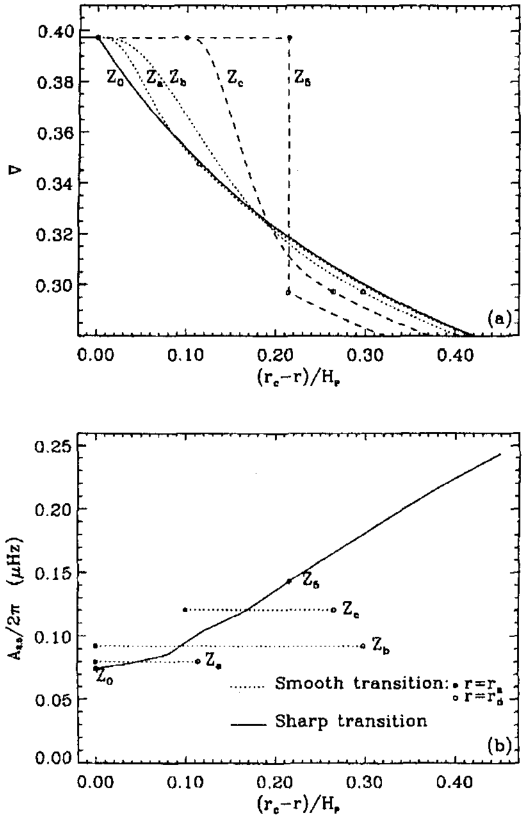

Figure 4. Amplitude of the phase shift oscillation versus the penetration depth, for the temperature profiles shown the upper panel. The smooth profile Z_b is compatible with the observed amplitude, as would be a discontinuous profile with a penetration depth of $\approx 0.10 H_p$ (Christensen-Dalsgaard et al., 1995, courtesy MNRAS).

dient. The numerical simulations performed in the 90s showed clearly such a trend (Muthsam et al., 1995; Singh et al., 1998) and, although the temperature gradient still departed from adiabatic, this could be ascribed to the fact that the Peclet number characterizing the plumes was not that much greater than unity.

More recently numerical simulations have been carried out by Brummel et al. (2001) with much higher spatial resolution, up to $512 \times 512 \times 575$. This allowed them to reach a Peclet number of 10^3, and thus to explore the asymptotic regime for $Pec \gg 1$. Their results are shown in Fig. 5, where it appears that the penetration depth scales as $Pec^{-1/2}$.

Why does this scaling differ from that derived in Sect. 4, where the extent of penetration does not depend on the Peclet number? The reason is that the plumes are much sparser in these 3-D simulations than was expected, based on the 2-D calculations. Therefore, in spite of their high Peclet number, they are

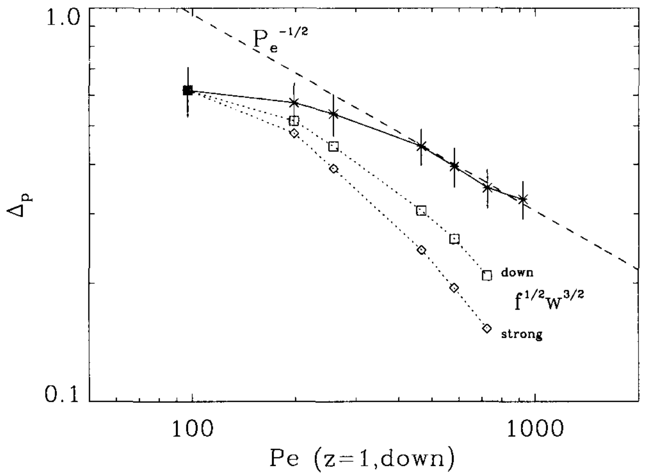

Figure 5. Penetration depth as a function of the Peclet number, in the high resolution 3-D simulations performed by Brummel et al. (2001). The penetration depth scales as $Pec^{-1/2}$

not able to enforce a nearly adiabatic stratification in the penetration region, as was assumed to establish that scaling.

Such simulations are extremely costly in computer resources, and therefore it makes sense to experiment with other methods, in which the small scales are modeled in some way. The first attempts were made by Xiong (1985) and Kuhfuss (1986), who integrated the horizontally averaged equations, with a prescription for the higher moments, assumed to represent the effect of turbulence. These simulations again displayed substantial penetration. Work is in progress by Kupka (1999), based on a formalism recently derived by Canuto (1997). To validate the moment equations which he uses, he compares the results with ab initio 3-dimensional simulations, and the agreement is rather encouraging. However the resolution of these 3-D calculations is still rather low, compared with what Brummel et al. have achieved, and it is too early to draw definite conclusions.

8. Conclusion

Fifteen years ago Renzini (1987) summarized the situation in a pessimistic formula: "(convective) overshoot is found small if supposed small, large if supposed large". How much progress have we made since ?

Let us first consider convective penetration below the solar convection zone. If we rely on acoustic sounding alone, we must confess that we find appreciable penetration when we assume that the temperature gradient changes smoothly from adiabatic to radiative, and much less penetration when we allow for a

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discontinuous transition. At present, this diagnostic tool has not been able to distinguish between the two solutions, but we have seen that the most recent numerical simulations by Brummel et al. favor the smooth profile. This would mean that the convective eddies would penetrate right into the tachocline, that thin layer where the rotation rate changes abruptly from differential in the convection zone to uniform below. The consequences of this are yet to be examined.

As for penetration above a convective core, some mixing is clearly required there, but it can be produced by another process, such as a combination of meridional circulation and turbulence generated by differential rotation (Zahn, 1992). Evolutionary sequences built with such rotational mixing are in good agreement with the observations, and they need no convective penetration (Meynet & Maeder, 2000). In all likelihood both convective penetration and rotational mixing are operating there, and we must find means to model them together, preferably without adjustable parameters.

References

Brummel, N., Clune, T., & Toomre, J. 2001, ApJ, in press

Canuto, V.M. 1997, ApJ, 482, 827

Castellani, V., Giannone, P., & Renzini, A. 1971, Ap&SS, 10, 340

Christensen-Dalsgaard, J., Monteiro, M., & Thompson, M. 1995, MNRAS, 276, 283

Cogan, B. 1975, ApJ, 201, 637

Duvall, T.L. 1982, Nature, 300, 242

Freytag, B., Ludwig, H.-G., & Steffen, M. 1996, A&A, 313, 497

Hurlburt, N.E., Toomre, J., & Massaguer, J.M. 1986, ApJ, 311, 563

Hurlburt, N.E., Toomre, J., Massaguer, J.M., & Zahn, J.-P. 1994, ApJ, 421, 245

Kuhfuss, R. 1986, A&A, 160, 116

Kupka, F. 1999, ApJ, 526, L45

Latour, J., Toomre, J., & Zahn, J.-P. 1981, ApJ, 248, 1081

Maeder, A. 1975, A&A, 43, 61

Massaguer, J.M., Latour, J., Toomre, J., & Zahn, J.-P. 1984, A&A, 140, 1

Meynet, G. & Maeder, A. 2000, A&A, 361, 101

Moore, D. R. & Weiss, N.O. 1973, J. Fluid Mech., 61, 553

Muthsam, H.J., Göb, W., Kupka, F., Liebich, W., & Zöchling, J. 1995, A&A, 293, 127

Prather, M.J. & Demarque, P. 1974, ApJ, 193, 109

Renzini, A. 1987, A&A, 188, 49

Roxburgh, I.W. 1978, A&A, 65, 281

Roxburgh, I.W. & Simmons, J. 1993, A&A, 277, 93

Roxburgh, I.W. & Vorontsov, S.V. 1994, MNRAS, 268, 880

Saslaw, W. C. & Schwarzschild, M. 1965, ApJ, 142, 1468

Schmitt, J., Rosner, R., & Bohn, H.U. 1984, ApJ, 282, 316

Shaviv, G. & Salpeter, E.E. 1973, ApJ, 184, 191
Singh, H.P., Roxburgh, I.W., & Chan, K.L. 1998, A&A, 340, 178
Veronis, G. 1963, ApJ, 137, 641
Xiong, D.R. 1985, A&A, 150, 133
Zahn, J.-P. 1991, A&A, 252, 179
Zahn, J.-P. 1992, A&A, 265, 115
Zahn, J.-P., Toomre, J. & Latour, J. 1982, GAFD, 22, 159

Discussion

D. O. Gough : Because the descending plumes are far from being space-filling, they transport heat globally through the tachocline yet they do mix material. Consequently their effect is likely to mimic that of the much slower Ekman circulation which is bound to be present. The issue of the transport of angular momentum is less clear.

J.-P. Zahn : Indeed our picture of the tachocline would be radically changed with the intrusion of these plumes. But the problem would remain of finding which mechanism prevents the spread of differential rotation into the radiative interior.