Ends Against the Middle: Measuring Latent Traits when Opposites Respond the Same Way for Antithetical Reasons

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Abstract

Standard methods for measuring latent traits from categorical data assume that response functions are monotonic. This assumption is violated when individuals from both extremes respond identically, but for conflicting reasons. Two survey respondents may "disagree" with a statement for opposing motivations, liberal and conservative justices may dissent from the same Supreme Court decision but provide ideologically contradictory rationales, and in legislative settings, ideological opposites may join together to oppose moderate legislation in pursuit of antithetical goals. In this article, we introduce a scaling model that accommodates ends against the middle responses and provide a novel estimation approach that improves upon existing routines. We apply this method to survey data, voting data from the U.S. Supreme Court, and the 116th Congress, and show that it outperforms standard methods in terms of both congruence with qualitative insights and model fit. This suggests that our proposed method may offer improved one-dimensional estimates of latent traits in many important settings.

Keywords: measurement, Bayesian statistics, item response

1 Introduction

Item response theoretic (IRT) models are now standard tools for measurement tasks in political science across substantive domains including survey research (e.g., Caughey and Warshaw 2015; Treier and Hillygus 2009), courts (e.g., Bafumi *et al.* 2005; Martin and Quinn 2002), legislators (e.g., Clinton, Jackman, and Rivers 2004; Jackman 2001), international bodies (Bailey, Strezhnev, and Voeten 2017), democratic institutions (e.g., Treier and Jackman 2008), and more (e.g., Quinn 2004). However, a common problem with these models is that individuals can *respond* to some survey item or roll-call vote in an identical fashion while having differing *motivations*. Two survey respondents may indicate that they "strongly disagree" with an item, but do so for opposite reasons. Both liberal and conservative justices may dissent from the same Supreme Court decision, but provide ideologically contradictory rationales. Moreover, in legislative settings, ideological opposites may join together to oppose moderate legislation in pursuit of antithetical goals.

When this happens, and it often does, standard models can produce estimates for latent traits that are misleading or just wrong (e.g., Spirling and McLean 2007). This is because IRT models—as well as related techniques (e.g., Poole 2000; Tahk 2018)—assume that response functions are monotonic. Monotonicity means that the probability of any given response must be increasing (or decreasing) as a function of the latent space.[1] More concretely, the probability of choosing "strongly disagree" should be associated with individuals who are *either* high or low on the latent

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1 The NOMINATE procedure is a special case where *limited* nonmonotonicity is allowed (Carroll *et al.* 2009; Poole and Rosenthal 1985). We discuss this in more detail in our Congress example below and in Appendix E of the Supplementary Material. We note here, however, that NOMINATE is not appropriate for our other applications since it demands much more data than is provided in, for example, survey applications.

trait, but *not* both. If two justices vote the same way on a case, monotonicity implies that they share a common ideological motivation. Furthermore, if a member of Congress often votes with conservative Republicans, monotonicity assumes that it must be because she is a conservative. In short, monotonicity assumes that similar observed *responses* also have similar *motivations*—an assumption not always consonant with the true data-generating process.

In this article, we introduce a modification to traditional IRT models that *allows for* "ends against the middle" behavior while recovering near identical estimates as standard IRT models when such behavior is absent. The method, the generalized graded unfolding model (GGUM), was first proposed by Roberts, Donoghue, and Laughlin (2000) to accommodate moderate survey items. We introduce the method to political science, develop a novel estimation method that outperforms existing algorithms in the GGUM literature, and provide an open-source R package, `bggum`, for applied scholars (Duck-Mayr and Montgomery 2020). We apply the model to survey data, voting data from the U.S. Supreme Court, and roll calls from the 116th Congress, and show that it outperforms standard IRT models in important settings and can provide superior measures of latent constructs.

In the next section, we provide a basic intuition about the GGUM and then contextualize it within the constellation of existing measurement models. We then present the GGUM and provide a novel parameter estimation method, Metropolis-coupled Markov chain Monte Carlo (MC3), which significantly outperforms existing routines for estimating the GGUM model (e.g., de la Torre, Stark, and Chernyshenko 2006) in terms of accuracy and convergence to the proper posterior. We then test the robustness of the method via simulation. We show that MC3-GGUM gives essentially identical estimates as standard scaling methods in the absence of ends against the middle responses. We also address the potential (but incorrect) criticism that the MC3-GGUM is simply picking up on a second dimension and provide a brief discussion of the advantages and disadvantages of our approach relative to standard IRT models. Finally, we apply MC3-GGUM to survey responses as well as voting data in two settings. We conclude with a discussion of future directions for this research as well as the substantive interpretation of the resulting estimates.

2 Ends Against the Middle

For over four decades, political methodologists have worked to accurately measure latent traits for voters, legislators, and other political elites based on categorical responses. The broad goal is to take a large amount of data (e.g., survey responses or roll calls) and reduce it to a low-dimensional representation of some latent concept.

After gaining wide acceptance in the 1990s and 2000s, this work expanded to accommodate dynamics (Bailey 2007; Martin and Quinn 2002), ordered responses (Treier and Jackman 2008), nominal data (Goplerud 2019), and bridging institutions (Shor and McCarty 2011) and voters (Caughey and Warshaw 2015). Methodologically, approaches span the spectrum of statistical philosophies including Bayesian inference (Jackman 2001), parametric (Poole and Rosenthal 1985), and nonparametric models (Duck-Mayr, Garnett, and Montgomery 2020; Poole 2000; Tahk 2018). As data sources expanded, researchers incorporated more kinds of evidence including social media activity (Barbará 2015), campaign giving (Bonica 2013), and word choice (Kim, Londregan, and Ratkovic 2018; Lauderdale and Clark 2014).

The GGUM fits into this dizzying array of methods by providing an *unfolding* model designed for use with *categorical* data. To understand this intuitively, consider a survey respondent asked to indicate her support or disapproval for a set of survey items. Most survey items ask respondents about extreme statements. For instance, in a battery measuring immigration attitude, we might ask respondents if they agree or disagree with the statement, "All undocumented immigrants currently living in the United States should be required to return to their home country." For this item, responses are unambiguous; agreement indicates a more conservative position on

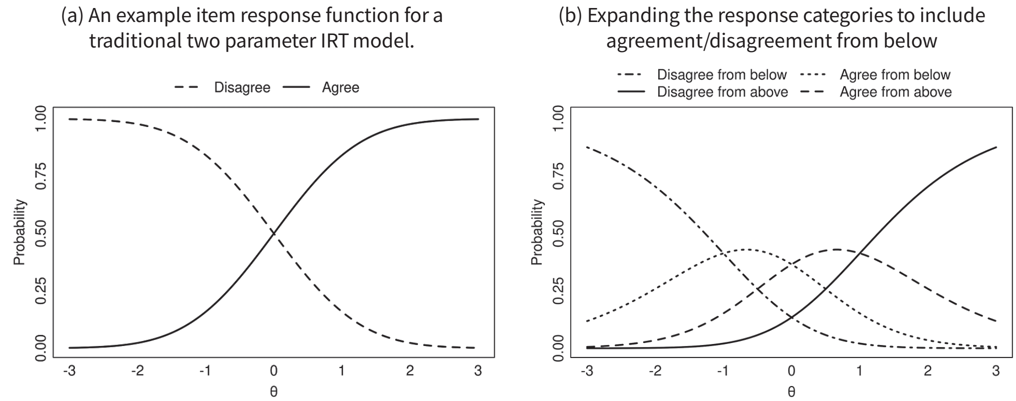


Figure 1. Example response functions linking standard IRT model to the GGUM.

immigration. We would thus expect to see response patterns like Figure 1a, where the probability of an "agree" response increases monotonically from liberal (left) to conservative (right).

However, for some kinds of questions, the meaning of observed responses can be far from plain. For example, we might ask respondents whether or not they agree with the statement, "I am fine with the current level of enforcement of U.S. immigration laws." From the analyst's perspective, question items like this are problematic. We can safely assume that respondents who agree with the statements are probably moderates. But what can we say about individuals who disagree?[2] Conservatives might reject the *status quo* on the grounds that we need stronger borders and more aggressive internal enforcement. Liberal respondents, on the other hand, might disagree on the grounds that current enforcement is already too stringent and deportations should be dramatically reduced. Thus, we can get "disagreement from above" and "disagreement from below" such that the same *observed* response corresponds with opposite rationales. Indeed, as illustrated in Figure 1b, we might think of all respondents as falling into one of four categories: disagreeing from below, agreeing from below, agreeing from above, and disagreeing from above. Here, we are mapping out the probability of each of these four hypothetical responses as a function of ideology.

The key intuition of the GGUM is that we can combine these four *hypothetical* responses into the two *observed* responses as depicted in Figure 2a.[3] Here, we see that the probability of agreeing with the item is nonmonotonic and reaches a maximum at the so-called "bliss point," δ. The closer a respondent's ideology is to this point, the more likely they are to "agree." Meanwhile, respondents who are far from this point (whether to the left *or* to the right) are increasingly likely to disagree.

Unfolding models such as the GGUM date back at least to Coombs (1950) and assume that responses reflect a *single-peaked* (symmetric) preference function. That is, facing any particular stimuli, respondents prefer options that are "closer" to themselves in the latent space. A common form of data that exhibits this feature is "rating scales," where respondents are asked to evaluate various politicians, parties, and groups on a 0–100 thermometer. Unfolding models for ratings scales date back to Poole (1984). Indeed, unfolding models generally capture the intuitions and assumptions behind spatial voting (Enelow and Hinich 1984), wherein individuals prefer policy options that are closer to their ideal point in policy space. The response function in Figure 2a is an example of a response function consistent with an unfolding model. In this case, it is individuals

2 An implicit assumption of this discussion is that there is only a single underlying dimension. In theory, GGUM could be extended to a multidimensional latent space, but we are aware of no existing work that does this. We provide a more extensive discussion of the role of GGUM models in a multidimensional setting in Section 4 and Appendix F of the Supplementary Material.

3 As we explain below, the model generalizes to cases with categorical response options. We begin with the binary case merely for ease of exposition.

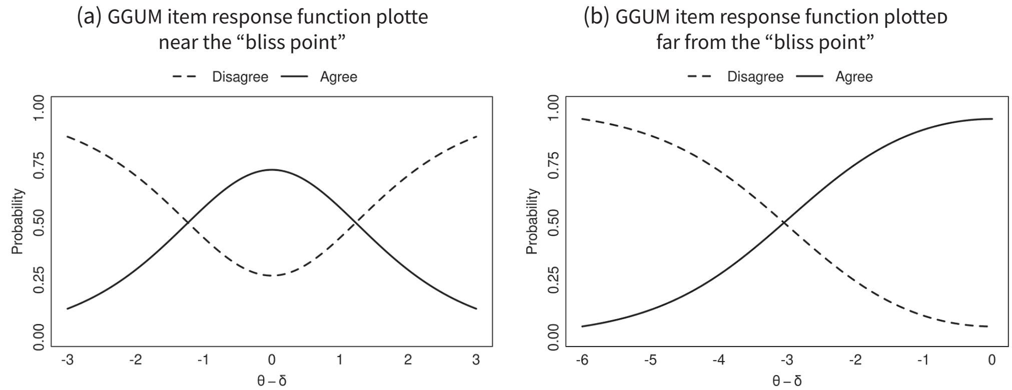


Figure 2. Example item response functions for the GGUM.

near δ who are most likely to "agree" and individuals at the most extreme are expected to behave the same ("disagree") despite being dissimilar on the underlying trait.

Unfolding models stand in contrast to "dominance models," which are more common in both psychology and political science. Figure 1a provides an example of a monotonic response function common to dominance models (in this case, a two-parameter logistic response model). These models assume that there is a monotonic relationship between the latent trait and observed responses. In Figure 1a, the probability of agreement always increases as respondents' ideology measure increases. Thus, the *least likely* individuals to "disagree" are those at the extreme right. Examples of dominance models include factor analysis, Guttman scaling, and the various forms of IRT models discussed above.

One reason many scholars are unaware of the distinction between dominance and unfolding models is that single-peaked preferences, the basis for the unfolding models, result in monotonic response functions consistent with dominance models in one important situation: when individuals with concave (e.g., quadratic) preferences make a *choice* between *two* options. A key example of when this equivalence holds is a member of Congress deciding between a proposed policy change and the *status quo* (Armstrong II *et al.* 2014).[4]

It is for this reason that standard models of roll-call behavior that derived from the unfolding tradition result in monotonic response functions nearly identical to dominance models. So, optimal classification (Poole 2000) is motivated theoretically via single-peaked preferences consistent with the unfolding tradition, but assumes monotonicity. Therefore, in our discussion below, we include all models that result in monotonic item response functions as dominance models regardless of their theoretical motivation. We provide additional discussion of the NOMINATE model, which is a special case of an unfolding model based on Gaussian preference functions, in Appendix E of the Supplementary Material.[5]

Thus, the value of the GGUM is in settings where (i) we anticipate single-peaked preferences, but (ii) where actors may not (always) perceive they are choosing between exactly two alternatives and (iii) where responses are categorical. Furthermore, the method will be most appropriate in settings where it is the behavior of extreme individuals who are poorly explained by more traditional dominance models. As in our immigration battery example above, identifying the position of moderates is (relatively) unproblematic. For items with extreme bliss points (as shown in Figure 2b), responses are unambiguous for all respondents and correspond nearly identically

4 See Clinton *et al.* (2004) for a succinct proof of this equivalence.

5 Our discussion here focuses only on latent trait models where the input is a set of categorical responses by respondents. This excludes multidimensional scaling (Armstrong II *et al.* 2014; Bakker and Poole 2013), which assumes that the data are in the form of "similarity" between units. Likewise, we do not discuss ratings scale models which are unfolding models appropriate for continuous responses.

to monotonic response functions. (Indeed, as we illustrate below, the GGUM is able to easily accommodate monotonic items by estimating the δ parameters to be relatively extreme.) The ambiguity only arises for moderate items—and the resulting disagreement arises primarily for extreme individuals.

Where in practice might this occur? As already discussed, GGUM might be useful for survey batteries where two-sided disagreement can occur. However, GGUM may also be valuable in studies of political elites where the choice set is not always between two options. For instance, in Supreme Court decision-making, justices are *not* always presented with a binary choice, but instead can select among several options to either join opinions, join dissents, concur, or write their own opinions. Indeed, it is widely understood that votes relate only to the disposition of the lower court ruling, while justices may be more interested in doctrine. So we observe *responses* (votes) to either support or oppose the lower court opinion. However, the *motivations* behind identical votes often do not match up at all—something we know from the written opinions themselves.

Another motivation for GGUM is illustrated by the U.S. House of Representatives. Here, it may seem unneeded given our discussion of the strong link between dominance and unfolding models in legislative voting. However, recent history suggests that members do not always vote in ways consistent with monotonic response functions (cf. Kirkland and Slapin 2019). Members do not seem to be simply comparing the *status quo* and the proposal before them. Instead, members—especially ideologically extreme members—may refuse to support bills that move the *status quo* in their direction because they are still "too far" from their ideal point (Slapin *et al.* 2018).

Finally, a significant portion of the methodological work on latent scaling has focused on the U.S. context characterized by a strong two-party tradition that extends across institutions. In other settings, scholars have noted that models assuming binary agenda setting perform poorly (Spirling and McLean 2007; Zucco and Lauderdale 2011). In the online appendix, we therefore also consider the model's performance in a comparative setting building on the analysis of Mexico's *Instituto Federal Electoral* in Estévez, Magar, and Rosas (2008).

3 MC3-GGUM

More formally, we begin by modeling the full set of "hypothetical" response options as described above. GGUM is itself an extension of the general partial credit model (GPCM) (Bailey *et al.* 2017; Muraki 1992), which extends the dichotomous IRT models for categorical responses where the order is not known *a priori*. For respondent $i \in \{1, \dots, n\}$ on item $j \in \{1, \dots, m\}$, let $k^* \in \{0, \dots, K_j^* - 1\}$ indicate the hypothetical choice set where K_j^* is the number of hypothetical categories available for item j including, for example, agreeing from above and below.

Specifically, we denote the probability of i choosing option k^* for item j as $P(z_{ij} = k^* | \theta_i) = P_{jk^*}(\theta_i)$, where z_{ij} are the hypothetical response categories, and

$$P_{jk^*}(\theta_i) = \frac{\exp(\alpha_j [k^*(\theta_i - \delta_j) - \sum_{l=0}^{k^*} \tau_{jl}])}{\sum_{k^*=0}^{K_j^*-1} \exp(\alpha_j [k^*(\theta_i - \delta_j) - \sum_{l=0}^{k^*} \tau_{jl}])}. \tag{1}$$

This response probability derives directly from Muraki's graded response model (GRM). Here, α_j is the usual "discrimination" parameter common to IRT model, and indicates the degree to which the item corresponds to the underlying dimension (similar to a factor loading). As described above, δ_j is the "bliss point," which indicates the point in the latent space around which the item response function will be folded.

Finally, the τ_{jk} parameters determine where the hypothetical response probabilities cross.[6] Figure 3 shows a two-category item, which implies four hypothetical categories. Assuming $\alpha_j = 1$,

6 Appendix A of the Supplementary Material provides additional information on the parameters and how they can be interpreted.

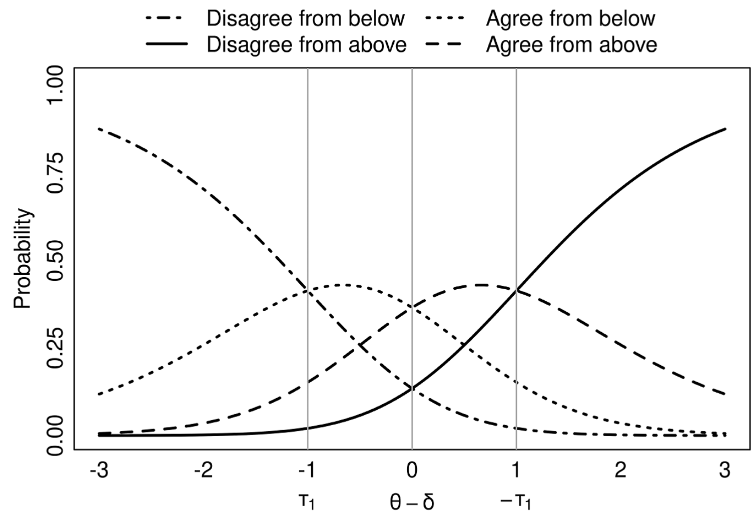


Figure 3. Probability of hypothetical responses as a function of $\theta - \delta$ where $\alpha = 1$ and $\boldsymbol{\tau} = (0, -1)$.

τ_{jk} values determine how far away from δ_j the item response functions for each hypothetical category of response will cross. The model is identified by setting $\tau_{j0} = 0$ and $\sum_{k^*=1}^{K_j^*} \tau_{jk^*} = 0$.

The final step is to also combine the probabilities for *hypothetical* response options into the *observed* response categories. Thus, the probability that a respondent will "agree" is the sum of the probability they will "agree from below" and "agree from above." We also assume that the τ parameters are symmetric around the point $(\theta_i - \delta_j) = 0$. Thus, for each τ_{jk} parameter in the model, there exists an equivalent hypothetical response corresponding with $-\tau_{jk}$. Substantively, this assumption means that we assume preferences to be symmetric and single-peaked around δ_j.

This last step involves some tedious algebra as explicated in Roberts *et al.* (2000), but the result is

$$P(y_{ij} = k|\theta_i) = \frac{\exp(\alpha_j[k(\theta_i - \delta_j) - \sum_{m=0}^{k} \tau_{jm}]) + \exp(\alpha_j[(2K_j - k - 1)(\theta_i - \delta_j) - \sum_{m=0}^{k} \tau_{jm}])}{\sum_{l=0}^{K-1}[\exp(\alpha_j[l(\theta_i - \delta_j) - \sum_{m=0}^{l} \tau_{jm}]) + \exp(\alpha_j[(2K_j - l - 1)(\theta_i - \delta_j) - \sum_{m=0}^{l} \tau_{jm}])]}, \tag{2}$$

where $P(y_{ij} = k|\theta_i) = P_{jk}(\theta_i)$ is the probability for the *observed* response y_{ij} and K_j is the number of observed response options. While unwieldy, this equation is actually a modest modification of the GPCM IRT model to allow for the "folding" of various hypothetical responses around δ_j to create the observed responses. Appendix A of the Supplementary Material provides additional discussion on how to interpret each parameter. We emphasize here, however, that although this parameterization appears ungainly, the total number of parameters estimated increases by only one parameter per item relative to standard IRT models. The primary difference is the assumed functional form.

With this equation, the likelihood for a set of responses **Y** is

$$L(\mathbf{Y}) = \prod_i \prod_j \sum_k P_{jk}(\theta_i)^{\mathcal{I}(y_{ij}=k)}.$$

Note that the summation here is over all possible responses to item j. Roberts *et al.* (2000) outline a procedure whereby item parameters are estimated using a marginal maximum likelihood (MML) approach and the θ parameters are then calculated by an expected *a posteriori* estimator. de la Torre *et al.* (2006) provide a Bayesian approach to estimation via Markov chain Monte Carlo (MCMC).

However, there are a few aspects to the surface of the likelihood (and posterior) that make parameter estimation difficult. First, the construction of the model allows the likelihood to be multimodal. The model is designed, after all, to reflect the fact that the same behavior (e.g., voting against the bill) can be evidence of two underlying states of the world (e.g., being extremely conservative or extremely liberal). Example profile likelihoods are shown in Appendix B of the Supplementary Material.

Second, like many IRT models, the GGUM is subject to reflective invariance; the likelihood of a set of responses $\mathbf{Y}$ given $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$ vectors is equal to the likelihood of $\mathbf{Y}$ given vectors $-\boldsymbol{\delta}$ and $-\boldsymbol{\theta}$ (Bafumi *et al.* 2005). However, unlike standard IRT models, simply restricting the sign of one (or even several) θ or δ parameters is not sufficient to shrink the reflective mode and identify the model. That is, because the likelihood is multimodal, constraining a few parameters will not eliminate the reflective invariance.

The consequence of these two facts together mean that both maximum likelihood models and traditional MCMC approaches struggle to fully characterize the likelihood/posterior surface absent the imposition of many strong *a priori* constraints. Furthermore, both are sensitive to starting values and may focus on one mode—sometimes a reflective mode.

3.1 Estimation via Metropolis-Coupled Markov Chain Monte Carlo

To handle these issues, we offer a new Metropolis-coupled MC3 approach, and implement this algorithm in our `R` package.[7] To begin, we follow de la Torre *et al.* (2006) in using the following priors:

$$\begin{array}{rclrcl} P(\theta_i) & \sim & \mathcal{N}(0,1), & P(\alpha_j) & \sim & Beta(\nu_\alpha,\omega_\alpha,a_\alpha,b_\alpha), \\ P(\delta_j) & \sim & Beta(\nu_\delta,\omega_\delta,a_\delta,b_\delta), & P(\tau_{jk}) & \sim & Beta(\nu_\tau,\omega_\tau,a_\tau,b_\tau), \end{array}$$

where $Beta(\nu,\omega,a,b)$ is the four-parameter Beta distribution with shape parameters ν and ω, with limits a and b (rather than 0 and 1 as under the two-parameter Beta distribution). These priors have been shown to be extremely flexible in a number of settings allowing, for instance, bimodal posteriors (Zeng 1997). However, the priors censor the allowed values of the item parameters to be within the limits a to b. As discussed in Appendix C of the Supplementary Material, researchers must take care that the prior hyperparameters are chosen, so they do not bias the posterior via censoring.

We utilize an MC3 algorithm (Gill 2008, 512–523; Geyer 1991) for drawing posterior samples, and the complete algorithm is shown in Appendix C of the Supplementary Material. In MC3 sampling, we use N parallel chains at inverse "temperatures" $\beta_1 = 1 > \beta_2 > \cdots > \beta_N > 0$. Parameter updating for each chain is done via Metropolis–Hastings steps, where new parameters are accepted with some probability p that is a function of the current value and the proposed value (e.g., $p\left(\theta^*_{bi},\theta^{t-1}_{bi}\right)$). The "temperatures" modify this probability by making the proposed value more likely to be accepted in chains with lower values of β_b. Formally, the probability p of accepting a proposed parameter value becomes p^{β_b}, so that chains become increasingly likely to accept all proposals as $\beta \longrightarrow 0$.

The goal here is to have higher temperature chains that will more quickly explore the posterior and therefore be more likely to move between the various modes in the posterior. We then allow adjacent chains to "swap" states periodically as a Metropolis update. Since only draws from the

7 We emphasize that our focus in this subsection is exclusively on the approach to estimation and not the model itself. The MC3 procedure offers considerable advantages to alternative estimation schemes for the GGUM model as discussed more fully below as well as in Appendix C of the Supplementary Material. However, the advantages of the GGUM relative to standard IRT models is a function of the model and not the estimation procedure *per se*. Any proper MCMC routine should, in theory, return the same posterior. As we show in the Supplementary Material, however, prior MCMC algorithms routinely fail to fully characterize the posterior as they become stuck in local modes.

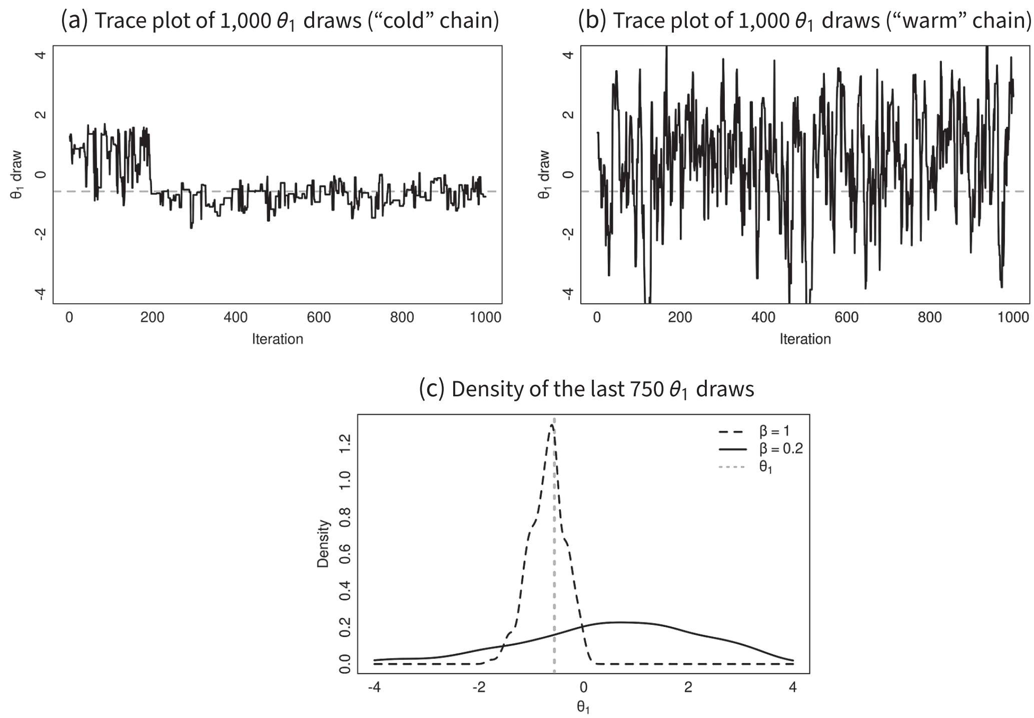


Figure 4. θ_1 draws for chains with inverse temperatures 1 and 0.2. Panel (a) shows draws from the cold chain with inverse temperature of 1, panel (b) shows draws from the hot chain with inverse temperature of 0.2, and in all panels the dashed gray line shows the true value of θ_1.

first “cold” chain are recorded for inference, the result is a sampler that will simultaneously be able to efficiently sample from the posterior around local modes while also being able to jump between modes that are far apart. Intuitively, the idea is to use the “warmer” chains to fully explore the space to create a somewhat elaborate proposal density for a standard Metropolis–Hastings procedure.

To illustrate the difference in propensity to accept proposals between colder and hotter chains, we simulated data from 100 respondents and 10 items with four options each and ran two chains for 1,000 iterations from the MC3 sampler, one with an inverse temperature of 1, the other with an inverse temperature of 0.2 (no swapping between chains was permitted).[8] The results are shown in Figure 4.[9] Figure 4a shows the draws for the latent trait parameter for the first respondent for the “cold” chain and Figure 4b for the “hot” chain, and Figure 4c shows the density plots for the last 750 draws. You can see that the hotter chain explores the posterior space more freely, and more proposals are accepted; the acceptance rates were 0.29 and 0.73 for the cold and hot chains, respectively. While the density of draws for the cold chain is a single peak concentrated around a small range of values in one posterior mode, the heated chain freely explores a “melted” posterior surface. Critically, these “warm” chains are not preserved for inference. Rather, they simply propose new values for colder chains and only the proper chain ($\beta = 1$) is ultimately used.

In Appendix C of the Supplementary Material, we compare our proposed estimation methods with both the MML routine proposed in Roberts *et al.* (2000) and the MCMC approach outlined in de la Torre *et al.* (2006). We find that the MC3 algorithm significantly reduces the root-mean-squared

8 For the simulation, the respondents’ latent trait parameters were drawn from a standard normal, the item discrimination parameters were distributed *Beta*$(1.5, 1.5, 0.5, 3.0)$, the item location parameters were distributed *Beta*$(2.0, 2.0, -3.0, 3.0)$, and the option threshold parameters were distributed *Beta*$(2.0, 2.0, -2.0, 0.0)$, and the responses were selected randomly according to the response probabilities given by Equation (2).

9 Replication code for this article is available in Duck-Mayr and Montgomery (2022) at https://doi.org/10.7910/DVN/HXORK9.

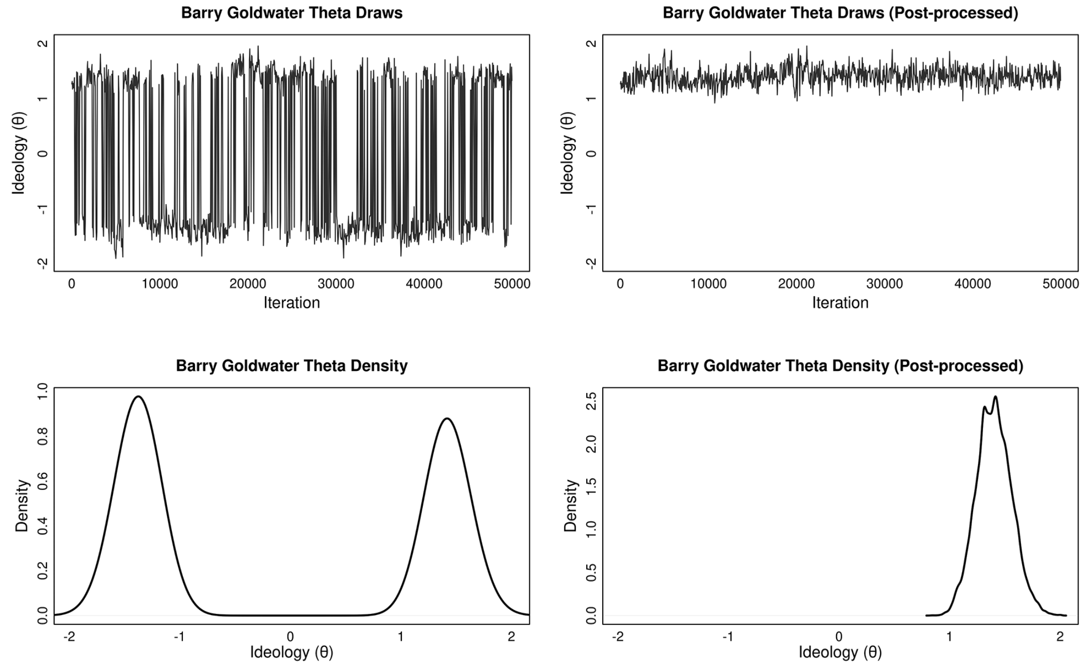


Figure 5. Posterior θ draws for Sen. Goldwater (R - AZ) before and after post-processing.

error for key parameters in finite samples relative to the MML algorithm and avoids becoming stuck in single modes as is common with the extant MCMC algorithm.

3.2 Identification

Most Bayesian IRT models rely on constraints placed on specific parameters to achieve identification during the actual sampling process. We follow this procedure in part by identifying the *scale* of the latent space via a standard normal prior on θ. For the reasons discussed above, however, standard constraints will not prevent an MCMC or MC3 sampler from visiting reflective modes. To avoid this problem, we instead allow the MC3 algorithm to sample the posterior without restriction, then impose identification constraints post-processing.[10] Since for this model the only source of invariance is rotational invariance, restricting the sign of one relatively extreme item location or respondent latent trait parameter is sufficient to separate samples from the reflective mode.

For example, we post-process the output of our MC3 algorithm on the voting data from the 92nd Senate (see Appendix F of the Supplementary Material) using Sen. Ted Kennedy's θ parameter (restricting its sign to be negative). Figure 5 shows the traceplot and posterior density for two independent chains for the famous conservative Sen. Barry Goldwater (R-Arizona). Before post-processing, the chains jump across reflective modes. Once we impose our constraint on Ted Kennedy, the posterior for Goldwater is restricted to the positive (conservative) side.

4 Advantages and Disadvantages of MC3-GGUM

In the next section, we turn to three applications to illustrate the advantages of the method in a variety of settings. However, it is worth pausing first to briefly consider the potential limitations of our approach relative to alternative methods already in the literature.

First, we may be worried that while the MC3-GGUM performs well when its assumptions are met, it may perform worse than standard methods in cases where the usual monotonicity assumptions

10 This approach is available, for example, in the popular `pscl` R package (Jackman 2017). For a mathematical proof that post-processing constraints are just as valid as *a priori* constraints, see Proposition 3.1 and Corollary 3.2 of Stephens (1997).

Table 1. Comparing log likelihood for the Clinton–Jackman–Rivers (CJR) monotonic IRT model and the MC3-GGUM for responses simulated under the Clinton–Jackman–Rivers model. The log likelihoods are near-identical for monotonic response functions; the respondent parameters correlate at 0.999.

Model	Log likelihood ($\mathcal{L}$)	$\mathcal{L}/N$	Mean θ s.d.
CJR	−18,989	−0.47	0.11
GGUM	−19,021	−0.48	0.11

Note: N is the number of nonmissing responses in the data (here, $N = nm$ as no responses were simulated as missing).

hold. While it is true that standard models will always perform better when their assumptions are met, in practice the MC3-GGUM performs well (if not identically) even when a standard IRT model is exactly correct. To show this, we simulated responses from 100 individuals to 400 binary items according to the model described in Clinton *et al.* (2004) and estimated using the R package `MCMCpack` Martin, Quinn, and Park 2011. We then estimate the GGUM from these data and compare the in-sample fit statistics in Table 1.[11]

The results show that in the presence of monotonic response functions, the MC3-GGUM recovers ideological estimates that are nearly (if not exactly) identical in terms of fit. Indeed, the θ estimates from the two approaches are correlated at 0.999. This is because for items with strictly increasing response functions, the nonmonotonic gradient is estimated to occur outside of the support of the θ estimates meaning that the nonmonotonicity has no effect. An example of this case is shown in Figure 2b, which shows the IRF far from the "bliss point" δ_j.

A second consideration is that the MC3-GGUM is a unidimensional model, and we are aware of no implementations that allow for more than one dimension. As we show below, the model is still very useful for better understanding political behaviors in many important settings, but the GGUM would not be an appropriate choice in settings where we anticipate multiple dimensions *a priori*.

A related concern is conflating nonmonotonic responses with a second (monotonic) dimension. This is salient to our application to Congress below. To explore this, we simulate a roll-call record with 100 respondents and 400 items from a standard IRT model assuming the presence of a second dimension. We fit both an MC3-GGUM model and a two-dimensional CJR model to these data. Estimates from both the MC3-GGUM and a two-dimensional IRT model are essentially identical (correlations are greater than 0.99), indicating that the mere presence of a second dimension should not lead MC3-GGUM to confuse ends against the middle voting with two-dimensional voting.[12] Thus, it is not true that the GGUM is simply picking up on a latent second dimension. We demonstrate this further in Appendix F of the Supplementary Material with simulated and real-world data. If there is no GGUM-like behavior and member ideologies are two-dimensional, MC3-GGUM simply measures the first dimension. It is not so easily confused.

One can of course construct instances where the MC3-GGUM would mistake a second dimension for ends against the middle voting. A particularly salient example might be if there was a second dimension correlated with extremity on the first dimension. So, for instance, we could imagine a second dimension representing "party loyalty" that declines for extreme members of a caucus. This argument is similar in flavor to arguments proposed by Spirling and McLean (2007) and Zucco

11 Often in political science for such data, fit statistics such as aggregate proportional reduction in error, percent correctly classified, area under the receiver operating characteristic curve (AUC), or Brier score are used to compare models. However, for these models, we can directly compare the log likelihood of the data given the model, which is what we report in Table 1. We also report these other fit statistics in Appendix D of the Supplementary Material.

12 These results are also replicated using the W-NOMINATE model. Likewise, the GGUM scores are essentially uncorrelated with the second NOMINATE dimension, or with extremity of second dimension estimates. See Appendix F of the Supplementary Material for details.

and Lauderdale (2011). But the general argument that the GGUM and a multidimensional model are in some way equivalent representations of the same data-generating process is simply untrue.

Furthermore, there are obvious computational costs associated with running multiple chains at differing temperatures that work to increase the computational burden and the time the model takes to run. This is particularly true considering the much faster implementations of standard models proposed in Imai, Lo, and Olmsted (2016) that do not rely on sampling. However, our custom implementation of MC3-GGUM generates posterior samples in a reasonable amount of time given the additional computational overhead. For example, in our Supreme Court application in Section 5.2, the `MCMCpack` Martin *et al.* 2011 implementation of the Martin and Quinn (2002) model generated approximately 246 posterior samples per second, whereas our MC3-GGUM implementation produced 87 posterior samples per second despite running six chains; that is, despite doing six times the work, we were able to streamline our implementation enough so that it only required a little less than three times the run time as the Martin and Quinn (2002) model. (This resulted in a 14-minute 56-second run time for the Martin and Quinn (2002) model and a 42-minute 8-second run time for the MC3-GGUM model in this application.)

Finally, as noted above, researchers need to examine the posteriors to ensure that there is no censoring at the outer bounds for the item parameters resulting from the Beta priors. For instance, we found this to be an issue for some of the more extreme (lopsided) votes in our analysis of congressional voting below. In these cases, researchers will need to try alternative hyperparameters.

In general, MC3-GGUM is most appropriate and useful when attempting to scale political actors in a unidimensional ideological space when ends against the middle behavior is present for at least some of the votes (or cases, or survey items). In the next section, we show that this behavior is indeed present in a wide variety of political contexts and using MC3-GGUM in those cases improves the substantive insights we glean from our data.

5 Applications

In this section, we provide three applications of MC3-GGUM to political science data. These examples serve to illustrate the strengths of the method and highlight the substantive insights that the model can provide. We begin simply by analyzing a survey battery where some items exhibit two-sided disagreement. Then we analyze votes by justices in the U.S. Supreme Court and finally the study of voting in the U.S. House of Representatives.[13] While we do note that MC3-GGUM offers superior model fit to the data, our primary motivation remains offering superior substantive insights. That is, we argue that the substantive conclusions reached based on the item characteristic curves and ability estimates are more in line with the empirical realities and thus more valid.

5.1 Immigration Survey Battery

To illustrate the basic properties of MC3-GGUM, we developed and fielded a 10-item battery consisting of statements related to immigrants and immigration policy and offering respondents a standard five-item Likert scale with options ranging from "strongly disagree" (1) to "strongly agree" (5).[14] Some items represented extreme statements designed to elicit "one-sided" disagreement. However, we also included items that could draw "two-sided" disagreement in a way that is inconsistent with traditional IRT models (see Figure 6). The complete inventory and additional information about this survey are shown in Appendix G of the Supplementary Material.

13 In the Supplementary Material, we provide another application outside the United States: Studying votes by Mexico's Federal Electoral Institute.

14 We received 2,621 responses after removing respondents who failed attention checks or who "straight-lined" their responses to the battery.

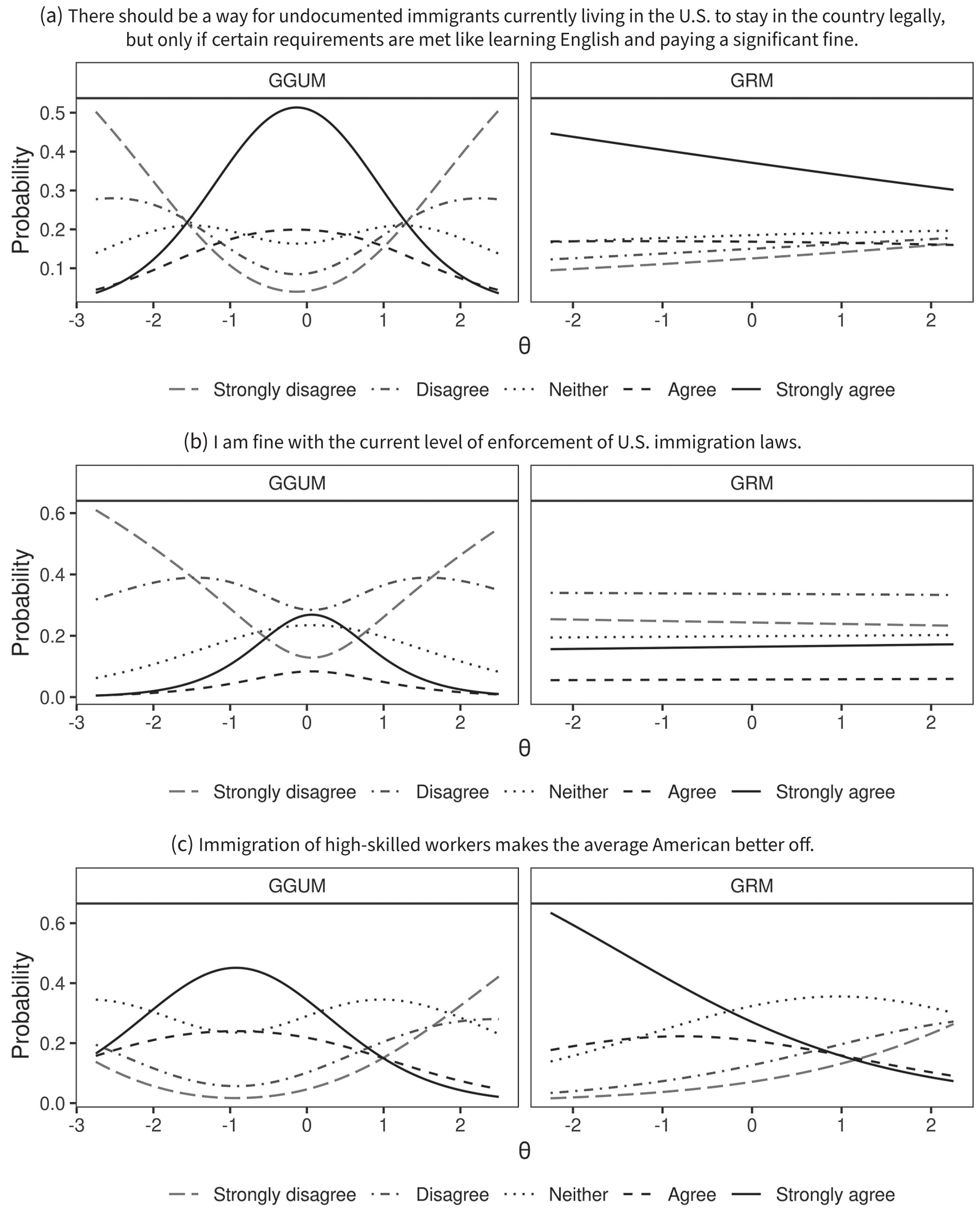


Figure 6. Item response functions for two moderate items and one more extreme item measuring immigration attitudes. The full inventory is shown in Appendix G of the Supplementary Material.

We fit our MC3-GGUM model[15] and compare it to a graded response model (the GRM is a standard IRT model for ordered categorical data) using the `ltm` package in R. Figure 6 shows item response functions for two moderate survey items in the battery and one extreme item. It shows that while MC3-GGUM identifies the two-sided disagreement in the survey responses, the GRM views them as essentially providing no information about the underlying latent trait (shown

15 We produced two recorded chains, each obtained by running six parallel chains at the inverse temperature schedule $(1.00, 0.97, 0.94, 0.92, 0.89, 0.86)$ for 10,000 burn-in iterations and 10,000 recorded iterations. The temperature schedule was determined using the optimal temperature finding algorithm from Atchadé, Roberts, and Rosenthal (2011), which is implemented and available for use in our package. Convergence of all posteriors in this paper was assessed using the Gelman and Rubin (1992) criteria and reached standard levels near 1.1 or below. Mixing in this model is generally quite high, and no other issues with the sampler were detected. Acceptance rates for the Metropolis–Hastings steps are near 0.23.

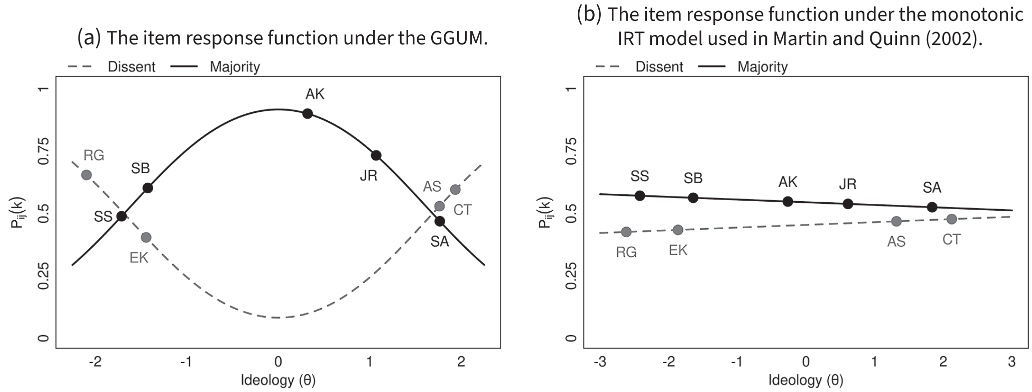


Figure 7. Item response functions for *Comptroller of the Treasury of Maryland v. Wynne* (2015). The probability of each justice's actual response is marked and labeled with the justice's initials.

by the flat slopes for the lines). The final figure shows that the GGUM also identifies the more extreme items as being one-sided (although there is some nonmonotonicity on the far left of the distribution).

As a consequence, the MC3-GGUM provides a slightly different measure of respondents' latent position on immigration policy. While they are strongly (if imperfectly) correlated with each other ($r = 0.936$), the MC3-GGUM was more strongly correlated with self-reported ideology than the GRM measure ($r = 0.627$ vs. $r = 0.618$, respectively) and more predictive of the underlying responses.[16]

5.2 The U.S. Supreme Court

For our Supreme Court application, we analyze all nonunanimous cases from the 1704 natural court, or the period beginning when Justice Elena Kagan was sworn in and ending with the death of Justice Antonin Scalia. We treat each case as a single "item" with two observable responses: voting for the outcome supported by the majority, or with the dissent. Under this coding scheme, we have 203 nonunanimous cases.[17]

The results illustrate several advantages of the GGUM over monotonic IRT models (Clinton *et al.* 2004; Martin and Quinn 2002) commonly used to analyze Supreme Court voting. Most importantly, we gain the ability to concisely explain disjoint voting coalitions. An example is *Comptroller of the Treasury of Maryland v. Wynne*, a case about the dormant Commerce Clause of the Constitution as applied to a tax scheme by the state of Maryland. A centrist majority opinion drew dissents from both sides of the Court. The majority opinion ruled the law was unconstitutional as it violated existing jurisprudence by discriminating against interstate commerce. Justices Scalia and Thomas authored a dissents on the grounds that the dormant Commerce Clause does not exist. At the other end, Justice Ruth Bader Ginsburg authored a separate dissent (joined by Justice Elena Kagan) that while the dormant Commerce Clause does exist, it should not be interpreted so stringently as to disallow Maryland's tax scheme.

Figure 7 shows the item response functions from both the Martin–Quinn model and GGUM with the estimated positions of the Justices. Due to the monotonicity assumption, the standard IRT model treats this case as if it provides essentially no information about ideology; voting in the case appears to be *entirely nonideological*. This is shown by the flat lines shown in Figure 7(b). On the other hand, the GGUM item response function, shown in Figure 7(a), indicates that the model

16 MC3-GGUM accurately predicted 45% of cases correctly and had a sensitivity of 0.68 and 0.72 for the 1 (strongly disagree) and 5 (strongly agree) response options. This compares to 43%, 0.54, and 0.63 for the standard GRM.

17 We produced two recorded chains, each obtained by running six parallel chains at the inverse temperature schedule (1.00, 0.89, 0.79, 0.71, 0.63, 0.56) for 5,000 burn-in iterations and 25,000 recorded iterations.

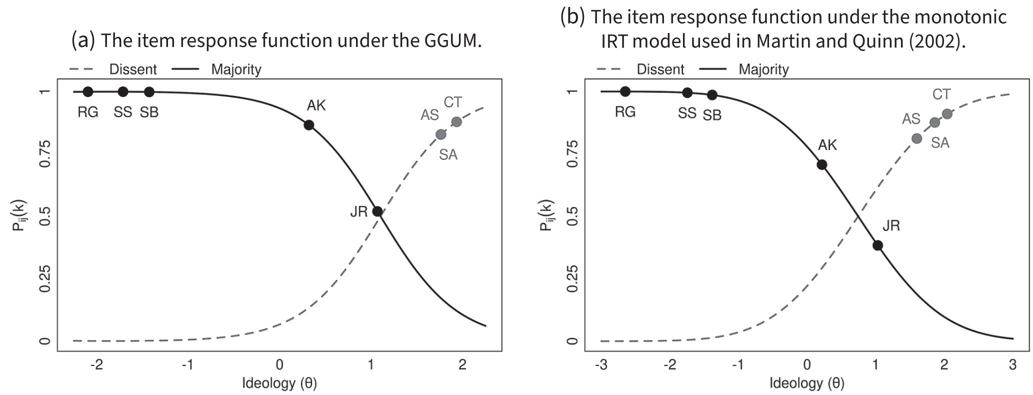


Figure 8. Item response functions for *Arizona v. United States* (2012). The probability of each justice's actual response is marked and labeled with the justice's initials.

Table 2. Log likelihood for all models in the U.S. House of Representatives and Supreme Court applications.

	Model	Log likelihood ($\mathcal{L}$)	$\mathcal{L}/N$	Mean θ s.d.
U.S. Supreme Court	MC3-GGUM	−540	−0.30	0.22
	CJR	−563	−0.31	0.26
	MQ	−554	−0.31	0.37
U.S. House	MC3-GGUM	−34595	−0.10	0.08
	CJR	−37308	−0.11	0.12

Note: N is the number of nonmissing responses in the data.

can learn from such disagreement since the dissents are joined by two ideologically opposed but (somewhat) coherent groups. That is, we are able to adequately account for these voting coalitions based on justices' ideologies and provide more accurate predictions for their voting decisions.

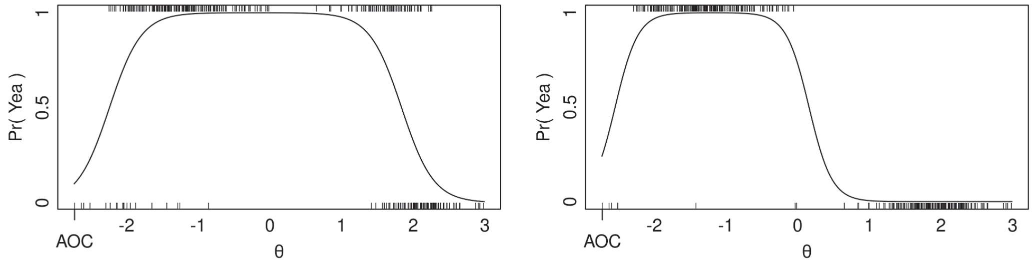
However, for many decisions, a monotonic item response function is completely appropriate. This is exemplified by *Arizona v. United States*, where the majority coalition consisted of Justices Roberts, Kennedy, Ginsburg, Breyer, and Sotomayor, with partial dissents coming from Justices Scalia, Thomas, and Alito. In this case, with a clear left–right divide on the court, Figure 8 shows that both GGUM and Martin–Quinn scores result in very similar monotonic response functions.

We also compare fit in Table 2. The result shows that GGUM provides a modest improvement over standard methods, meaning we get estimates that are both more precise and more accurate.[18] It also shows that the posterior variance for our estimates is lower, resulting from the higher amount of information (in a statistical sense) that we derive from items when the IRFs are less flat. In summary, we are able to simultaneously provide more accurate predictions, with less uncertainty, while also being more consonant with our substantive understanding of the data-generating process.

5.3 The House of Representatives

During the 116th Congress, scholars began to notice an irregularity. Even after the entire Congress was over, ideology estimates for several of the newest members of the Democratic caucus seemed unusually inaccurate. As of this writing, for instance, Poole and Rosenthal's DW-NOMINATE identifies Rep. Alexandria Ocasio-Cortez (D-NY) as one of the most *conservative* Democrats in the

18 This difference is more pronounced when focusing only on cases with more than one written dissent ($N = 45$), where it is more likely that we will observe disparate coalitions. The Brier score is 0.095 for Martin–Quinn and 0.087 for MC3-GGUM. In Appendix H of the Supplementary Material, we use a k-fold cross-validation and find no evidence of overfitting.



(a) H.J. Res. 31, the funding bill passed February 14, 2019 to avoid a partial government shutdown.

(b) H.R. 2740, a bill funding several federal government departments and agencies for the 2020 fiscal year.

Figure 9. Item response functions for two votes in the 116th House of Representatives. The solid line indicates the item response function for this vote. The rugs indicate the estimated ideology (θ) for all members where "Yea" votes are shown at the top and "Nay" votes are shown at the bottom.

chamber (the 90th percentile, just to the left of the chamber median; Lewis *et al.* 2019). This contrasts strongly with her wider reputation as an extreme liberal. She is not alone in having unusual estimates. Three members of the so-called "squad" (Reps. Ilhan Omar, Ayanna Pressley, and Rashida Tlaib) are estimated as being on the conservative side of the Democratic caucus.

This is because ends against the middle voting confuses many standard scaling methods. In the case of Rep. Ocasio-Cortez, the problem is that she regularly voted against the majority of the Democratic party and *with* Republican members. From public statements, it is clear that she does this because the proposals being considered are *not liberal enough*, while Republicans oppose the same bills because they are *not conservative enough*.

To show this, we use all nonunanimous roll-call votes in the 116th House for which the minority vote was at least 1% of the total vote. We omit from analysis members who participated in less than 10% of these roll calls.[19] This results in 438 total "respondents" (House members) and 846 "items" (roll-call votes); we used as observable response categories "Yea" votes and "Nay" votes. We obtained member ideology and item parameters using our MC3 algorithm for the MC3-GGUM, producing two recorded chains, each obtained by running six parallel chains for 10,000 burn-in iterations and 100,000 recorded iterations.[20] We compare our estimates to the standard two-parameter IRT model (Clinton *et al.* 2004).[21]

The results of the MC3-GGUM analysis indicate that while ends against the middle votes are not the modal case, they are nonetheless common. One example occurs about 1 month into the 116th Congress, on a vote designed to prevent a(nother) partial government shutdown. Republicans opposed the bill because it did not include funding for the border wall. Liberal Democrats, however, opposed it because it did not sufficiently reduce funding for border detention facilities (McPherson 2019). In both cases, the proposed bill was not sufficiently proximate to members' preferences. The item response function from the MC3-GGUM is shown in Figure 9a. As it clearly shows, MC3-GGUM captures the tendency of some members to vote in objectively similar ways (in this case Nay) for subjectively different reasons (opposition from the right and from the left).

Figure 9b shows the item response function for a bill to appropriate funds for fiscal year 2020. For Republicans, it provided too much domestic spending, representing "an irresponsible and unrealistic $176 billion increase above our current spending caps" while "imposing cuts to our military" (Flores 2019). Extreme Democrats did not support it because it gave the "military industrial complex another $733B windfall" while not bringing "economic opportunities we need"

19 We also omitted Rep. Justin Amash, who left the Republican party during this terms because the literature is inconsistent as to whether such members should be treated differently before and after they formally leave their caucus.

20 The parallel chains' inverse temperature schedule was $(1, 0.96, 0.92, 0.88, 0.85, 0.81)$.

21 In the main text, we focus on IRT models as these have a proper likelihood and are used in a wider array of settings (as shown in our other examples). We provide a more detailed comparison to the popular `wnominate` software in Appendix E of the Supplementary Material.

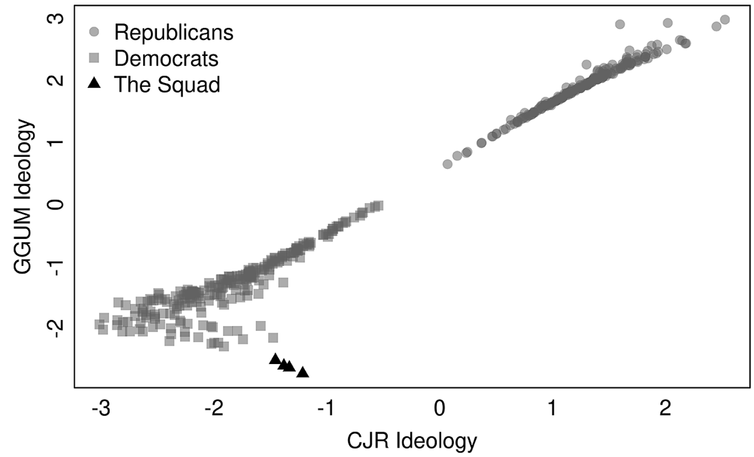


Figure 10. Comparing ideology estimates for members of the 116th House of Representatives from the MC3-GGUM and CJR IRT models. Estimates for Republicans are depicted with circles, and estimates for Democrats with squares, except that estimates for Reps. Ocasio-Cortez, Omar, Pressley, and Tlaib are depicted with triangles.

(Tlaib 2019). Members at both ideological extremes opposed the bill while providing exactly opposite rationales. Detailed discussions of additional examples of nonmonotonic item response functions on key bills in the 116th Congress are shown in Appendix I of the Supplementary Material.

The ability of the MC3-GGUM to capture ends against the middle behavior allows it to outperform IRT in terms of fit. Table 2 shows that while both models fit the data very well, MC3-GGUM has lower log-likelihood scores while at the same time providing narrower posterior standard deviations. It is again, therefore, both more accurate and more precise.

Perhaps more importantly, because it can accommodate votes that should have nonmonotonic item response functions, we can more accurately scale extremists who vote against their party. As shown in Figure 10, ideology estimates from MC3-GGUM and the CJR IRT model largely agree, but the dominance model scales the Squad as moderates, while MC3-GGUM correctly identifies them as the most liberal House members.[22] They also disagree on other notable progressives. The next three largest disagreements are for Rep. Pramila Jaypal, the chair of the Congressional Progressive Caucus (CPC), Rep. Peter DeFazio (founding member of the CPC), and Rep. Rohit Khanna (CPC member and national co-chair of the Bernie Sanders presidential campaign). In each case, MC3-GGUM identifies them as being far to the left, whereas CJR identifies them as moderates.

Before moving on, it is worth briefly discussing why this is occurring. While we cannot provide a comprehensive answer to this question here, the evidence suggests that some members—especially ideologically extreme members—may refuse to support bills that move the *status quo* in their direction because the proposal is still "too far" from their ideal point (Gilmour 1995). For instance, in discussing the Republican bill to replace the Affordable Care Act in 2017, Rep. Andy Biggs (R-AZ) explained that he opposed the bill (thus joining every Democrat) because it fell short of his promise of full repeal (Biggs 2019). In short, the bill was not conservative enough.

The literature explaining this behavior is unsettled. Kirkland and Slapin (2019) argue extreme members "rebel" against leadership as an electoral strategy to mark themselves as ideologues. They hypothesize that ideological extremity should be paired with voting against party leadership, but largely within the majority party. Or, perhaps members are engaged in a dynamic strategy holding out for more favorable eventual policy outcomes (in the flavor of Buisseret and Bernhardt 2017). Spirling and McLean (2007) offer a differing argument in the context of Westminster systems,

22 On the Republican side, the major outliers are Rep. Thomas Massie and Rep. Charles Roy. These are two extreme conservatives who regularly vote against their Republican colleagues when proposals are not sufficiently conservative.

arguing majority-party rebels vote sincerely against policies they dislike, whereas the opposition party votes strategically against nearly all government proposals. This debate cannot be resolved here. However, if these questions are to be pursued, at the very least, we need a measurement technique that does not conflate expressive disagreement with ideological moderation.

6 Conclusion

In this paper, we introduce the MC3-GGUM to the political science literature. The model accounts for and leverages ends against the middle responses—disagreement from both sides—when estimating latent traits. We provide a novel estimation and identification strategy for the model that outperforms existing routines for estimating the GGUM as well as open-source software, so researchers can implement the MC3-GGUM in their own work.

We illustrate this method with survey data, and votes in two institutional settings. We show that we gain the ability to treat survey responses with two-sided disagreement, court cases with discontinuous sets of dissenting justices, or roll-call votes with nay votes from both sides of the ideological spectrum, as informative for estimating latent traits. As a consequence, we recover more accurate estimates that better capture the underlying data.

However, it is worth noting that GGUM will not always be the correct choice in all settings. To our knowledge the GGUM model has not been extended to handle multidimensional latent scales. Furthermore, although the model is more flexible, in some settings (e.g., a multiparty legislature such as Brazil), the multimodal posteriors can make identification and summary challenging. Like all measurement models, the GGUM will be more or less suitable in different settings depending on the structure of the data and the appropriateness of its assumptions.

Yet, as we show in our examples above, it can be useful in many important empirical settings. It may allow for more flexible development of survey batteries where disagreement may come from "both sides" of a latent dimension. As noted in our Supreme Court example above, judicial decision-making often involves disjoint ideological coalitions. Indeed, almost one out of four (45/203) nonunanimous cases in our analysis resulted in more than one dissent, indicating that the same behavior may arise from differing (if not always antithetical) ideological motivations. In Appendix J of the Supplementary Material, we also estimate that nearly 17% of all roll calls in the 116th House resulted in nonmonotonic item response functions. Broadening the scope of our analysis to the 110th–116th congresses (both House and Senate), this proportion ranges from approximately 1 in 10 to 1 in 3 roll calls. Other future application areas might include voting in the United Nations (Bailey *et al.* 2017) or co-sponsorship decisions where members can choose from a menu of bills to support.

Finally, it is worth considering what the latent trait estimates *mean*, especially when applied to voting data. After all, dominance models are embedded in a clear theoretical framework, especially as they pertain to Congress and the Court. They are, in some sense, structural parameters based on standard theories of voting. In moving away from this, one may worry the resulting measures are less valid indicators of the theoretical concept of ideology. We argue that MC3-GGUM is not a measure of a different concept, but a better measure of the same concept. When dominance models are appropriate, MC3-GGUM does a fine job, recovering the same latent parameters as dominance models. However, when individuals behave more expressively, GGUM *also* works to uncover their latent ideology. These are cases where votes serve to signal approval of (or proximity to) a specific policy or opinion; these are cases where spatial theories deviate from dominance models because actors are not just considering the *status quo* and proposal. Thus, we view MC3-GGUM not as a measure of a different ideology, but a more valid measure of the same ideology. To this end, we have provided evidence (both empirical and qualitative) that where dominance and unfolding models disagree, GGUM conforms better with our substantive understanding of *where* actors are in the ideological space and *why* they behave as we observe.

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Data Availability Statement
Replication code for this article is available in Duck-Mayr and Montgomery (2022) at https://doi.org/10.7910/DVN/HXORK9.

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