

# EXCITATION OF SOLAR ACOUSTIC OSCILLATIONS

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**Abstract.** The stochastic excitation of solar oscillations due to turbulent convection is reviewed. A number of different observational results that provide test for solar p-mode excitation theories are described. I discuss how well the stochastic excitation theory does in explaining these observations. The location and properties of sources that excite solar p-modes are also described. Finally, I discuss why solar g-modes should be linearly stable, and estimate the surface velocity amplitudes of low degree g-modes assuming that they are stochastically excited by the turbulent convection in the sun.

## 1. Introduction

It was realized about 25 years ago that the Sun, our nearest star, is a variable star. Millions of acoustic normal modes (p-modes) of the sun are seen to be excited with a typical surface velocity amplitude of only a few  $\text{cm s}^{-1}$ , whereas other pulsating stars have a few modes excited to large amplitudes. Considering this dramatic difference between the pulsation property of the sun and other variable stars, it should not be surprising that the solar oscillations are excited by a mechanism that is different from the overstability mechanism believed to be responsible for the pulsation of other stars (overstability can arise for instance when the radiative flux is converted to mechanical energy of pulsation due to an increase of opacity with temperature). A number of early papers in the field proposed that the solar p-modes are excited by some overstability mechanism (Ulrich 1970, Leibacher and Stein 1971, Wolf 1972, Ando and Osaki 1975). However, the margin of instability for solar p-modes is found to be small and different ways of handling radiative transfer and/or the interaction of convection with oscillation

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seems to change the sign of stability e.g. Goldreich and Keeley 1977a, Antia et al. 1982 and 1988, Christensen-Dalsgaard and Frandsen 1983, Balmforth and Gough 1990, Balmforth 1992 (the last two papers used a sophisticated version of the mixing length theory of Gough, 1977); see Cox et al. 1991 for a more complete list of references. If we assume that the solar p-modes are overstable then their amplitudes grow exponentially until some nonlinear mechanism limits their growth. By considering all possible 3-mode nonlinear couplings amongst overstable and stable p-modes in the sun, which is the most efficient process for saturating the amplitudes of overstable modes, Kumar and Goldreich (1989) and Kumar, Goldreich and Kerswell (1991) showed that the amplitudes of overstable modes saturate at a value that is several orders of magnitude larger than the observed value. This suggests that solar p-modes are linearly stable.

In this article we will assume that solar p-modes are stable, and describe how they can be excited by acoustic waves generated by turbulent convection. The basic idea is that the broad band acoustic noise generated by the turbulent flow in the convection zone is selectively amplified at frequencies corresponding to the normal mode frequencies of the sun. The process of wave generation by homogeneous turbulence was first studied systematically and in some detail by Lighthill (1952). Stein (1967) and Kulsrud (1955) applied it to the heating of the solar chromosphere/corona by acoustic and MHD waves respectively. Goldreich and Keeley (1977b) carried out a careful calculation of the stochastic excitation of solar normal modes by turbulent convection (for an excellent general review of wave generation due to turbulent fluid please see Crighton 1975). We describe the stochastic excitation process for the simple case of a homogeneous sphere below and later discuss its generalization to the Sun (§2). In §3 we describe various observations that any theory for excitation of solar p-modes must be able to explain and discuss how well the stochastic excitation theory performs when confronted with these observations. The estimate for the surface velocity amplitude of low degree g-modes, assuming that they are stochastically excited, is given in §4.

## 2. Stochastic Excitation

Let us consider a homogeneous gas sphere with a surface that reflects acoustic waves. Some fraction of the fluid inside this sphere is assumed to be in the state of turbulence which acts as a source of sound waves. Following Lighthill (1952) we write the perturbed mass and momentum equations as

$$\rho_1 + \nabla \cdot (\rho \xi) = 0, \quad (1)$$

and

$$\frac{\partial^2 \rho \xi_i}{\partial t^2} + c^2 \frac{\partial \rho_1}{\partial x_i} = -\frac{\partial T_{ij}}{\partial x_j}, \tag{2}$$

where  $c$  and  $\rho$  are unperturbed mean sound speed and density of the medium,  $\xi$  is fluid displacement, and

$$T_{ij} \equiv \rho v_i v_j + p \delta_{ij} - \rho c^2 \delta_{ij}. \tag{3}$$

These equations can be combined to yield the following inhomogeneous wave equation

$$\frac{\partial^2 \rho \xi_i}{\partial t^2} - c^2 \nabla^2 (\rho \xi_i) = -\frac{\partial T_{ij}}{\partial x_j}, \tag{4}$$

Expanding  $\xi$  in the terms of normal modes of the system

$$\xi = \frac{1}{\sqrt{2}} \sum_q A_q \xi_q \exp(-i\omega t) + c.c., \tag{5}$$

where  $\xi_q$  is displacement eigenfunction of mode  $q$  which is normalized to unit energy *i.e.*

$$\omega_q^2 \int d^3x \rho \xi_q \cdot \xi_q^* = 1, \tag{6}$$

and substituting this expansion into equation (4) we find the following equation for the mode amplitude  $A_q$

$$\frac{dA_q}{dt} \approx -\frac{i\omega_q}{\sqrt{2}} \exp(i\omega_q t) \int d^3x \xi_{qi} \frac{\partial T_{ij}}{\partial x_j} = \frac{i\omega_q}{\sqrt{2}} \exp(i\omega_q t) \int d^3x \frac{\partial \xi_{qi}}{\partial x_j} T_{ij}. \tag{7}$$

Turbulent flow is crudely described as consisting of critically damped eddies. The velocity  $v_h$  of an eddy of size  $h$  is related to the largest or the energy bearing eddy (size  $H$  and velocity  $v_H$ ) by the Kolmogorov scaling *i.e.*

$$v_h = v_H \left( \frac{h}{H} \right)^{1/3}. \tag{8}$$

Moreover, following Lighthill (1952), we take  $T_{ij} \approx \rho v^2 \delta_{ij}$ . Since the displacement eigenfunction, for low  $\ell$  modes, near the surface of the sphere is in the radial direction, therefore equation (7) reduces to

$$\frac{dA_q}{dt} \approx \frac{i\omega_q}{\sqrt{2}} \exp(i\omega_q t) \int d^3x \rho v^2 \frac{\partial \xi_{qr}}{\partial r}. \tag{9}$$

where  $\xi_{qr}$  is the radial displacement eigenfunction of mode  $q$ . The mean energy input rate in mode  $q$  can be obtained from the above equation and is given by

$$\frac{dE_q}{dt} \equiv \frac{d\langle |A_q|^2 \rangle}{dt} \approx 2\pi\omega_q^2 \int dr r^2 \rho^2 v_\omega^3 h_\omega^4 \left[ \frac{\partial \xi_{qr}}{\partial r} \right]^2, \quad (10)$$

where  $v_\omega$  and  $h_\omega$  are the velocity and size of the eddies which have characteristic time,  $\tau_h \equiv h_\omega/v_\omega$ , approximately equal to the mode period. This equation is valid not only for the homogeneous gas sphere considered here but also for more general systems including the excitation of solar p-modes by the Reynolds stress as discussed below. Of course, we must use the eigenfunction  $\xi_q$  and turbulent velocity appropriate for the system being considered.

It can be easily shown that the solution of the homogeneous wave equation (eq. [4] with right side set equal to zero), in the limit of large  $n$  (mode order) is

$$\xi_{qr} \approx B j_\ell(\omega_q r/c) \approx B \frac{\sin(r\omega_q/c - \pi\ell/2)}{r\omega_q}, \quad (11)$$

where  $j_\ell$  is spherical Bessel function, and  $B$  is a constant factor independent of mode frequency for properly normalized mode eigenfunction (condition expressed by eq. [6]). Substituting this into equation (10) we find

$$\dot{E}_q \equiv \frac{dE_q}{dt} \propto \omega_q^2 h_\omega^4 v_\omega^4. \quad (12)$$

Let us assume that the turbulent velocity field in the sphere is concentrated in a thin layer of thickness  $H$  located near the surface of the sphere. We shall take the size of the largest eddies to be  $H$  and their rms speed to be  $v_H$ . The p-modes of period greater than  $\tau_H = H/v_H$  are predominantly excited by the largest size eddies and the resultant energy input rate in these modes is proportional to  $\omega_q^2$ , as can be seen immediately from the above equation. Modes of higher frequency ( $\omega_q \gtrsim \tau_H^{-1}$ ) couple best to inertial range eddies which have characteristic time of order the wave period. Making use of the Kolmogorov scaling (eq. [8]) and equation (12) we see that  $\dot{E}_q$  scales as  $\omega_q^{-5.5}$ . Thus the energy input rate into p-modes of this homogeneous system shows a break at frequency  $1/\tau_H$  where the power law index changes by 7.5.

The generalization of above equations to describe the excitation of solar modes is not difficult. Equation (2) is replaced by the linearized momentum equation valid for a stratified medium i.e.

$$\rho \frac{\partial^2 \xi}{\partial t^2} + \nabla p_1 - \rho_1 g = -\nabla \cdot (\rho v v) \equiv F, \quad (13)$$

and the linearized equation of state is

$$p_1 = \frac{\partial p}{\partial \rho} \rho_1 + \frac{\partial p}{\partial s} s_1, \tag{14}$$

where

$$s_1 = \tilde{s} - (\boldsymbol{\xi} \cdot \nabla) s. \tag{15}$$

Here  $\nabla s$  denotes the background entropy gradient, and  $\tilde{s}$  is the entropy fluctuation associated with turbulent convection. Equation (15) is the Eulerian version of the statement that the Lagrangian entropy perturbation is due entirely to turbulent convection. In other words, we approximate the waves as adiabatic (these equations are adopted directly from Goldreich et al. 1994). Combining equations (1) and (13)-(15) we obtain the following inhomogeneous wave equation, which is the generalization of equation (4) and describes the stochastic excitation of solar oscillations:

$$\begin{aligned} \rho \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} - \nabla \left[ c^2 \nabla \cdot (\rho \boldsymbol{\xi}) + \rho \boldsymbol{\xi} \cdot \mathbf{g} - c^2 \rho \boldsymbol{\xi} \cdot \nabla \ln \rho \right] + \mathbf{g} \nabla \cdot (\rho \boldsymbol{\xi}) \\ = -\nabla \left( \frac{\partial p}{\partial s} \tilde{s} \right) - \nabla \cdot (\rho \mathbf{v} \mathbf{v}). \end{aligned} \tag{16}$$

This equation describes wave generation due to Reynolds stress as well as entropy fluctuation. As the entropy of a fluid element fluctuates, so does its volume. The fluctuating volume is a monopole source for acoustic waves. In a stratified medium the fluctuating buoyancy force adds a dipole source. By transferring momentum among neighboring fluid elements, the Reynolds stress acts as a quadrupole source.<sup>2</sup> The anisotropy of a stratified medium blurs the distinction between monopole, dipole, and quadrupole sources. It allows for destructive interference between the monopole and dipole amplitudes. Although the monopole and dipole amplitudes are individually larger than the quadrupole amplitude, their sum is of comparable size to that of the quadrupole. That this applies to energy bearing eddies follows directly from equations (16) and the relation between entropy and velocity fluctuation for convective eddies. The justification for inertial range eddies requires a subtle argument (cf. Goldreich and Kumar, 1990).

The new energy equation (which replaces eq. [10]) is given below

$$\dot{E}_\alpha \sim 2\pi\omega_\alpha^2 \int dr r^2 \rho^2 \left| \frac{\partial \xi_\alpha^r}{\partial r} \right|^2 v_\omega^3 h_\omega^4 (C_\alpha^2 \mathcal{R}^2 + 1) \mathcal{S}^2, \tag{17}$$

<sup>2</sup>We classify acoustic sources as monopole, dipole, or quadrupole according to whether they produce a change in volume, add net momentum, or merely redistribute momentum.

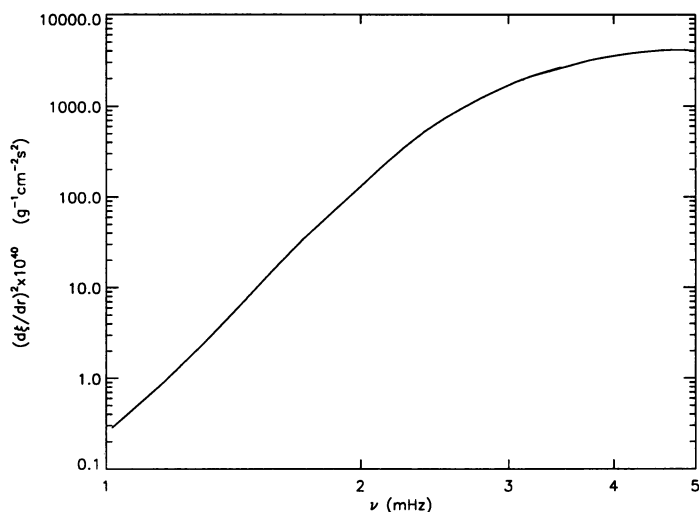


Figure 1. The plot of  $(d\xi_r/dr)^2$ , at the top of the solar convection zone, as a function of p-mode frequency (the mode degree is zero). The solar model used here is due to J. Christensen-Dalsgaard.

where  $C_\alpha$  is wave compressibility defined by

$$\nabla \cdot \xi_\alpha = C_\alpha \frac{\partial \xi_\alpha^r}{\partial r}, \quad (18)$$

the shape parameter  $\mathcal{S}$  describes the ratio of the horizontal to vertical correlation lengths of turbulent eddies, and  $\mathcal{R}$  is given by

$$\mathcal{R} \equiv \frac{4H}{\Lambda} \left( \frac{\partial \ln p}{\partial \ln \rho} \right)_s, \quad (19)$$

with  $\Lambda$  the mixing length and  $H$  the pressure scale height. The factor  $C_\alpha^2 \mathcal{R}^2$  measures the ratio of the excitation by entropy fluctuations to that due to fluctuating Reynolds stress. Note that the frequency spectra of waves excited due to entropy fluctuation and that due to Reynolds stress are identical except of course for an over all normalization factor. The compressibility,  $C_\alpha$ , for p-modes near the top of the convection zone, where the excitation takes place, is close to 1, and the value of  $\mathcal{R}^2$  in this region is about 10 (see Goldreich et al. 1994). Therefore, the excitation of p-modes is dominated by entropy fluctuations (Stein and Nordlund 1991, arrived at the same conclusion using their numerical simulations of solar convection). On the other hand f-modes are nearly incompressible ( $C_\alpha \approx 0$ ) and so they are not excited by entropy fluctuation. This is perhaps why the power in f-modes is observed to be smaller than a p-mode of similar frequency.

The observed rate of energy input into solar p-modes can now be readily understood. One of the differences between the homogeneous gas sphere system and the sun is in the shape of the eigenfunction especially near the surface where the wave excitation takes place. It can be shown that the radial derivative of the normalized radial displacement eigenfunction for p-modes just below the photosphere scales as  $\nu_q^{3.8}$  for  $\nu_q \lesssim 3.0\text{mHz}$  and as  $\nu_q^{1.1}$  for  $\nu_q \gtrsim 3.5\text{mHz}$  (see Figure 1). Substituting this scaling in equation (17), or equation (10), we find that the energy input rate in the p-modes, at a fixed degree, scales as  $\nu_q^7$  for  $\nu_q \lesssim 3.0\text{mHz}$  and as  $\nu_q^{-4.4}$  for  $\nu_q \gtrsim 3.5\text{mHz}$  which is in good agreement with the observations (Libbrecht and Woodard 1991); please see Goldreich et al. (1994) for a more detailed analysis and comparison with the observed energy input rate.

### 3. Observational constraints for the excitation theory of solar p-modes

A valid theory for the excitation of solar p-modes should be able to explain the observed rate of energy input in different modes. In addition, there are four other observational results that the theory must be able to explain and provide a fit to the data. These observations are: mode linewidth, the deviation of p-mode line profiles from symmetric Lorentzian shape, the statistics for the fluctuation of mode energy, and the presence of peaks in the power spectrum above the acoustic cutoff frequency ( $\nu \gtrsim 5.3\text{mHz}$ ).

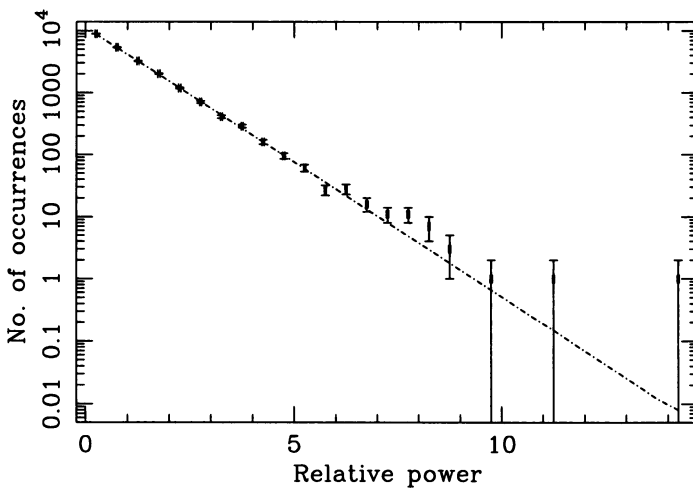
The agreement between the observed and the theoretically calculated energy input rate in solar p-modes due to stochastic excitation was described in the last section. We describe below the other four observations and compare them with the results of the stochastic excitation theory.

#### 3.1. MODE LINEWIDTH

A number of groups have measured p-mode linewidth as a function of mode frequency (cf. Duvall et al. 1988, Libbrecht 1988, Elsworth et al. 1990). It is found that the mode linewidth at a fixed degree increases monotonically with mode frequency. At 2 mHz the linewidth is about  $0.5 \mu\text{Hz}$ , or mode lifetime is 20 days ( $Q \approx 4 \times 10^3$ ) and at 4 mHz the linewidth is  $10 \mu\text{Hz}$ . The observed linewidth for  $2\text{mHz} \lesssim \nu \lesssim 3\text{mHz}$  increases as  $\nu^{4.2}$  whereas numerical calculations show that the mode linewidth due to radiative and turbulent damping increases as  $\nu^8$  for frequencies below  $\sim 4\text{mHz}$  (cf. Christensen-Dalsgaard and Frandsen 1983, Balmforth 1992; Goldreich and Kumar 1991, give a simple analytical derivation of these results). This suggests that the mode damping at frequencies greater than about 2 mHz is due to some process other than the radiative and turbulent viscosity. A number of alternate mechanisms have been suggested to account for

the observed mode linewidth. These include modulation of convective flux (Christensen-Dalsgaard et al. 1989), scattering of low degree p-modes by turbulent convection to high degree modes (Goldreich and Murray 1994), scattering of p-modes by magnetic fields (Bogdan et al. 1996). Goldreich and Murray (1994) have carried out a detailed calculation of the scattering process and find that an almost elastic scattering of p-modes by convective eddies is an important contributor to the mode linewidth at frequencies  $\nu \gtrsim 2\text{mHz}$ , and the computed linewidth has the same frequency dependence as the observed width (see also Murray, 1993).

Recent observational results indicate that the linewidth scales as  $\nu^7$  for  $\nu \lesssim 2\text{mHz}$  (Chaplin et al. 1996, and Tomczyk 1996). Perhaps below 2 mHz there are few modes available for p-modes to scatter into, and thus the linewidth falls off more rapidly with decreasing frequency. According to Jefferies (personal communication) the observed mode linewidth peaks at a frequency of about 5mHz, followed by a slight decline, and then remains constant at higher frequencies. This is a puzzling result for which as far as I know no explanation has been offered.



*Figure 2.* The statistics of power fluctuation in low degree p-modes. The straight line is the exponential distribution, which is the theoretically expected distribution if modes are stochastically excited due to turbulent convection. The data is kindly provided by BiSON (please see Chaplin et al. 1995 for details).



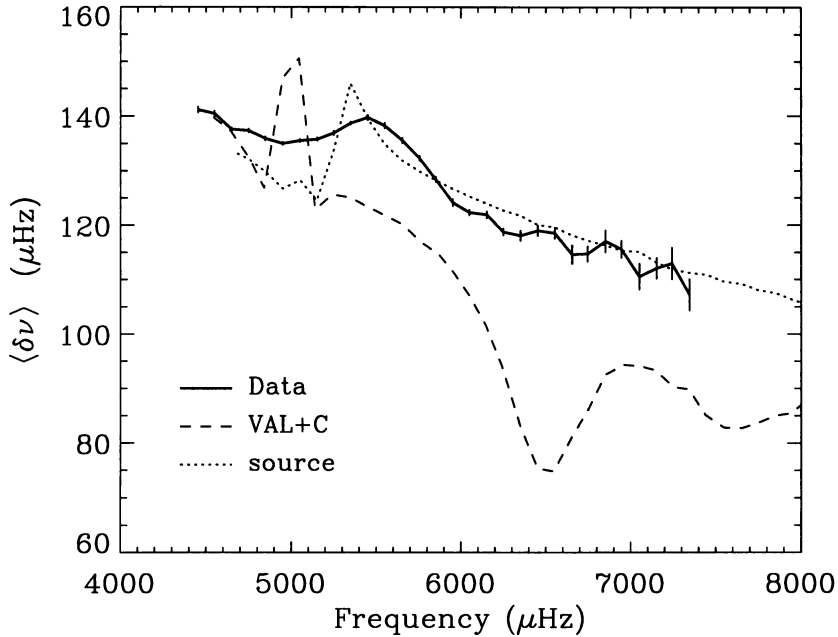
### 3.2. ENERGY STATISTICS

Modes excited by their interaction with a Gaussian random field (turbulent convection) have fluctuating amplitudes that follows the Gaussian distribution. The correlation time for mode amplitude, which is of order the mode lifetime, is typically much larger than the mode period (see §3.1) or the characteristic time of resonant eddies. This is because a mode interacts with a large number of eddies each of which contribute only a small fraction of the total energy in the mode. A good analogy is a pendulum placed in contact with a thermal heat bath of molecules. The mean energy in the pendulum is one third the mean kinetic energy of molecules, however it takes a large number of collisions (of order the ratio of the pendulum mass to molecular mass) in order for the amplitude of the pendulum to change. The statistics of energy fluctuation in a solar p-mode, if stochastically excited, like the energy of the pendulum placed in a heat bath, follows Boltzmann distribution (see Kumar et al. 1988 for a rigorous derivation of this result). At least two different groups (Toutain and Frohlich 1992; and Elsworth et al. 1995) have looked for the statistics of energy fluctuation in the solar p-modes and find it to be in good agreement with the theoretical expectation for stochastic excitation i.e. Boltzmann or exponential distribution (see fig. 2).

### 3.3. PEAKS AT HIGH FREQUENCIES

Acoustic waves of frequency less than about 5 mHz (the acoustic cutoff frequency at the temperature minimum) are reflected at the solar photosphere and thus trapped inside the sun. The reflectivity however drops off rapidly at higher frequencies; at 6mHz about 2% of the incident wave energy is reflected at the photosphere whereas at 7mHz the reflectivity drops to less than 0.3%. A number of observations indicate that high frequency acoustic waves (waves of frequency greater than about 5mHz) suffer little reflection at the chromosphere/corona as well (Duvall et al. 1993, Kumar et al. 1994, Jefferies 1996). If high frequency acoustic waves were significantly reflected at the chromosphere/corona boundary then the frequency spacing between modes of adjacent order would fluctuate with mode frequency (see figure 3); this is because of the interference of waves partially trapped between two cavities above and below the temperature minimum. The observations, however, show, no evidence for such a behavior (fig. 3) and thus provide an upper limit of about 10% to the reflection at the chromosphere/corona boundary (Kumar et al. 1991; Jefferies, personal communication).

In the absence of wave reflection at the solar surface these high frequency acoustic waves are not trapped in the sun, and thus it was expected that the power spectrum above the acoustic cutoff frequency should be featureless



*Figure 3.* Average frequency spacing between adjacent peaks in the power spectrum,  $\langle \delta\nu \rangle$ , as a function of frequency. The averaging over frequency bins of width  $100 \mu\text{Hz}$ , and  $\ell$  range of 80 and 150 has been carried out after subtracting a linear term in  $\ell$  ( $0.6981 \ell \mu\text{Hz}$ ) from  $(\nu_{n+1,\ell} - \nu_{n,\ell})$ . The observational data (thick solid curve) was obtained by Duvall et al. (1993) at the geographical South Pole in 1988. The dotted curve, labeled 'source' in the legend, is the result of calculation of peak frequencies in the theoretically computed power spectra for Christensen-Dalsgaard's solar model with sources lying about 140 km below the photosphere. The dashed curve labeled 'VAL+C' is the frequency spacing calculated for JC-D solar model that includes the "mean quiet sun" chromospheric structure of Vernazza et al. (1981) as well as an isothermal corona at a temperature of  $10^6 \text{ K}$ .

i.e. devoid of peaks. However, the observed spectra contain very regular peaks that are seen upto the Nyquist frequency of observations. One of the best recent data set obtained at the South Pole in 1994 shows peaks extending to almost 11 mHz which makes the length of the spectrum above the acoustic cutoff frequency larger than the observed spectrum below the cutoff frequency!

The existence of these high frequency peaks provides one of the strongest evidence that solar acoustic oscillations above 5 mHz are not excited by some overstability mechanism<sup>3</sup>, and since power spectrum varies smoothly

<sup>3</sup>Considering the poor reflectivity of high frequency waves at the chromosphere/corona, the energy flux in the solar atmosphere associated with them represents a net loss of their energy. So if these waves are to be excited due to an overstability

from below the acoustic cutoff frequency to above the cutoff frequency we infer that the trapped p-modes in the sun are also not overstable (Kumar et al. 1989).

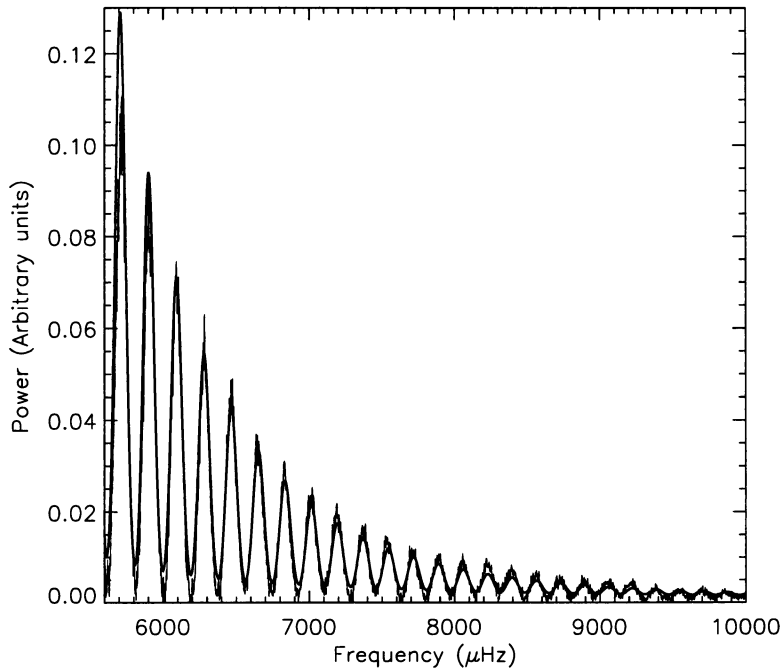
The peaks at high frequencies can be understood as arising quite naturally if waves are stochastically excited. These peaks form because of the constructive interference between waves propagating from the source (located in the convection zone) upward to the photosphere and waves traveling downward from the source that is refracted back up due to increasing sound speed and thus end up at the photosphere (Kumar and Lu 1991). Therefore the frequencies of peaks above the acoustic cutoff ( $\sim 5$  mHz for the sun) depend on the difference between these two paths or in other words on the depth of acoustic sources. A good fit to the high frequency power spectrum is obtained by placing sources (assumed to be quadrupole) approximately 140 km below the photosphere (Kumar 1994); please see figure 4. It should be emphasized that unlike the lower frequency trapped p-modes (frequency less than about 5 mHz) the frequencies of peaks at high frequencies is not a property of the equilibrium model of the sun alone but depends in a sensitive way on the location of sources that excite these oscillations.

As discussed in Kumar (1994) if the acoustic sources are assumed to be dipolar instead, then no matter where these sources are placed in the solar convection zone they do not provide a fit to the observed power spectrum. This suggests that the acoustic sources, at least for the high frequency waves, are not dipole but quadrupole, which is consistent with the work of Goldreich et al. (1994).

High frequency acoustic waves also provide information about the power spectrum of turbulent convection in the sun (Kumar 1994); we can constrain the spectrum of turbulent convection in the region where acoustic emission is significant. The theoretical power spectrum shown in figure 4 was computed using the Kolmogorov power spectrum of turbulence, *i.e.*,  $P(k) \propto k^{-5/3}$ . Evidently, this provides a good fit to the observed spectrum between 5.5 and 10 mHz. In order to determine the power law index  $\alpha$  of solar turbulence,  $P(k) \propto k^{-\alpha}$ , from the high frequency interference peaks, we relate the fluctuating velocity,  $v_h$ , of sub-energy bearing eddies to the velocity  $v_H$  of scale-height size eddies as follows:

$$v_h \approx v_H \left( \frac{h}{H} \right)^{(\alpha-1)/2},$$

mechanism, their  $e$ -folding time must be less than about an hour, and thus these waves can at best be amplified by a factor of  $\sim e$  as they make one passage through the solar interior. Thus we need a mechanism that provides a large seed amplitude, within a factor of  $e$  of the observed value, and clearly in this case it seems most natural that the same mechanism generates the full observed amplitude.



*Figure 4.* Observed power spectrum (thin solid line) from the 1994 South Pole observation (courtesy of S. Jefferies) for  $\ell=90$ , and theoretically calculated power spectrum for sources lying 140km below (thick solid line) the photosphere. The Nyquist frequency of the data is 11.9 mHz. Both the  $\ell$ -leakage and the Nyquist folding has been included in the theoretically computed spectrum; to model the  $\ell$ -leakage the theoretically calculated power spectra for  $\ell=88$  to 92 were added together with weighting factors of 0.147, 0.68, 1.0, 0.61, and 0.10 respectively which corresponds to the 1994 South Pole observations (Jefferies, personal communication). The radial extent of the sources is taken to be 50km, and the spectrum of turbulent convection is Kolmogorov. A frequency dependent background has been subtracted from the observed spectrum.

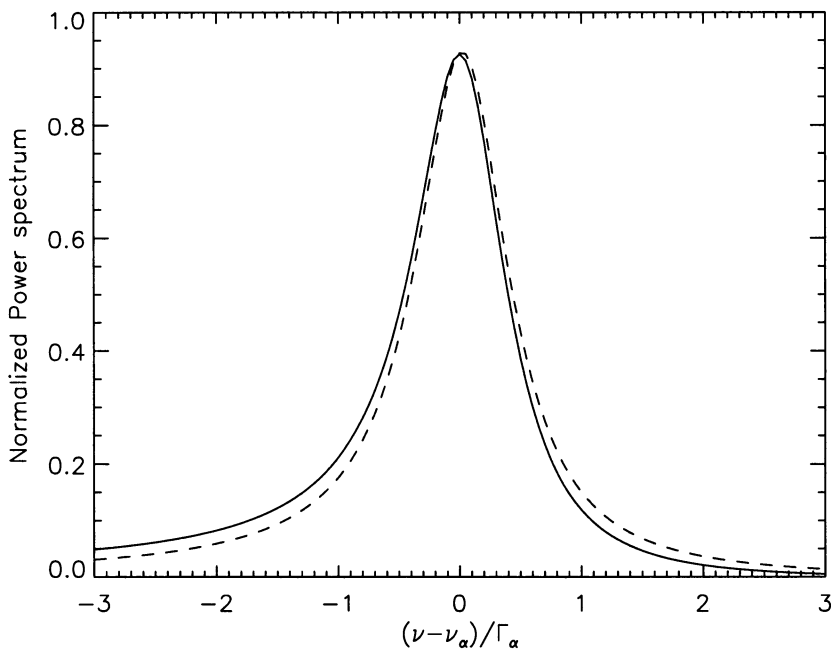
where  $h$  is the size of the eddy. This equation is a generalization of equation (8). Using this relation, we find that the frequency dependence of the source function is  $(\omega\tau_H)^{-(3\alpha+5)/(3-\alpha)}$  (the derivation is similar to the one leading to eq. [12]). Therefore, a change in the spectral index of turbulence from  $5/3$  to  $1.4$  decreases the dependence of the acoustic power spectrum on frequency by  $\omega^{1.75}$ , which results in a poor fit to the observed spectrum. We find that the observed high frequency power spectra suggest that the power law index for the solar turbulence lies between  $1.5$  and  $1.7$ .

We note that the energy input rate for p-modes in the frequency range between  $3.5$  and  $5$ mHz is proportional to  $\omega^{-4.4}$ , which is understood most naturally if the spectrum of turbulence near the top of the solar convection zone is taken to be Kolmogorov (Goldreich et al. 1993). The result described above extends this range to  $10$ mHz.

## 3.4. ASYMMETRIC LINE PROFILES OF P-MODES

The last observational result I would like to describe is the asymmetry of low frequency p-mode line profiles. The spectrum of individual p-modes is fitted very well by a Lorentzian profile. However, Duvall et al. (1993) discovered that the line profiles do not have perfect Lorentzian shape and in particular the power spectrum falls off more rapidly on one side of the peak than the other i.e. the lines are asymmetrical. The data from GONG and SOHO clearly show that line-profiles for low frequency modes are asymmetrical. Duvall et al. had also proposed in their original paper an explanation for why the lines are asymmetrical which is found to be basically correct by a number of independent investigations (Gabriel 1992, 1995; Abrams and Kumar 1996). The line profile for a p-mode of frequency 1.9 MHz and  $\ell = 1$ , calculated using JC-D solar model, is shown in figure 5.

The physical explanation for line asymmetry is simplest when sources lie in the region of the sun where acoustic waves can propagate (this case

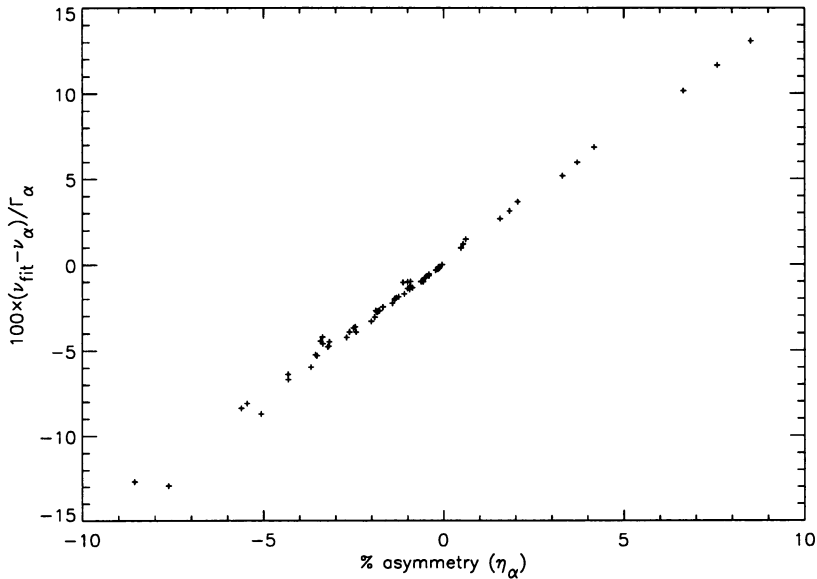


*Figure 5.* Line profiles of a p-mode of frequency 1.9 MHz and  $\ell = 1$  of a solar model due to J. Christensen-Dalsgaard. The power spectrum plotted using a continuous line corresponds to sources placed at the upper turning point of the mode. The other power spectrum, dashed curve, arises when sources are placed 300 km below the upper turning point. The radial extent of sources in both cases was taken to be 100 km.

does not seem to apply to solar oscillations however which are excited by sources that lie in the evanescent region). Consider a source lying close to the node of a p-mode. As the frequency of acoustic waves is varied in the neighborhood of this p-mode frequency the position of the node changes with respect to the source position. Thus waves of frequencies lying symmetrically on the opposite side of the p-mode frequency gets excited to different amplitudes making the resultant power spectrum asymmetrical. It is clear from this rather simple example that not all p-mode line profiles are expected to be equally asymmetrical (as is observed) and also that the degree of asymmetry depends on the location of sources. In fact Duvall et al (1993) had recognized this in their original paper and used this to determine the depth of sources that are exciting p-modes. The recent paper of Abrams and Kumar (1996) uses a realistic solar model due to Christensen-Dalsgaard to calculate the p-mode power spectrum and finds that in order to reproduce the magnitude of asymmetry observed by Duvall et al. (1993) the sources responsible for exciting low frequency p-modes should lie about 250 km below the photosphere. This might appear to be in conflict with the result obtained using high frequency solar oscillations described in §3. However, the result is in agreement with the theory of stochastic excitation which predicts that lower frequency oscillations are excited deeper in the convection zone where the characteristic eddy time is longer.

Line asymmetry causes a slight error to the observational determination of p-mode frequencies which are obtained by fitting a Lorentzian function to the power spectra. Abrams and Kumar (1996) find that this frequency error is proportional to the product of mode linewidth and a nondimensional measure of line asymmetry  $\eta_\alpha$  (see the figure below). The parameter  $\eta_\alpha$  is obtained by decomposing the observed power spectrum in the neighborhood of a peak corresponding to a mode  $\alpha$  into even and odd functions. Since the odd function is zero at the peak (by definition) and again far from the peak, its magnitude has a maximum at some intermediate distance from the peak, typically less than one linewidth. The ratio of the maximum magnitude of the odd function to the maximum magnitude of the even function is a dimensionless measure of the asymmetry which we denote by  $\eta_\alpha/100$  i.e.  $\eta_\alpha$  is the percentage line asymmetry of mode  $\alpha$ . The sign of  $\eta_\alpha$  is taken to be positive or negative according to whether there is more power on the high- or low-frequency side of the peak, respectively.

There is one feature of the observed line asymmetry that is very puzzling and for which there is no theoretical explanation. Duvall et al. (1993) reported that the sense of asymmetry reverses in the velocity and the intensity power spectra for the p-mode. This behavior has been confirmed by the most recent GONG data. The difference between the velocity and the intensity power spectra can arise as a result of line formation in the



*Figure 6.* Error in the measurement of mode frequency (expressed as percentages of corresponding linewidths) as a result of asymmetry of lines in the power spectrum; *fit* is the frequency obtained by fitting a Lorentzian function to the power spectrum,  $\nu_\alpha$  is the mode eigenfrequency, and  $\Gamma_\alpha$  is mode linewidth. The frequency error is shown as a function of a dimensionless measure of line asymmetry ( $\eta_\alpha$ ) defined in the text. The slope of the line is approximately 1.5. The power spectra were calculated by solving an inhomogeneous wave equation which included radiative damping of waves (see Abrams and Kumar 1996, for details). The solar model used in this calculation was kindly provided by J. Christensen-Dalsgaard.

presence of oscillations. However, it is not clear what process can cause a reversal of the sign of  $\eta_\alpha$  in the two spectra; the process has to be extremely frequency sensitive so that it can modify the spectrum in an interval of only a few  $\mu\text{Hz}$ .

#### 4. Can we detect gravity modes in the sun?

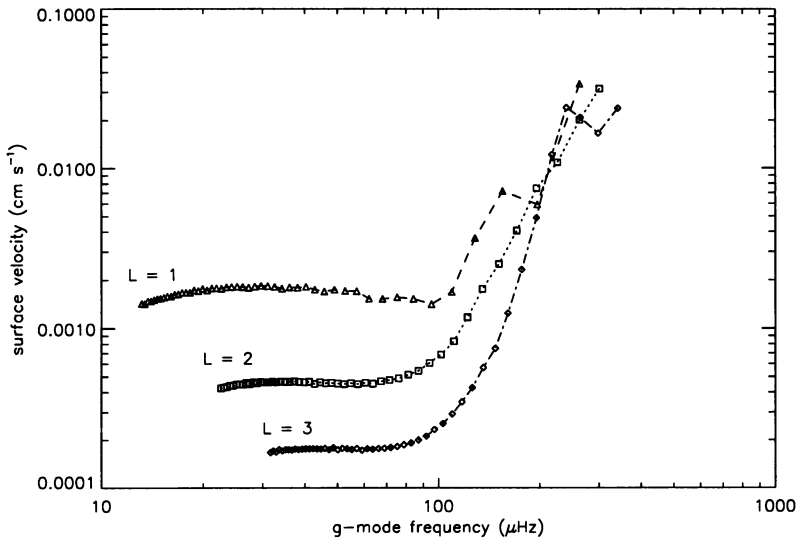
Gravity mode oscillations of the sun are primarily confined to its radiative interior and their observation would thus provide a wealth of information about the energy generating region which is poorly probed by the p-modes. In the past 20 years a number of different groups have claimed to detect g-modes in the sun (e.g. Brookes et al. 1976; Brown et al. 1978; Delache and Scherrer 1983; Scherrer et al. 1979; Severny et al. 1976, Thomson et al. 1995; for a detailed review of the observations please see the article by Pallé, 1991, and references therein), but thusfar there is no consensus that g-modes have in fact been observed. One of the objectives of the instruments

aboard SOHO (VIRGO, GOLF and SOI) is to search for solar g-modes. So it should be helpful to estimate the expected surface velocity amplitudes of g-modes in the sun, providing observations and data analysis programs a rough target number to shoot for. The first question we need to address in this respect is whether solar g-modes are self-excited (overstable) or not. This is dealt with in the following paragraph.

A number of people have investigated the linear stability of solar g-modes (e.g. Dilke and Gough 1972; Rosenbluth and Bahcall 1973; Christensen-Dalsgaard et al. 1974; Shibahashi et al. 1975; Bouchy et al. 1975; Saio 1980). All of these investigations find that g-modes of radial-order ( $n$ ) greater than 3 are stable. However, there is no general agreement about the stability of low order modes ( $n \leq 3$ ). If overstable, the g-mode amplitude will increase exponentially with time until nonlinear effects become important and saturate their growth. Kumar and Goodman (1995) have recently investigated 3-mode parametric interaction, a very efficient nonlinear process. Using their results we find that the low order overstable g-modes in the sun will attain an energy of at least  $10^{37}$  erg before they are limited by nonlinearities. The velocity at the solar surface corresponding to this energy is  $\sim 10^2$  cm s $^{-1}$ , which is an order of magnitude larger than the observational limit of Pallé (1991). Thus even low order g-modes of frequency greater than about 150  $\mu$ Hz are unlikely to be overstable.

However, g-modes can be stochastically excited. A number of people have estimated g-mode amplitude assuming that they are linearly stable and stochastically excited. Keeley (1980) applied his theory, developed with Goldreich in 1977, of the excitation of solar modes to estimate the amplitude of the 160 minute oscillation, and found the theoretical amplitude to be much smaller than claimed by the observations; much more sensitive observational searches since then have not detected this oscillation (cf. Pallé 1991). Gough (1985) carried out an application of the energy partition result of Goldreich and Keeley (1977) to solar g-modes and estimated the surface velocity amplitude of low  $n$  and  $\ell$  g-modes ( $n \leq 3$ ,  $\ell \leq 2$ ) to be about 1-2 mm s $^{-1}$ . Kumar et al. (1996) estimated the g-mode amplitude using the recent theoretical work of Goldreich et al. (1994) on stochastic excitation of waves, which reproduces the observed energy input rate into solar p-modes of all frequencies (see §2), and taking into account the radiative and viscous turbulent dampings. They find the surface velocity amplitude of low order g-modes to be about 0.4 mm s $^{-1}$  (see figure 7). Recently Anderson has carried out numerical simulation of g-mode excitation as a result of turbulent flow associated with penetrative convection. He finds that the transverse surface velocity amplitudes of g-modes of degree about 6 is  $\sim 0.2$  mm s $^{-1}$  in the case when he assumes that  $10^3$  modes are excited by this process (Anderson, 1996). Thus several different calculations suggest that the am-





*Figure 7.* Magnitude of the surface velocity amplitude as a function of frequency for low degree solar g-modes excited by coupling with turbulent convection. The surface velocity amplitude falls off rapidly with increasing  $\ell$ , thus only low degree g-modes are expected to be observable.

plitudes for low degree g-modes are of order  $0.5 \text{ mm s}^{-1}$ . The uncertainty in this estimate is at least a factor of a few. If the nature turns out to be cooperative and the actual amplitudes of solar g-modes are a factor of a few larger than these estimates, then instruments aboard SOHO have a good chance of detecting g-modes and opening up a new window in the study of the solar core.

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