# ROTATION AND FLATTENING OF GLOBULAR CLUSTERS

S. Michael Fall[1,3] and Carlos S. Frenk[2,3]
1) Institute of Astronomy, Madingley Road, Cambridge CB3 OHA England
2) Astronomy Centre, University of Sussex, Falmer, Brighton BN1 9QH England
3) Institute for Theoretical Physics, University of California, Santa Barbara, California 93106 USA

## 1. INTRODUCTION

Pease and Shapley (1917) first remarked on the apparent flattening of several Galactic globular clusters, a view that has been confirmed by many subsequent studies. Tidal stresses, internal rotation, and velocity anisotropies can cause deviations from sphericity in stellar systems. In general, we might expect globular clusters to have some angular momentum at the time of formation and, if they collapsed from flattened initial conditions, to have anisotropic pressure support. Since the velocity distributions within the clusters can be altered by a variety of internal and external processes, their shapes are expected to evolve. In this article, we review the methods for measuring ellipticities and the results that have emerged from such studies. Our main purpose, however, is to discuss the processes that determine the shapes of globular clusters and the ways in which they change with time.

Dynamical evolution may be observable in the rich star clusters of the Magellanic Clouds. Searle, Wilkinson, and Bagnuolo (1980, hereafter SWB) have classified these objects into seven types on the basis of their spectrophotometric properties. An approximate age-calibration of the SWB sequence, using Hodge's (1983) compilation of color-magnitude diagrams, gives

$$\log(\mathrm{age/yr}) \approx 0.5\ (\mathrm{SWB\ type}) + 6.6 \qquad (1)$$

The clusters in the Large Magellanic Cloud (LMC) with SWB types I-IV certainly are, and those with SWB types V-VII probably are, members of the disk population (Freeman, Illingworth, and Oemler 1983). However, the richest clusters of all ages are considerably more massive than the open clusters in our Galaxy. As is now customary, we refer to these objects as globular clusters even though very few of them may be members of the halo population in the Magellanic Clouds.



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## 2. MEASUREMENTS OF ELLIPTICITY

The shapes of globular clusters are usually quantified in terms of apparent ellipticities, $\varepsilon_a \equiv 1 - b/a$, where a and b are respectively the major and minor axes of the projected images. Since most of the light comes from a relatively small number of bright stars (red giants in old clusters and early-type stars in young clusters), the accuracy of the measurements is limited mainly by sampling errors. The ellipticities derived from star counts are consequently more reliable than those derived from smooth surface brightness maps. Kholopov (1953) used star counts for 19 Galactic globular clusters and we used them for 12 young LMC clusters (Frenk and Fall 1982). Geisler and Hodge (1980) estimated the ellipticities of 25 old LMC clusters from microdensitometer scans but, by undersampling the images, they appear to have introduced large systematic errors. Geyer and Richtler (1981) used Agfa contourfilm for 25 clusters of all ages in the LMC and Geyer, Hopp, and Nelles (1983) used the same method for 20 globular clusters in our Galaxy and 4 in the Magellanic Clouds.

The simplest way to measure ellipticities is by eye. Shapley and Sawyer (1927) used this method for 75 Galactic globular clusters and we used it for 93 globular clusters in the Galaxy and 52 in the LMC (Frenk and Fall 1982). Our measurements, which were made from polaroid enlargements of the Palomar and SRC Sky Surveys, apply to the regions near (1–2) $r_h$ where $r_h$ is the median radius of a cluster (i.e. the radius containing half of the light in three dimensions). Since the results are somewhat subjective, they need to be checked by other methods. Figure 1 is a comparison between our eye-measurements and determinations from star counts by Kholopov and by us in the same regions of the clusters. The standard deviation for individual ellipticities is about 0.05 and there is no systematic difference between the results from the two methods.

Our eye-measurements are compared in Fig. 2 with the ellipticities derived from Agfa contourfilm by Geyer and Richtler (1981) and Geyer, Hopp, and Nelles (1983). The agreement for most of the sample is reasonable but the systematic deviations for several of the flattest

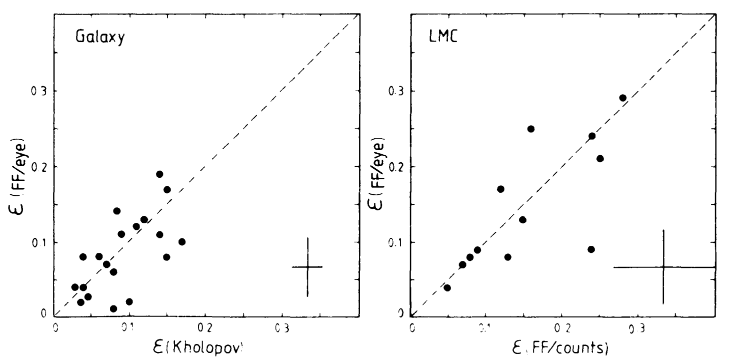


Figure 1. Comparison of ellipticities from eye-measurements and star counts in the same regions of the clusters.

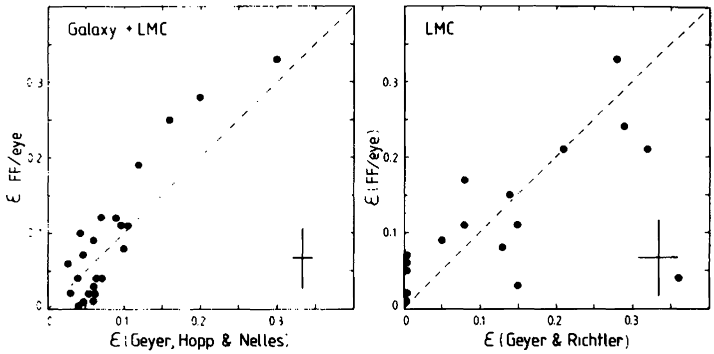


Figure 2. Comparison of ellipticities from eye-measurements and from Agfa contourfilm. The left-hand panel differs slightly from Fig. 3 of Geyer et al. because we have plotted the means of their measurements $\overline{\varepsilon}$ rather than their extrapolated values $<\varepsilon>$.

clusters led Geyer et al. to question the validity of our results. In fact, most of the difference arises because their ellipticities are averages over large regions of the clusters whereas the eye-measurements apply to a smaller range of radii. The discrepant clusters in the Galactic sample are 47Tuc (NGC104), ωCen (NGC5139), and M19 (NGC6273); and, in at least two of them, Geyer et al. found strong ellipticity gradients. At the radii where the eye-measurements were made, the agreement with the results derived from Agfa contourfilm is excellent. This is shown in Fig. 3 for the four clusters with the most extensive data, including two of the discrepant clusters.

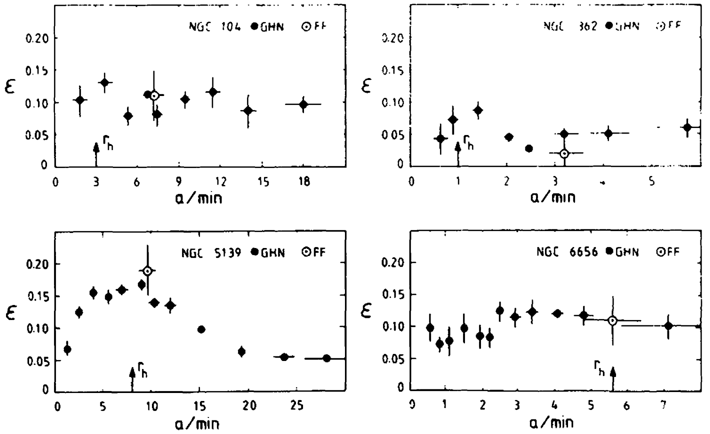


Figure 3. Radial dependence of ellipticity derived from Agfa countourfilm by Geyer et al. in four Galactic globular clusters. The open circles represent our eye-measurements.

While improved determinations of ellipticities would be highly desirable, the eye-measurements have allowed us to explore the statistical properties of large samples (Frenk and Fall 1982; Fall and Frenk 1983). The main results can be summarized as follows. In both the Galaxy and the LMC, globular clusters appear to be oriented at random. After deprojecting the apparent shapes of the clusters, the mean of the true ellipticities is $\langle\varepsilon\rangle = 0.12 \pm 0.01$ for the Galactic sample and $\langle\varepsilon\rangle = 0.18 \pm 0.02$ for the LMC sample. There is a general tendency for the ellipticities of the LMC clusters to decrease along the SWB sequence. Most of the change occurs near types III and IV, which corresponds to ages of a few times $10^8$ yr by Eq. (1). On average, the youngest LMC clusters are nearly as flat as elliptical galaxies and the oldest LMC clusters are nearly as round as the globular clusters in our Galaxy.

van den Bergh (1983) found that the flattest clusters in the LMC are generally among the brightest. Since the clusters become fainter as they get older, this may be in part a reflection of the correlation between ellipticity and age. Without a theoretical model for the origin and evolution of the clusters, it is not possible to say which relation has the greatest physical significance. van den Bergh (1984) also found a correlation between the visual extinction of Galactic globular clusters and their apparent ellipticities. He suggests, rather arbitrarily, that the clusters with $A_V > 1.0$ are distorted by patchy obscuration. This is true for M19, which appears kidney-shaped at visual wavelengths and suffers differential redding across the image (Shapley 1930; Harris, Racine, and deRoux 1976). A smooth contribution to $A_V$ would not affect the shape of a cluster and, apart from M19, there is no evidence that the other clusters in our sample are distorted by patchy obscuration.

The absence of alignment with the structural features of the Galaxy and the LMC rules out tidal stresses as the cause of flattening in globular clusters. This is not surprising since the densities are too high in the regions where ellipticities are measured for tidal stresses to be important (King 1961). The remaining possibility is that globular clusters owe their non-spherical shapes to internal rotation, velocity anisotropies, or some combination of both effects. Rotation appears to be the dominant cause of flattening in four of the Galactic globular clusters for which stellar radial velocities are available: ωCen (Freeman and Seitzer 1984; Meylan and Mayor 1984), 47Tuc (Mayor et al. 1984; Freeman and Da Costa 1984), M13 and M92 (Lupton, Gunn, and Griffin 1984). The other cluster with extensive data, M3, is nearly round and shows no signs of rotation. Using proper motions, Cudworth (1979) has found direct but weak evidence for velocity anisotropies in the outer parts of this cluster. The radial velocities measured by Gunn and Griffin (1979) are also fitted best by a distribution function that is elongated toward the center of M3.

## 3. THE TENSOR VIRIAL THEOREM

The tensor virial theorem provides a useful connection between the global rotation, velocity anistropy, and shape of a self-gravitating system (Chandrasekhar 1969). In a steady state, this takes the form

$$2T_{kl} + \Pi_{kl} + W_{kl} = 0 \ , \tag{2}$$

where $T_{kl}$, $\Pi_{kl}$, and $W_{kl}$ are respectively the kinetic energy, pressure, and potential energy tensors. For an axisymmetric system with no radial or vertical streaming motions, the only nontrivial components of Eq. (2) are

$$\begin{aligned} &M(v^2 + \sigma_r^2 + \sigma_\phi^2) + W_{11} + W_{22} = 0 \ , \\ &M\sigma_z^2 + W_{33} = 0 \ , \end{aligned} \tag{3}$$

where M is the total mass, v is the root mean square rotation velocity, and $\sigma_r$, $\sigma_\phi$, and $\sigma_z$ are the velocity dispersions in the radial, azimuthal, and vertical directions when integrated over all positions.

To proceed further, we impose the additional restriction that the system be stratified on similar oblate ellipsoids with an axial ratio $q \equiv 1 - \varepsilon$ but with an otherwise arbitrary density profile. Equations (3) can then be combined into the form

$$(3 + \alpha + \beta)(T_{rot}/T_{rand}) + (\alpha+\beta) = \frac{(1+2q^2)\cos^{-1}q - 3q(1-q^2)^{1/2}}{q(1-q^2)^{1/2} - q^2\cos^{-1}q}$$

$$\approx \frac{8}{5}\,\varepsilon \text{ for } \varepsilon << 1, \tag{4}$$

where, by definition,

$$\alpha = (\sigma_r/\sigma_z)^2 - 1, \quad \beta = (\sigma_\phi/\sigma_z)^2 - 1,$$

$$T_{rot} = \frac{1}{2}Mv^2, \quad T_{rand} = \frac{1}{2}M(\sigma_r^2 + \sigma_\phi^2 + \sigma_z^2) \tag{5}$$

(Binney 1978). For a fixed value of $T_{rot}/T_{rand}$, the ellipticity depends on anisotropic stresses only through the combination $\alpha+\beta$ and the case $\alpha=\beta=0$ corresponds to a rotationally supported system. [The approximation suggested by King (1961) differs from Eq. (4) by a factor of 3 in this limit.] If the phase space distribution is a function only of the energy E and the angular momentum $J_z$ about the symmetry axis, then $\sigma_r = \sigma_z$. The converse is not necessarily true for global velocity dispersions but in practice $\sigma_r \neq \sigma_z$ will usually imply the existence of non-classical integrals. The price we pay for the simple description given by the tensor virial theorem is the loss of information about local variations in the ellipticity, rotation, and velocity anisotropy.

Two-body diffusion causes a system with initial anisotropies to evolve quasistatically toward a state with isotropic pressure support. The characteristic time-scale for this process is the deflection time, defined as $\tau_D \equiv \langle u^2\rangle/\langle(\Delta u_\perp)^2\rangle$ where $\langle u^2\rangle^{1/2}$ is the local velocity dispersion and $\langle(\Delta u_\perp)^2\rangle$ is the diffusion coefficient for a test star in the direction perpendicular to its motion (Spitzer 1956). This is comparable with the relaxation time $\tau_r \equiv \langle u^2\rangle/3\langle(\Delta u_{\parallel})^2\rangle$ where $\langle(\Delta u_{\parallel})^2\rangle$ is the diffusion coefficient for a test star in the direction parallel to its motion. Thus, we expect the velocity distribution at each point in a cluster to isotropize at nearly the same rate as it relaxes. Similarly, we expect the global anisotropy parameters $\alpha$ and $\beta$ and the ellipticity near the median radius to change on small multiples of the reference relaxation time $\tau_{rh}$, which is defined as the value of $\tau_r$ at the mean density within $r_h$ (Spitzer 1975). The N-body models and the Fokker-Planck integrations discussed below confirm this expectation.

Since most of the old LMC and Galactic globular clusters have ages greater than $\tau_{rh}$, they should have nearly isotropic velocity distributions in the regions where ellipticities are measured. Setting $\alpha = \beta = 0$ in Eq. (4) and deprojecting the ellipticities measured by eye, we find $\langle(v/\sigma)^2\rangle^{1/2} = 0.47 \pm 0.03$ for Galactic globular clusters and $\langle(v/\sigma)^2\rangle^{1/2} = 0.56 \pm 0.06$ for the LMC clusters with SWB types V-VII (Fall and Frenk 1983). As the result of residual anisotropies, these estimates may be slightly too large but they are in rough agreement with the values of $v/\sigma$ derived from stellar radial velocities in the four clusters mentioned in the previous section. If globular clusters form by aspherical collapse and incomplete violent relaxation, we would expect them to have anisotropic velocity distributions when they first reach dynamical equilibrium (Aarseth and Binney 1978). This may account for the flattened shapes of the LMC clusters with ages smaller than their relaxation times. Unfortunately, a direct test of this conjecture, which would require proper motions for inidividual stars, is virtually impossible.

## 4. N-BODY MODELS

To simulate the evolution of globular clusters with initially anisotropic velocity distributions, we computed a series of models with equal-mass particles using Aarseth's (1984) code. The validity of this approach can be justified as follows. In a typical globular cluster, the dynamical, relaxation, and close-encounter times are in the ratios $\tau_{dh}:\tau_{rh}:\tau_{ch} \approx 1:7\times10^3:4\times10^5$. In an N-body model, there is some freedom to alter the characteristic time-scales by adjusting the number of particles and the softening parameter in the interaction potential $\phi(r) = -G/(r^2+s^2)^{1/2}$. We chose N = 400 and $s = r_h/16$, which implies $\tau_{dh}:\tau_{rh}:\tau_{ch} = 1:14:700$. Although these time-scales do not differ by as much as in globular clusters, they are in the correct sequence and are fairly well separated. Thus, the global properties of a globular cluster are modelled accurately by integrating the equations of motion for many fewer particles.

As the initial conditions for the N-body simulations we chose a slowly rotating, oblate spheroid with a density profile resembling that of a King model. The initial concentration parameter we adopted, $c \approx 1.3$, is typical of the globular clusters in the LMC (Chun 1978). The initial velocities of the particles were assigned randomly in such a way that the distribution function was the same throughout the system and approximately in a steady state. We computed five models with different statistical realizations of two sets of initial conditions and then combined the results into two ensembles. Ensemble A (with 3 models) has an initial ellipticity of $\varepsilon_i = 0.4$ and initial anisotropy parameters of $\alpha_i = 0.3$ and $\beta_i = 0.5$; ensemble B (with 2 models) has $\varepsilon_i = 0.6$, $\alpha_i = 0.8$, and $\beta_i = 1.2$. In both cases, the initial streaming motion consists of solid-body rotation about the z axis with $T_{rot}/T_{rand} = 0.05$, which corresponds to $v/\sigma \gtrsim 0.4$. Thus, rotation plays only a minor role in the early stages but it becomes more important as the velocity anisotropies decay.

The N-body simulations were stopped after a time equal to $175\tau_{dh}$ or equivalently $13\tau_{rh}$, which is not long enough for the core to collapse in isolated systems with equal-mass particles (Cohn 1980). Since only one particle escaped from one of the five models, evaporation has no influence on their evolution. The models remained axisymmetric at all times and we obtained a global estimate of the ellipticity $\varepsilon_{ML}$ by a maximum likelihood method and an estimate for the inner half of the mass $\varepsilon_h$ by White's (1978) ranking method. These techniques and other properties of the simulations are described in detail in a separate paper (Fall and Frenk 1984). Figure 4a shows the ellipticities as a function of time in units of the instantaneous value of $\tau_{rh}$. In both ensembles, $\varepsilon_{ML}$ and $\varepsilon_h$ decrease by a factor of 2 on time-scales of about $5\tau_{rh}$. After that, the evolution is slower and, by the end of the simulations, the ellipticities appear to be close to their asymptotic values. The arrows in Fig. 4a indicate the times equal to the local relaxation time for radii containing 30, 50, and 70 percent of the mass. As successive shells relax, weak ellipticity gradients develop, which are reflected in the differences between $\varepsilon_{ML}$ and $\varepsilon_h$.

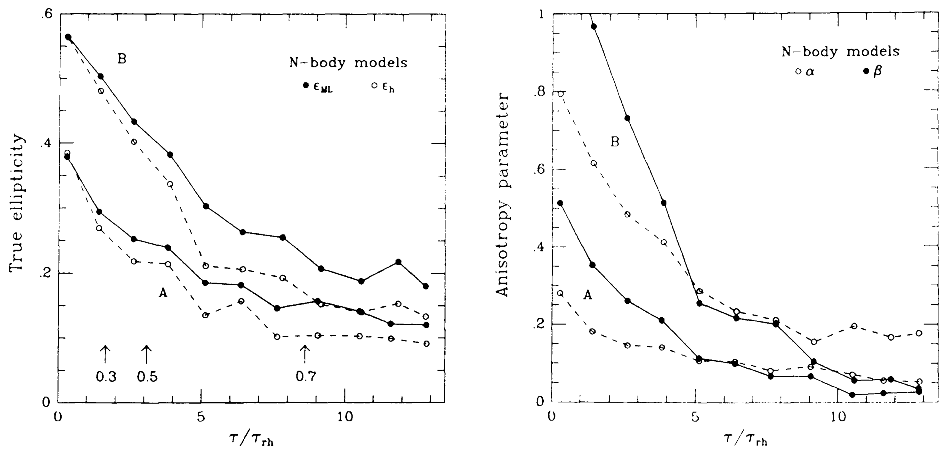


Figure 4. Evolution in the two ensembles of N-body models: (a) ellipticities $\varepsilon_{ML}$ and $\varepsilon_h$ from the maximum likelihood and ranking methods, (b) anisotropy parameters $\alpha$ and $\beta$ in the radial and azimuthal directions. Each symbol represents an average over 24 different times separated by $0.5\tau_{rh}$.

The evolution of the anistropy parameters for both ensembles is shown in Fig. 4b. As expected, $\alpha$ and $\beta$ decrease at nearly the same rate as the clusters become rounder although some residual anisotropy remains at the end of the simulations. This probably explains why the two ensembles, which have the same total angular momentum, do not have exactly the same final shapes. Despite the development of ellipticity gradients, the tensor virial theorem for similar ellipsoids provides a remarkably good description of the N-body models. The ratio $v/\sigma$ remains approximately constant but, as a result of residual anisotropies in the final state, it is smaller than the value that would be inferred from Eq. (4) with $\alpha = \beta = 0$. The discrepancy of 30 percent for ensemble A and nearly 100 percent for ensemble B gives a rough indication of the possible systematic errors in the estimates of $\langle(v/\sigma)^2\rangle^{1/2}$ mentioned in the previous section.

## 5. SIMPLE EVAPORATION MODEL AND FOKKER-PLANCK INTEGRATIONS

Agekian (1958) suggested that the shape of a rotating cluster would change as stars escape by two-body diffusion. As an approximation, he assumed that all of the stars in the unbound tail of a Maxwellian distribution would leave the system with zero energy in each relaxation time. Since the low-velocity part of the distribution is isotropic in the rotating frame, more stars escape in the direction of rotation and the angular momentum per unit mass of the cluster decreases. Agekian applied this simple evaporation model to a Maclaurin spheroid and found that it becomes rounder if the initial ellipticity is less than 0.74. Shapiro and Marchant (1976) obtained the same result and suggested that globular clusters could have been much flatter at the time of formation than they are at present.

The previous arguments can be applied even more simply to a system with an arbitrary density profile if the ellipticity is small. This limit is more appropriate for globular clusters than Agekian's assumptions of arbitrary ellipticity and uniform density. For an oblate system with $\varepsilon \ll 1$ and $\alpha = \beta = 0$, the tensor virial theorem in the form of Eq. (4) gives $\varepsilon \approx 15T_{rot}/8T_{rand}$. In general, we can write

$$T_{rot}/T_{rand} = g(\varepsilon)J^2(-E)G^{-2}M^{-5} , \tag{6}$$

where M, E, and J are respectively the total mass, energy, and angular momentum of the system and $g(\varepsilon)$ is some function that approaches a constant value in the limit $\varepsilon \to 0$. Thus, at constant energy, the evolution is governed by

$$d(\ln\varepsilon)/dt \approx 2d(\ln J)/dt - 5d(\ln M)/dt , \tag{7}$$

to lowest order in the ellipticity. By definition, we can also write

$$d(\ln M)/dt = - f_M(\varepsilon)/\tau_r , \quad d(\ln J)/dt = - f_J(\varepsilon)/\tau_r , \tag{8}$$

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where $f_M(\varepsilon)$ and $f_J(\varepsilon)$ are respectively the fractions of the mass and angular momentum lost in each relaxation time $\tau_r$. These functions tend to constant values in the limit $\varepsilon \to 0$ and, in the simple evaporation model, are given by $f_M \approx 7 \times 10^{-3}$ and $f_J \approx 3 \times 10^{-2}$ (Agekian 1958). Thus, the e-folding time for local changes in the ellipticity is

$$\tau_\varepsilon \equiv - dt/d(\ln\varepsilon) \approx \tau_r/(2f_J - 5f_M) \approx 40\tau_r \ . \qquad (9)$$

For changes in the ellipticity near the median radius, the reference relaxation time would seem appropriate and we therefore expect $\tau_\varepsilon \approx 40\tau_{rh}$. This is an order of magnitude larger than the time-scale for the decay of velocity anisotropies and is also larger than the ages of most globular clusters. Shapiro and Marchant's suggestion was based on central relaxation times, which are generally an order of magnitude smaller than reference relaxation times. They therefore overestimated the influence of evaporation on the overall shapes of globular clusters by a similar factor.

A more accurate description of the effects of two-body diffusion in a stellar system is given by the orbit-averaged Fokker-Planck equation. Goodman (1983) used this method to study the evolution of clusters supported either by rotation or pressure. He assumed that the distribution function depends only on E, $J_z$, and time, which reduces the number of dimensions in the problem from seven to three. Thus, in contrast to the N-body simulations, his method neglects the effects of non-classical integrals. Goodman calculated the evolution of two non-rotating models with anisotropic velocity distributions and equal-mass particles. They were tidally truncated with an initial concentration parameter of c = 1.9 and were stopped when the cores collapsed at times equal to $(2\text{-}3)\tau_{rh}$. Over this limited interval, the ellipticities in the Fokker-Planck integrations decreased at roughly the same rate as in the N-body simulations, despite the different assumptions.

Goodman also calculated the evolution of two rotating models with isotropic velocity distributions. In this case, the shapes changed more rapidly than predicted by the simple evaporation model. However, Goodman attributed most of the evolution to two effects other than the escape of stars. The first, which has also been discussed by Aarseth (1966), is the relaxation-driven transfer of angular momentum from the inner to the outer parts of a cluster when the angular velocity decreases with radius. This is the analog of viscous transport in a rotating fluid but is not a local phenomenon because the mean-free path is much longer in a stellar system. The second reason for the faster evolution in Goodman's models is that their density profiles changed. As the cores contract, they release energy and rotate more rapidly while the envelopes, which absorb the energy and expand, rotate more slowly. These effects are essentially absent in our N-body models because they have solid-body rotation and nearly constant density profiles.

## 6. CONCLUSIONS

The arguments given here suggest that globular clusters become rounder as they get older. Two-body diffusion causes the decay of velocity anisotropies, the evaporation of stars, the viscous transport of angular momentum and the redistribution of matter during core collapse. We therefore expect the ellipticities near the median radii of the clusters to be functions of $\tau/\tau_{rh}$ where $\tau$ is the age and $\tau_{rh}$ is the reference relaxation time. For the Galactic globular clusters, which have $\tau \approx 1.5 \times 10^{10}$ yr, the values of $\tau_{rh}$ range from $10^8$ yr to $10^{10}$ yr (Spitzer 1975). The N-body and Fokker-Planck models discussed above have equal-mass particles and evolve on small multiples of $\tau_{rh}$; the inclusion of a realistic mass spectrum would probably lead to somewhat faster rates. Thus, globular clusters could be much rounder now than they were at the time of formation, especially if their initial velocity distributions were anisotropic. Evaporation has almost certainly played a less important role since the characteristic time-scale for changes in the overall shapes by this process alone is about $40\tau_{rh}$.

Unfortunately, the masses and radii needed to derive relaxation times are available for only a few of the brightest clusters in the LMC. From the data given by Chun (1978) and Elson and Freeman (1984), we estimate values of $\tau_{rh}$ between $10^8$ yr and $10^9$ yr but the dispersion in the whole LMC sample is likely to be at least as large as in the Galactic sample. Since the ages of the clusters with SWB types III and IV are a few times $10^8$ yr, they correspond, very roughly, to $\tau \approx \tau_{rh}$. Thus, the observed decrease in the ellipticities of LMC clusters could be explained by the decay of initial velocity anisotropies; viscous transport of angular momentum could have also played a role. We cannot, however, rule out the possibility that the initial distribution of shapes has varied over the life-time of the LMC. There is some evidence that the youngest clusters are less massive than the oldest and variations in their other properties would not be surprising. Nevertheless, the young LMC clusters will become rounder with time and some contribution to the relation between ellipticity and age from the processes discussed here seems almost inevitable. More accurate estimates of the ages, ellipticities and relaxation times of the LMC clusters would undoubtedly lead to a better understanding of these problems.

## ACKNOWLEDGMENTS

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## REFERENCES

Aarseth, S.J. 1966, M.N.R.A.S., **132**, 35.
Aarseth, S.J. 1984, in Methods of Computational Physics, eds. J.U. Brackbill and B.I. Cohen (New York: Academic Press), p. 1.

Aarseth, S.J. and Binney, J. 1978, M.N.R.A.S. 185, 227.
Agekian, T.A. 1958, Soviet Astr. - A.J., 2, 22.
Binney, J. 1978, M.N.R.A.S., 183, 501.
Chandrasekhar, S. 1969, Ellipsoidal Figures of Equilibrium (New Haven: Yale University Press).
Chun, M.S. 1978, A.J., 83, 1062.
Cohn, H. 1980, Ap. J., 242, 765.
Cudworth, K.M. 1979, A.J., 84, 1312.
Elson, R.A.W. and Freeman, K.C. 1984, Ap. J., submitted.
Fall, S.M. and Frenk, C.S. 1983, A.J., 88, 1626.
Fall, S.M. and Frenk, C.S. 1984, in preparation.
Freeman, K.C. and Da Costa, G. 1984, in preparation.
Freeman, K.C., Illingworth, G., and Oemler, A. 1983, Ap. J., 272, 488.
Freeman, K.C. and Seitzer, P. 1984, in preparation.
Frenk, C.S. and Fall, S.M. 1982, M.N.R.A.S., 199, 565.
Geisler, D. and Hodge, P. 1980, Ap. J., 242, 66.
Geyer, E.H., Hopp, U., and Nelles, B. 1983, Astr. Ap., 125, 359.
Geyer, E.H. and Richtler, T. 1981, in Astrophysical Parameters for Globular Clusters, eds. A.G.D. Philip and D.S. Hayes (Schenectady: L. Davis Press), p. 239.
Goodman, J. 1983, Ph.D. Thesis, Princeton University.
Gunn, J.E. and Griffin, R.F. 1979, A.J., 84, 752.
Harris, W.E., Racine, R., and deRoux, J. 1976, Ap. J. Suppl. 31, 13.
Hodge, P.W. 1983, Ap. J., 264, 470.
Kholopov, P.N. 1953, Publ. Astr. Sternberg Inst., 23, 250.
King, I. 1961, A.J., 66, 68.
Lupton, R., Gunn, J.E., and Griffin, R.F. 1984, in preparation.
Mayor, M. et al. 1984, Astr. Ap., 134, 118.
Meylan, G. and Mayor, M. 1984, preprint.
Pease, F.G. and Shapley, H. 1917, Contr. Mt. Wilson Obs., 129.
Searle, L., Wilkinson, A., and Bagnuolo, W.G. 1980, Ap. J. 239, 803.
Shapiro, S.L. and Marchant, A.B. 1976, Ap. J., 210, 757.
Shapley, H. 1930, Star Clusters (New York: McGraw-Hill).
Shapley, H. and Sawyer, H. 1927, Harvard Obs. Bull., 852.
Spitzer, L. 1956, Physics of Fully Ionized Gases (New York: Inter-Science).
Spitzer, L. 1975, in Dynamics of Stellar Systems, ed. A. Hayli (Dordrecht: Reidel), p. 3.
van den Bergh, S. 1983, P.A.S.P., 95, 839.
van den Bergh, S. 1984, Observatory, in press.
White, S.D.M. 1978, M.N.R.A.S., 184, 185.

DISCUSSION

MAYOR (Comment to Fall): The $v_0 / \sigma_0$ values derived from our measurement of 47 Tuc and $\omega$ Cen are strongly in favor of a quasi-isotropic distribution of velocities in these two clusters. In spite of its large relaxation time $\omega$ Cen does not show any global evidence of anisotropy.

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