



Mikhail Birman, 17.01.1928 - 2.07.2009

## OBITUARY

On July 2, 2009, Professor Mikhail Birman, the renowned specialist in theory of operators in Hilbert spaces and spectral theory of differential operators, passed away after a long and grave illness.

M. Birman was born in Leningrad on January 17, 1928. He finished high school in 1944 and graduated from the Leningrad University in 1950. His teachers were M. K. Gavurin and L. V. Kantorovich. M. Birman defended his Ph.D. thesis in 1954 and got his doctoral degree in 1962. Already as a university student M. Birman took a part time job in the research team led by L. V. Kantorovich. Between 1950 and 1956 M. Birman worked as an assistant professor in the Leningrad Mining Institute. Since 1956 he has been employed by the Physics Faculty (Department of Mathematical Physics) of the Leningrad (since 1991, St. Petersburg) University, first as an associate professor and then as a full professor. M. Birman retained this position until the end of his life. His lectures to students specializing in mathematical and theoretical physics quickly

gained him the reputation of an outstanding teacher. They attracted students by both the careful choice of material and the clear and artistic presentation.

M. Birman belongs to the famous Petersburg school of mathematical physics founded by V. I. Smirnov. Many results of M. Birman, especially in spectral theory of differential operators, mathematical scattering theory and theory of functions, became classics. They are world renowned and to a large extent determined the further development of several branches of mathematics. M. Birman created his own widely recognized scientific school. More than 20 of his students defended their Ph.D. theses, and 8 of them became Doctors of science. Now they are professors in reputed Russian and Western Universities. During almost 60 years M. Birman was one of the leading persons of the seminar in mathematical physics organized by V. I. Smirnov and now named after him. M. Birman edited 13 volumes of “Topics in Mathematical Physics” published by the Leningrad University and translated in USA as well as several volumes of “Advances in Mathematical Sciences” published by the AMS.

First papers (1950-1952) by M. Birman were devoted to numerical analysis. In the middle of the 50's, M. Birman was influenced by works of M. G. Krein and began his research on the theory of operators in Hilbert spaces which eventually became one of his main scientific interests. He started with extension theory of symmetric operators. Later his interests shifted to spectral theory. Here M. Birman found new powerful methods leading to numerous important results in spectral theory of differential operators and in the variational approach to solutions of boundary value problems. Particularly famous is the Birman–Schwinger principle in the study of the discrete spectrum. This principle is now a starting point in many papers on spectral theory, especially in problems arising in quantum mechanics.

Of great importance are M. Birman's papers on mathematical scattering theory. To a large extent, they determined the development of this modern branch of mathematics and mathematical physics. A series of powerful and convenient conditions guaranteeing the existence and completeness of wave operators is known as the Kato–Birman theory. M. Birman found the famous invariance principle in the theory of wave operators. Another famous result is the Birman–Krein formula relating the scattering matrix and the spectral shift function. The methods created by M. Birman are systematically used by other researchers working in scattering theory.

In connection with the research on scattering theory, the need arose to work out the theory of double operator integrals. To construct a rigorous theory of such integrals, M. Birman together with M. Solomyak developed the scheme of piecewise polynomial approximations for functions from Sobolev classes. Their approach turned out to be adequate for many problems lying beyond the original target. In particular, M. Birman and M. Solomyak obtained the precise estimates for the  $\varepsilon$ -entropy of the embeddings of the Sobolev classes in the cases where linear methods do not apply. The same approach allowed the authors to find sharp estimates and asymptotics of eigenvalues for wide classes of differential and integral operators.

In an important series of papers, M. Birman with his colleagues and students studied the Maxwell operator with non-smooth coefficients in domains with non-smooth boundaries. In particular, they justified the Weyl asymptotic formula for eigenfrequencies of oscillations of a filled electromagnetic resonator in the non-smooth case.

Another of Birman's research interests was the study of the discrete spectrum appearing in

a spectral gap of a self-adjoint operator under perturbation. Typical examples are Dirac and Schrödinger operators with periodic potentials. In this field, M. Birman obtained important results giving new (non-Weyl) asymptotic formulas for the spectrum in a gap in the large coupling constant limit.

During his last years, M. Birman contributed significantly to the spectral theory of periodic operators of mathematical physics. Together with T. Suslina he studied the problem of absolute continuity of the spectrum of periodic differential operators. In particular, they solved the long standing problem of the absolute continuity of the spectrum of the periodic magnetic Hamiltonian in dimension two. This result stimulated numerous papers of different authors on closely related topics.

Recently, M. Birman and T. Suslina suggested and developed a new (spectral) method in the study of threshold effects and homogenization problems for periodic differential operators. This method allowed the authors to obtain results of a novel nature in homogenization theory. In particular, they found approximations of the resolvent of an elliptic operator with rapidly oscillating coefficients in operator norms with order-sharp estimates of the remainder.

M. Birman was one of the leaders in spectral theory of differential operators. His papers have original conception and are exceptionally well written. Usually, they were motivated by applications to adjacent fields of mathematics and mathematical physics (quantum theory, numerical methods, electrodynamics, mechanics of continuous media, etc.). Some of his papers founded new directions of research which continue to be successfully developed. He is the author of more than 160 papers largely cited in the scientific literature. Together with M. Solomyak, he wrote a textbook and a monograph translated into English.

M. Birman continued his work until the very last days of his life. His impact on mathematics and mathematical physics has inspired and continues to inspire many scientists who will always keep M. Birman's name in their hearts.

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