

THE DESIGN OF MODERN MERIDIAN CIRCLES FOR THE OBSERVATION OF FAINT OBJECTS

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ABSTRACT

Definitions of Instantaneous Instrumental Coordinates and Local Horizontal Coordinates of the photoelectric meridian circle of Tokyo Astronomical Observatory are given, and a method to determine the positions of stars by using a scanning double slits micrometer are presented. It is found that stars up to 12.2 mag are observable by making a composite photon intensity. Discussions on the application of observations of faint objects by meridian circles are presented.

1. INTRODUCTION

A new photoelectric meridian circle of the Tokyo Astronomical Observatory named as PMC 190 has just started observations for celestial objects including the Sun and other objects in the solar system. A detailed introduction of the system of PMC 190 is given by Yoshizawa and Yasuda (1982) (,see also Kühne (1983)).

PMC 190 has an impersonal photoelectric micrometer, and photons coming from light sources are detected by photon counting system through oscillating V-shaped double slits. The Carlsberg meridian circle at Brorfelde observatory, Denmark, has also a similar photoelectric scanning micrometer (Helmer et al. 1983 and references cited there). A theory of photoelectric multislit micrometer is given by Høg (1970).

The operation of PMC 190 is performed automatically through dual computer (host computer + process computer) control. The most important properties of modern photoelectric automatized meridian circles are (a) high accuracy, (b) high efficiency, and (c) being able to observe faint objects up to, say, 12-th mag. In this article we present a brief description of the coordinate

system of PMC 190 and the application of photoelectric meridian circles to observations of faint objects.

2. LOCAL COORDINATES OF PMC 190

2.1 Definition of the Instantaneous Instrumental Coordinates

Let R_I and R_{II} be, respectively, the readings of the slit position at the instant when peak intensity of the photon count-rates is recorded through slit I and slit II. In figure 1 ξ axis represents the direction of the instantaneous horizontal rotation axis of the instrument, and η axis direction perpendicular to ξ and the optical axis ζ . The direction of the

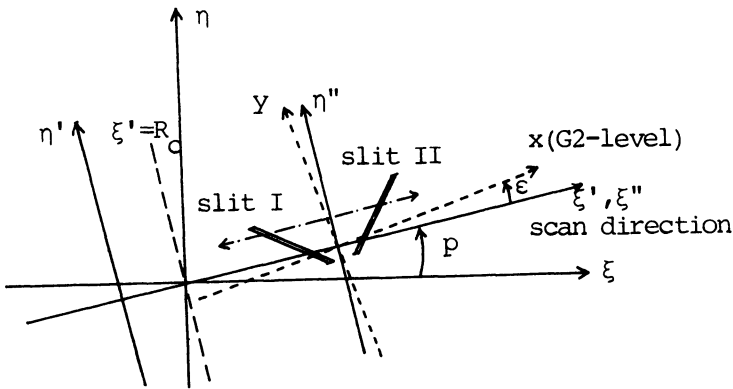


Fig. 1 The relation between the Instantaneous Instrumental Coordinates (ξ, η) and the slit system. Scan direction is the direction along which the slit plate moves, whereas G2-level represents a reference line of V-shaped double slits I and II. R_C stands for the slit position reading at the collimation point.

movement of the oscillating slit plate (ξ') is not parallel to ξ axis and is inclined by a small angle p , due to imperfect setting of the micrometer. Besides, a reference line (G2-level) X of the V-shaped double slits is also inclined to the scan direction by an angle ϵ . Thus the relation between the position of the light source in the instrumental coordinates (ξ, η) and the slit position readings R_I and R_{II} are given by

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = P(-p) \begin{pmatrix} \frac{1}{2}(R_I + R_{II}) - R_C \\ 0 \end{pmatrix}$$

$$+\frac{1}{2} \begin{pmatrix} -\sin(2\epsilon+p) & \sin(\epsilon+p) \\ \cos(2\epsilon+p) & -\cos(\epsilon+p) \end{pmatrix} \begin{pmatrix} R_I - R_{II} \\ a_o \end{pmatrix}, \quad (1)$$

where

$$P(-p) = \begin{pmatrix} \cos p & -\sin p \\ \sin p & \cos p \end{pmatrix},$$

a_o stands for the distance between the slits I and II at G2-level, and R_c for the collimation point.

2.2 Local Horizontal Coordinates

In order to reduce the instantaneous instrumental coordinates to a reference system, we introduce a fiducial coordinate system

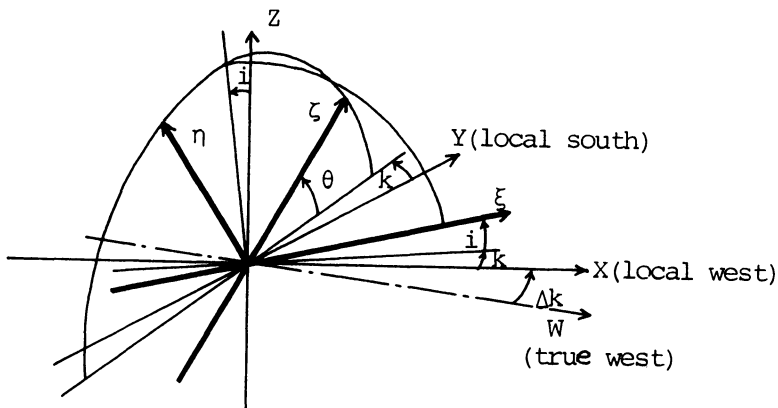


Fig. 2 The relation between the Instantaneous Instrumental Coordinates (ξ, ζ, η) and the Local Horizontal Coordinates (X, Y, Z). Y-axis is directed towards the azimuth mark, whereas X-axis represents the local west, which is shifted from the true west by an angle Δk , the azimuth of the (X, Y, Z) system.

(X, Y, Z). (Figure 2) The X-Y plane is assumed to be parallel to the local horizon; Y axis is directed towards an artificial azimuth mark on the ground about 80 m away from the main telescope, whereas X axis is shifted from the true west by an angle Δk , the azimuth of the (X, Y, Z) coordinate system. We call the (X, Y, Z) system as the Local Horizontal Coordinates; it is considered to be rather stable within a short period of the order of days.

During the observations the instantaneous instrumental coordinates has a level error $i(t)$ and a relative azimuth $k(t)$

with respect to the (X, Y, Z) coordinates. The relation between the positions of a star in coordinates (ξ, ζ, η) and (X, Y, Z) are given by the following equation:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = -R_Z(-k(t))R_Y(i(t))R_X(-\vartheta) \begin{pmatrix} \xi \\ \zeta \\ \eta \end{pmatrix}, \quad (2)$$

where R_Z , R_Y , and R_X represent the rotation matrices around Z , Y , and X axes, respectively, and ϑ the elevation angle of the optical axis above the local horizon.

3. OBSERVATION OF NORMAL BRIGHT STARS

Normally all necessary operations of PMC 190 to observe the stars are executed automatically through dual computer control. The nominal value of the scanning time is two minutes. We can observe one star within three minutes including a necessary operation time like positioning, data analysis, and data storage.

The selection of stars to be observed in the next time is also achieved automatically by host computer through dynamic scheduling over whole program stars according to a most appropriate scheme of selection for reliable catalogue making.

For stars up to 10-th mag the analyses of the observed photon countrates are made by applying the trimmed median method (cf. Høg, 1970) to individual slit passages of the star. Then the position of the slit plate is determined at the instant when peak intensity is detected. Knowing the value of collimation point R_C we can finally obtain the position of the star in the instantaneous instrumental coordinates.

In order to get positions of stars in the local horizontal coordinates, we have to determine the level error, relative azimuth, and the true elevation angle of the optical axis.

3.1 Definitions of Instrumental Constants

In addition to the usual supplementary equipments like mercury basin, meridian collimators, and azimuth marks, we have in PMC 190 system also a zenith mirror to determine the zenith point and axis collimators to watch the movements of the horizontal rotation axis (see in detail, Yoshizawa and Yasuda, 1982). Having those new accessories, we can define a more complete set of instrumental constants of a meridian circle than the one adopted in the conventional instruments (cf. Podobed, 1965).

Now we define collimation point, flexure, zero (fiducial) point of the circle, relative azimuth, and level error as follows:

a) Collimation point:

$$R_{C,\psi} = R_{OO} + g(\psi) \quad , \quad (3)$$

where

$$g(\psi) = a'\cos\psi + b'\sin\psi + A'\cos2\psi + B'\sin2\psi$$

and ψ is the zenith distance of the optical axis. The vertical collimation point, $R_{C,V}$, and the horizontal collimation point, $R_{C,H}$, are given by

$$R_{C,V} = R_{OO} + A' \quad \text{and} \quad R_{C,H} = R_{OO} - A' \quad . \quad (4)$$

b) Flexure:

$$f_{\psi} = a \cos\psi + b \sin\psi \quad ; \quad (5)$$

a and b are designated as vertical flexure and horizontal flexure, respectively.

c) Zero point of the circle readings :

$$z_o = C_{\psi} - f_{\psi} - \psi \quad , \quad (6)$$

where C_{ψ} is the value of the circle reading corrected for division errors.

d) Relative azimuth:

$$k_{\psi} = \bar{k} + AC_{\psi,H} \quad , \quad (7)$$

where

$$\begin{aligned} AC_{\psi,H} &= \text{the value of the axis collimator reading in the horizontal direction corrected for the displacement of the origin, } x_o, \text{ and the tilting, } r, \text{ of the half-reflection mirror} \\ &= \frac{1}{2}(AC_{\psi,H}^W + AC_{\psi,H}^E) - x_o - r \sin(\psi_o + \psi). \end{aligned}$$

e) Level error:

$$i_{\psi} = \bar{i} + AC_{\psi,V} \quad , \quad (8)$$

where

$$AC_{\psi,V} = \frac{1}{2}(AC_{\psi,V}^W - AC_{\psi,V}^E) - y_o - r \cos(\psi_o + \psi).$$

Constants appearing in equations (3) to (8) must be determined by measurements against nadir point and zenith point, meridian collimators, and azimuth marks. Although we can not determine individual constant separately from those measurements, pseud-constant constructed by combining two or more constants can be expressed by measurable quantities alone.

3.2 Position of stars in Local Horizontal Coordinate System

The position of a star in the local horizontal coordinates is given by equations (1) and (2). Assuming that $|i(t)| \leq 20''$ and $|k(t)| \leq 20''$, we obtain within the accuracy of $0''.001$ the following equation

$$\begin{aligned}
 X &= -[\xi + \{ i(t)\cos\psi + k(t)\sin\psi \}] \\
 &= -[\left[\frac{1}{2}(R_I + R_{II}) - \frac{1}{2}(R'_{C,V} + R'_{C,H}) - \frac{1}{2}(R'_{C,V} - R'_{C,H})\cos 2\psi \right] \cos p \\
 &\quad - \frac{1}{2}(R_I - R_{II})\sin(2\epsilon + p) + \frac{1}{2}a_0 \sin(\epsilon + p) \\
 &\quad + \{ L' + \frac{1}{2}(AC_V^W - AC_V^E) \} \cos\psi + \{ K' + \frac{1}{2}(AC_H^W + AC_H^E) \} \sin\psi \\
 &\quad \quad \quad - B' \cos p \sin 2\psi] , \quad (9)
 \end{aligned}$$

where $R'_{C,V} \equiv R_{C,V} + r \cos\psi_0$, $R'_{C,H} \equiv R_{C,H} + r \cos\psi_0$,

$L' \equiv \bar{l} - a' - y_0$, and $K' \equiv \bar{k} - b' - \frac{x_0}{\sin\beta}$ are pseud-

constants and are all measurable quantities except B' . The details of the derivations of those quantities will be given in a separate paper.

As for the zenith distance of the star, ψ_*' we get the relation

$$\begin{aligned}
 \psi_*' &= \psi + \eta + \Delta R \\
 &= C + \Delta C - z'_0 - (f'_V \cos\psi + f'_H \sin\psi) + \Delta R \\
 &\quad + \left[\frac{1}{2}(R_I + R_{II}) - \frac{1}{2}(R'_{C,V} + R'_{C,H}) - \frac{1}{2}(R'_{C,V} - R'_{C,H})\cos 2\psi \right] \sin p \\
 &\quad + \frac{1}{2}(R_I - R_{II})\cos(2\epsilon + p) - \frac{1}{2}a_0 \cos(\epsilon + p) - B' \sin p \sin 2\psi , \quad (10)
 \end{aligned}$$

where

ΔC = division correction to the circle reading,
 ΔR = correction to the astronomical refraction,
 $f'_V \equiv a + a' \sin p$, $f'_H \equiv b + b' \sin p$, and
 $z'_0 \equiv z_0 - r \cos\psi_0 \sin p$. The values of f'_V , f'_H , and z'_0 are also determined by the measurements.

Having expressions for $X(t)$ and $\psi_*'(t)$, we can finally

determine the transit time of the star through the local meridian, t_m , as the time when X becomes to zero. The zenith distance of the star at the meridian transit, ψ'_m , is given by using the relation

$$\psi'_*(t) = \frac{1}{2}X^2(t)\tan\delta \sin 1'' + \psi'_m \quad . \quad (11)$$

4. OBSERVATION OF FAINT OBJECTS

We show in Table 1 the expected numbers of photon countrates for stars of different magnitude when they are observed by PMC 190. It can be seen from the table that photon noises become serious for stars fainter than, say, 11-th mag. In order to reduce the effect

Table 1. Expected photon countrates for stars

m_V	9	10	11	12	13
i^\dagger	38	15	6	2.5	1

†) i is given in counts/50msec.
 $i_{\text{dark+sky}} \approx 5$ counts/50msec for slits of 7" width and 20" height (, corresponds roughly to 17.5 mag/□").

of the photon noises and to get accurate position of faint stars it is necessary to integrate the photon countrates over some period.

A method to integrate individual photon intensities over several scans is shown in figure 3 schematically. We assume that X position of the center of star image runs according to the formula $X(t) = X_0 + 15\cos\delta t$. The positions of the split plate at the time when X_0 star image passes through the slit I and slit II are given by

$$R_I(t) = X(t) + \left(\Delta H + \frac{1}{2}a_0 \right)$$

and

$$R_{II}(t) = X(t) - \left(\Delta H + \frac{1}{2}a_0 \right)$$

respectively, where ΔH is the height of the star image above G2-level, and a_0 the distance between the slit I and slit II at the G2-level.

Now we consider two different scanning paths IJ and IJ' (cf. figure 3). Due to not exactly synchronized oscillation, the

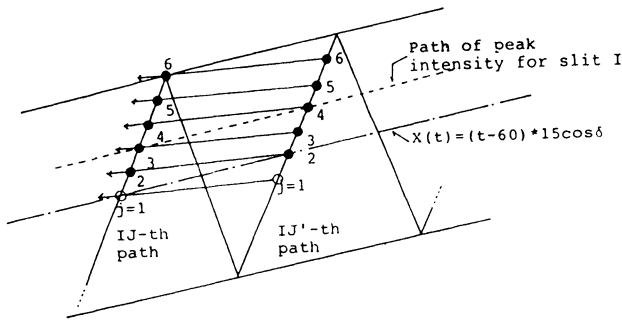


Fig. 3 A schematic diagram to show how a composite photon intensity is constructed. Due to not exactly synchronized oscillations, the recordings of individual photon intensities start at different relative positions to the path of star image.

recordings of individual photon intensities start at different relative positions to the path of star image. After constructing a composite photon intensity, we find, nevertheless, that the mean X position of the slit plate where the integrated photon intensity comes to its peak is just the position that the slit plate should have in individual scans when actual peak intensities are recorded. After all the mean position and height of the path of the star image are given by

$$\begin{aligned} \bar{X}(t) &= \frac{1}{2} (R_I(t) + R_{II}(t)) \\ &= \frac{1}{2} (\bar{R}'_I + \bar{R}'_{II}) + (t - 60) 15\cos\delta \quad , \end{aligned} \tag{12}$$

and

$$\begin{aligned} H &= \frac{1}{2} (R_I(t) - R_{II}(t) - a_0) \\ &= \frac{1}{2} (\bar{R}'_I - \bar{R}'_{II} - a_0) \quad , \end{aligned} \tag{13}$$

where \bar{R}'_I and \bar{R}'_{II} are the mean X positions of the slit plate relative to the oscillation center.

Within a single observation of two minutes duration, about twelve complete oscillations are executed, or 24 scanning paths. Then it is expected that S/N ratio is improved by constructing the composite photon intensity by factor of about 5, or equivalent to the improvement of about 1.7 mag. From our experiences 10.5 mag stars can be observable without any integrations. Thus stars up to 12.2 mag are expected to be observable when composite photon intensities are constructed.

A study on the effect of the photon noises in the multislit micrometer was made by Høg (1970). In figure 4 are shown the expected mean errors of single observation as a function of total observation time (cf. Høg, 1970). In the same figure is also represented a relation between the expected mean errors due to image motions and the observation time given by Høg (1968).

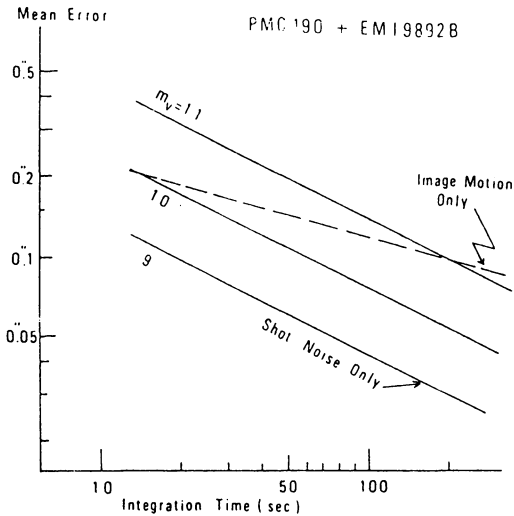


Fig. 4 Expected mean error of star position due to photon noises as a function of total integration time.

From table 1 and figure 4 we understand that the total expected mean error of single observation of two minutes duration for stars fainter than 11-th mag are roughly determined by the effect of photon noises alone.

5. DISCUSSIONS

An interesting application of faint star observations by meridian circle is to determine positions of faint stars that are scattered around extragalactic radio sources. Already we have selected a preliminary list of such radio sources; those radio sources are the most probable objects for which VLBI positions are determined very accurately in near future (cf. Vejt and Gehlich, 1982, and Johnston et al., 1980). Within a few years we shall give positions of faint stars (up to, maybe, 12.5 mag) around individual radio sources in the list.

To observe a minor planet as longer elongation as possible around the opposition is another interesting subject by PMC 190. The importance of minor planets to determine the positions of

equinox and equator are discussed by several authors (,see Duncombe and Hemenway, 1982, and references cited there). The theoretical investigation by Branham (1980) shows that, by using four minor planets, Vesta, Ceres, Eunomia, and Iris, we can determine the equinox correction with a mean error of the order of 0!1. Branham assumed that a minor planet can not be observable if its visual magnitude became fainter than 10. The mean error of determining the equinox correction will be reduced further if we can observe the minor planet until its magnitude being around 12.

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Discussion:

HUGHES: What kind of terrain are you looking over when you look at your meridian marks?
YOSHIZAWA: A grassy plain.