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AUTOMORPHISMS AND RANGE FAMILIES OF TRANSFORMATION SEMIGROUPS

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The problem of describing all automorphisms of a given semigroup of transformations of a set X has interested a number of mathematicians in the past fifty years. In 1937 Schreier [10] showed that every automorphism of the full transformation semigroup T_X is *inner* (that is, acts as a conjugation by some bijection of X). In 1952 Mal'cev [7] generalized this result by showing that every ideal of T_X has only inner automorphisms. More recently Symons [11] showed that all automorphisms of any G_X-*normal* semigroup (that is, invariant under a conjugation by any bijection of X) over a *finite* set X are inner, while Schein [9] produced the same result for G_X-normal semigroups of one-to-one transformations over an *infinite* set X. (See [2] for the special base of Baer-Levi semigroups.)

Chapters 2 and 3 of this thesis constitute a contribution towards the solution of the problem of describing all automorphisms of a given semigroup of transformations of an *infinite* set X. In Chapter 2 (see also [4]) we extend the well-known result from group theory, namely that any normal group of bijections of an infinite set X has only inner automorphisms, to an analogous one in semigroup theory. We show that any G_X-normal semigroup of transformations of an infinite set X has only inner automorphisms. Our purpose in Chapter 3 (see also [3]) is to offer a complete description of all automorphisms of an arbitrary *Croisot-Teissier*

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semigroup [1], a task suggested by Schein. In this joint work with O'Meara and Wood a rich variety of automorphisms is found, ranging from inner, to "locally" inner, to thoroughly outer. We also present a description of Green's relations on Croisot-Teissier semigroups.

In Chapter 4 (and in [5]) we are concerned with the problem of a characterisation of all subsets of the power set of X, P_X, which serve as sets of ranges of semigroups of transformations of X. This problem was suggested by Schein and to our knowledge has been solved only for the case of monogenic semigroups of partial transformations by Olonichev [8]. We define a *normal* subset of P_X and characterise all normal subsets of P_X which serve as sets of ranges of semigroups of total transformations of X. In particular, we give necessary and sufficient conditions for a subset of P_X to be the set of ranges of a G_X-normal and a constant-free G_X-normal semigroup of total transformations.

In Chapter 5 (and in [6]) for a particular normal subset of P_X we give necessary and sufficient conditions for an order-automorphism to be determined by a bijection of X (that is, *induced*). We then characterise those normal subsets of P_X for which all order-automorphisms are induced. Apart from being of independent interest, this problem is connected with the study of automorphisms of transformation semigroups. For if an automorphism ϕ of a transformation semigroup S is inner, then ϕ produces an induced order-automorphism of the set $R(S)$ of ranges of all transformations in S. On the other hand, in instances where an automorphism ϕ of S yields an order-automorphism of $R(S)$, the knowledge that all order-automorphisms of $R(S)$ are induced can be a first step in showing that ϕ is inner.

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