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WARPED PRODUCT SEMI-SLANT SUBMANIFOLDS IN LOCALLY RIEMANNIAN PRODUCT MANIFOLDS

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Abstract

In this paper, we prove that there are no warped product proper semi-slant submanifolds such that the spheric submanifold of a warped product is a proper slant. But we show by means of examples the existence of warped product semi-slant submanifolds such that the totally geodesic submanifold of a warped product is a proper slant submanifold in locally Riemannian product manifolds.

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1. Introduction

The differential geometry of slant submanifolds has shown an increasing development since B.-Y. Chen defined slant immersion in complex geometry as a natural generalization of both holomorphic and totally real immersions [2–6].

In [7], Lotto introduced the notion of slant immersion of a Riemannian manifold into an almost contact metric manifold. Recently, in [12], Li and Li defined and studied the geometry of a semi-slant submanifold in locally Riemannian product manifolds. The class of proper semi-slant submanifolds appears as a particular case of the class of warped product semi-slant submanifolds because the class of proper semi-invariant submanifolds is a particular case of the proper warped product semi-invariant submanifolds.

Let M be an m-dimensional manifold with a tensor of type (1, 1) such that $F^2 = I$ and $F \neq \pm I$. Then M is said to be an almost product manifold with almost product structure F. If an almost product manifold M has a Riemannian metric g such that $g(FX, Y) = g(X, FY)$, for any $X, Y \in \Gamma(TM)$, then M is called an almost Riemannian product manifold. We denote the Levi-Civita connection on M by $\overline{\nabla}$ with respect to g. If $(\overline{\nabla}_X F)Y = 0$, for any $X, Y \in \Gamma(TM)$, then M is called a locally Riemannian product manifold [12].

Let M be a Riemannian manifold with almost Riemannian product structure F and let N be an isometrically immersed submanifold in M. For each $x \in N$, we denote by D_x the maximal invariant subspace of the tangent space T_xN of N. If the dimension of D_x is the same for all x in N, then D_x gives an invariant distribution D on N.

A submanifold N in a locally Riemannian product manifold is called a semi-invariant submanifold if there exists on N a differentiable invariant distribution D whose orthogonal complement $D^\perp$ is an anti-invariant distribution, that is, $F(D^\perp) \subset TN^\perp$. A semi-invariant submanifold is called an anti-invariant (invariant) submanifold if $\dim(D_x) = 0$ ($\dim(D_x^\perp) = 0$). On the other hand, it is called proper semi-invariant if it is neither invariant nor anti-invariant.

A semi-invariant submanifold in the form $N = N_T \times N_\perp$ of a locally Riemannian product manifold M is called a Riemannian product if N_T and $N_\perp$ are totally geodesic submanifolds of N, where N_T is an invariant submanifold and $N_\perp$ is an anti-invariant submanifold of M. The notion of semi-invariance in a locally Riemannian product manifold was introduced in [1, 9, 11].

The above definitions have been generalized as follows.

(1) The submanifold N is called a semi-invariant submanifold if there exists a differentiable distribution $D : x \longrightarrow D_x \subset T_xN$ such that D is invariant and the complementary distribution $D^\perp$ is anti-invariant distribution.

(2) The submanifold N is called a slant submanifold if for each nonzero vector field $X \in \Gamma(TN)$, the angle $\theta(x)$ between FX and T_xN is constant, that is, it does not dependent on of the choice $x \in N$ and $X \in \Gamma(T_xN)$.

(3) The submanifold N is referred to as semi-slant if it has two orthogonal distributions such as D and D' such that D is an invariant distribution and D' is a slant distribution.

It is well known that the notion of warped products plays an important role in differential geometry as well as in physics. For a recent survey on warped products as Riemannian submanifolds, we refer to [4, 5, 8].

Let N_1 and N_2 be two Riemannian manifolds with Riemannian metrics g_1 and g_2, respectively, and f be differentiable function on N_1. Consider the product manifold $N_1 \times N_2$ with its projection $\pi : N_1 \times N_2 \longrightarrow N_1$ and $\eta : N_1 \times N_2 \longrightarrow N_2$. The warped product manifold $N = N_1 \times_{f^2} N_2$ is the manifold $N_1 \times N_2$ equipped with the Riemannian metric structure such that

$$\|X\|^2 = \|\pi_* X\|^2 + f^2(\pi(x))\|\eta_* X\|^2,$$

for any $X \in \Gamma(TN)$. Thus we have $g = g_1 + f^2 g_2$, where f is called the warping function of the warped product. The warped product manifold $N = N_1 \times_{f^2} N_2$ is characterized by the fact that N_1 and N_2 are totally geodesic and spheric foliations of N, respectively. If the warping function is constant, a warped product is said to be the Riemannian product [10].

The purpose of this paper is to investigate a new class of submanifolds of locally Riemannian product manifolds, that is, warped product semi-slant submanifolds.

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We shall focus our attention mainly on warped product semi-slant submanifolds which contain warped product semi-invariant submanifolds and Riemannian product semi-slant submanifolds as a general case.

2. Preliminaries

If N is an isometrically immersed submanifold in a Riemannian manifold M, then the formulas of Gauss and Weingarten for N in M are given, respectively, by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y) \tag{1}$$

and

$$\overline{\nabla}_X V = -A_V X + \nabla^{\perp}_X V, \tag{2}$$

for any $X, Y \in \Gamma(TN)$ and $V \in \Gamma(TN^{\perp})$, where $\overline{\nabla}$ and ∇ denote the Riemannian connections on M and N, respectively, h is the second fundamental form of N in M, $\nabla^{\perp}$ is the normal connection on the normal bundle and A is the shape operator of N in M. The second fundamental form and the shape operator are related by

$$g(A_V X, Y) = g((h(X, Y), V), \tag{3}$$

where g denotes the Riemannian metric on M as well as N. For any a submanifold N of a Riemannian manifold M, Gauss's equation is given by

$$\overline{R}(X, Y)Z = R(X, Y)Z + A_{h(X,Z)}Y - A_{h(Y,Z)}X + (\overline{\nabla}_X h)\,(Y, Z) - (\overline{\nabla}_Y h)\,(X, Z), \tag{4}$$

for any $X, Y, Z \in \Gamma(TN)$, where $\overline{R}$ and R denote the Riemannian curvature tensors of M and N, respectively. The covariant derivative of h is defined by

$$(\overline{\nabla}_X h)\,(Y, Z) = \nabla^{\perp}_X h(Y, Z) - h(\nabla_X Y, Z) - h(\nabla_X Z, Y). \tag{5}$$

We recall the following general lemma from [10] for later use.

LEMMA 2.1. *Let $N = N_1 \times_f N_2$ be a warped product manifold with warping function f. Then:*

(1) $\nabla_X Y \in \Gamma(TN_1)$ *for each* $X, Y \in \Gamma(TN_1)$*;*
(2) $\nabla_X Z = \nabla_Z X = X(\ln f)Z$*, for each* $X \in \Gamma(TN_1)$*,* $Z \in \Gamma(TN_2)$*;*
(3) $\nabla_Z W = \nabla^{N_2}_Z W - g(Z, W)\,((\mathrm{grad} f)/f)$*, for each* $Z, W \in \Gamma(TN_2)$.

Here ∇ and ∇^{N_2} denote the Levi-Civita connections on N and N_2, respectively.

3. Warped product semi-slant submanifolds of a locally Riemannian product manifold

Now, let $N = N_1 \times_f N_2$ be an immersed submanifold of a locally Riemannian product manifold M and denote the orthogonal complementary of $F(TN)$ in $TN^{\perp}$ by V. Then we have the direct sum

$$TN^{\perp} = F(TN) \oplus V. \tag{6}$$

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We can easily see that V is an invariant sub-bundle with respect to F. Furthermore, for any nonzero vector X tangent to N, we put

$$FX = TX + \omega X, \tag{7}$$

where TX and ωX denote the tangential and normal components of FX, respectively. For each nonzero vector X tangent to N at x, the angle $\theta(x)$, $0 \leq \theta(x) \leq (\pi/2)$, between FX and $T_x N$ is called the slant angle. If the slant angle is constant, then the submanifold is also called the slant submanifold. Invariant and anti-invariant submanifolds are slant submanifolds with slant angle $\theta = 0$ and $\theta = (\pi/2)$, respectively. A slant submanifold is said to be proper slant if it is neither invariant nor anti-invariant.

In the same way, for any vector V normal to N, we put

$$FV = tV + nV, \tag{8}$$

where tV and nV denote the tangential and normal components of FV, respectively.

THEOREM 3.1. *Let N be a submanifold of a locally Riemannian product manifold M. Then N is a slant submanifold if and only if there exists a constant $\lambda \in [0, 1]$ such that $T^2 = \lambda I$. In this case, if θ is the slant angle of N, then it satisfies $\lambda = \cos^2\theta$* [12].

DEFINITION 3.1. N is called a semi-slant submanifold of a locally Riemannian product manifold M if there exist two orthogonal distributions such as D and D' such that:

(1) TN has the orthogonal direct sum $TN = D \oplus D'$;
(2) the distribution D is an invariant distribution, that is, $F(D) = D$;
(3) the distribution D' is a slant with angle $\theta \neq 0$ and $\theta \neq (\pi/2)$ [2].

THEOREM 3.2. *Let D be a distribution on N. Then D is a slant distribution if and only if there exists a constant $\lambda \in [0, 1]$ such that $(P_1 T)^2 X = \lambda X$ for any $X \in \Gamma(D)$. In this case, if θ is the slant angle of D, then it satisfies $\lambda = \cos^2\theta$, where P_1 denotes the orthogonal projection on D* [12].

Furthermore, if N is a slant submanifold of a locally Riemannian product manifold M with slant angle θ, then

$$g(TX, TY) = \cos^2\theta g(X, Y) \quad \text{and} \quad g(\omega X, \omega Y) = \sin^2\theta g(X, Y), \tag{9}$$

for any $X, Y \in \Gamma(TN)$.

In this section, we study warped product semi-slant submanifolds, with warped product in the form $N = N_1 \times_f N_2$, in a locally Riemannian product manifold M. First, we suppose that N_1 is an invariant and N_2 is a semi-slant of M with slant angle $\theta \neq (\pi/2)$, 0. Later, N_1 will be an anti-invariant submanifold and N_2 will be a semi-slant submanifold of M with respect to F.

THEOREM 3.3. *Let M be a locally Riemannian product manifold and N be a submanifold of M. Then there exist no warped product semi-slant submanifolds $N = N_T \times_f N_\theta$ in M such that N_T is an invariant submanifold and N_θ is a proper slant submanifold of M.*

PROOF. We suppose that $N = N_T \times_f N_\theta$ is a warped product proper semi-slant submanifold of a locally Riemannian product manifold M such that N_T is invariant and N_θ is a proper slant submanifold of M. We denote the projections onto $\Gamma(TN_T)$ and $\Gamma(TN_\theta)$ by P_1 and P_2, respectively. Then for any vector $Z \in \Gamma(TN)$, we can put

$$Z = P_1 Z + P_2 Z, \tag{10}$$

and using (7) gives

$$FZ = FP_1 Z + FP_2 Z = TP_1 Z + TP_2 Z + \omega P_2 Z. \tag{11}$$

By using the Gauss–Weingarten formulas, (7), (8) and considering Lemma 2.1(2) we obtain

$$\begin{aligned} \overline{\nabla}_U FX &= F\overline{\nabla}_U X, \\ TX \ln(f)U + h(U, TX) &= X \ln(f)TP_2 U + X \ln(f)\omega P_2 U \\ &\quad + th(U, X) + nh(U, X), \end{aligned} \tag{12}$$

for any $X \in \Gamma(TN_T)$ and $U \in \Gamma(TN_\theta)$. Then, comparing tangential and normal components in (12) respectively, we obtain

$$TX \ln(f)U = X \ln(f)TP_2 U + th(U, X) \tag{13}$$

and

$$h(U, TX) = X \ln(f)\omega P_2 U + nh(U, X). \tag{14}$$

In the same way, we arrive at

$$\begin{aligned} \overline{\nabla}_X FU &= F\overline{\nabla}_X U, \\ \overline{\nabla}_X TP_2 U + \overline{\nabla}_X \omega P_2 U &= F\nabla_X U + Fh(U, X), \\ \nabla_X TP_2 U + h(X, TP_2 U) - A_{\omega P_2 U} X + \nabla^{\perp}_X \omega P_2 U &= F(X \ln(f)U) + Fh(X, U) \\ &= X \ln(f)TP_2 U \\ &\quad + X \ln(f)\omega P_2 U \\ &\quad + th(X, U) + nh(X, U), \end{aligned} \tag{15}$$

for any $X \in \Gamma(TN_T)$ and $U \in \Gamma(TN_\theta)$. Taking into account the tangential and normal components of (15) respectively, we obtain

$$A_{\omega P_2 U} X = -th(U, X) \tag{16}$$

and

$$h(X, TP_2U) + \nabla^{\perp}_X \omega P_2U = X\ln(f)\omega P_2U + nh(X, U). \tag{17}$$

By using (3) and (16), it is easily seen that

$$g(A_{\omega P_2U}X, U) = -g(th(U, X), U) = -g(Fh(U, X), U) = -g(h(U, X), FU),$$
$$g(h(U, X), \omega P_2U) = -g(h(U, X), \omega P_2U),$$

that is,

$$g(h(U, X), \omega P_2U) = 0. \tag{18}$$

On the other hand, replacing X by TX in (14) and taking into account TN_T being invariant, we obtain

$$\begin{aligned} TX\ln(f)g(\omega P_2U, \omega P_2U) &= g(h(U, X) - nh(U, TX), \omega P_2U) \\ &= g(h(U, X), \omega P_2U) - g(nh(U, X), \omega P_2U) \\ &= g(h(U, X), \omega P_2U) = 0, \end{aligned}$$

for any $X \in \Gamma(TN_T)$ and $U \in \Gamma(TN_\theta)$. Thus,

$$TX\ln(f)\sin^2\theta g(P_2U, P_2U) = 0.$$

Since N_θ is a proper slant submanifold, g is a Riemannian metric and P_2U is a nonnull vector, we arrive at $TX\ln(f) = 0$, that is, the warping function f is constant. Hence, the proof is complete. □

THEOREM 3.4. *Let M be a locally Riemannian product manifold and N be a submanifold of M. Then there exist no warped product semi-slant submanifolds $N = N_\perp \times_f N_\theta$ in M such that $N_\perp$ is an anti-invariant submanifold and N_θ is a proper slant submanifold of M.*

PROOF. We suppose that $N = N_\perp \times_f N_\theta$ is a warped product semi-slant submanifold such that $N_\perp$ is an anti-invariant submanifold and N_θ is a proper slant submanifold of a locally Riemannian product manifold M. Then for any vectors X, Y tangent to $N_\perp$ and U tangent to N_θ,

$$\overline{\nabla}_U FX = F\overline{\nabla}_U X,$$
$$-A_{\omega X}U + \nabla^{\perp}_U \omega X = F(X\ln(f)U) + th(U, X) + nh(U, X). \tag{19}$$

From the tangential components of (19), we obtain

$$-A_{\omega X}U = X\ln(f)TP_2U + th(U, X). \tag{20}$$

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Furthermore, from equations (1), (2), (7), (8), (11) and considering Lemma 2.1,

$$\begin{aligned}
\overline{\nabla}_X FU &= F\overline{\nabla}_X U,\\
\overline{\nabla}_X TP_2U + \overline{\nabla}_X \omega P_2 U &= F\nabla_X U + Fh(X, U),\\
\nabla_X TP_2U + h(X, TP_2U) - A_{\omega P_2 U}X + \nabla^{\perp}_X \omega P_2 U &= F(X\ln(f)U) + th(X, U)\\
&\quad + nh(X, U)\\
&= X\ln(f)TP_2U\\
&\quad + X\ln(f)\omega P_2 U\\
&\quad + th(X, U) + nh(X, U).
\end{aligned} \tag{21}$$

From the tangential components of (21),

$$A_{\omega P_2 U}X = -th(X, U). \tag{22}$$

In the same way, making use of (1), (2), taking account of $N_\perp$ being anti-invariant in M and totally geodesic in N, we obtain

$$\begin{aligned}
\overline{\nabla}_Y FX &= F\overline{\nabla}_Y X,\\
-A_{\omega X}Y + \nabla^{\perp}_Y \omega X &= F\nabla_Y X + th(X, Y) + nh(X, Y),
\end{aligned}$$

which gives

$$A_{\omega X}Y = -th(X, Y),$$

which is also equivalent to

$$A_{\omega X}Y = A_{\omega Y}X. \tag{23}$$

On the other hand, (3) and the symmetry of F and A lead to

$$\begin{aligned}
g(A_{\omega X}Y, W) &= g(h(Y, W), \omega X) = g(h(Y, W), FX) = g(\overline{\nabla}_W Y, FX)\\
&= g(\overline{\nabla}_W FY, X) = g(\overline{\nabla}_W \omega Y, X) = -g(A_{\omega Y}X, W),
\end{aligned}$$

for any $X, Y \in \Gamma(TN_\perp)$ and $W \in \Gamma(TN)$, which implies that

$$A_{\omega X}Y = -A_{\omega Y}X. \tag{24}$$

From (23) and (24), we conclude that

$$A_{\omega X}Y = 0 \quad \text{and} \quad th(X, Y) = 0, \tag{25}$$

for any $X, Y \in \Gamma(TN)$. Thus, from (22) and (25), we obtain

$$g(h(U, X), \omega P_2 U) = 0 \quad \text{and} \quad g(h(X, Y), \omega P_2 U) = 0.$$

Furthermore, making use of (22), by direct calculations, we obtain

$$A_{\omega P_2 U} X = A_{\omega X} T P_2 U = th(X, U) = 0. \tag{26}$$

From (20) and (26),

$$\begin{aligned} -X \ln(f) g(T P_2 U, T P_2 U) &= g(A_{\omega X} U, T P_2 U) + g(th(U, X), T P_2 U) \\ &= g(h(U, T P_2 U), \omega X) + g(th(U, X), T P_2 U) \\ &= g(th(U, X), T P_2 U) = 0. \end{aligned} \tag{27}$$

From (9) and (27) we conclude that

$$X \ln(f) g(T P_2 U, T P_2 U) = X \ln(f) \cos^2 \theta g(P_2 U, P_2 U) = 0.$$

Since N_θ is a proper slant submanifold, g is a Riemannian metric and $P_2 U$ is a nonzero vector, we can derive $X \ln(f) = 0$, that is, the warping function f is constant. Hence the proof is complete. □

CONCLUSION 3.1. It is easy to see from Theorems 3.3 and 3.4 that there exist no warped product semi-slant submanifolds $N = N_1 \times_f N_\theta$ in a locally Riemannian product manifold M such that N_1 is invariant (anti-invariant) and N_θ is proper slant submanifold of M. But we can find the warped product semi-slant submanifolds $N = N_\theta \times_f N_T$ (see Example 3.1) ($N = N_\theta \times_f N_\perp$ (see Example 3.2)) such that N_θ is proper slant and N_T is invariant ($N_\perp$ is anti-invariant) in a locally Riemannian product manifold M.

Next, to illustrate these cases, we shall give two examples.

EXAMPLE 3.1. Let N be a submanifold of $\mathbb{R}^8 = \mathbb{R}^4 \times \mathbb{R}^4$ with coordinates $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ given by

$$\phi(\beta, \alpha, v, u) = (u + v, u - v, u \cos \alpha, u \sin \alpha, \sqrt{5} u, 2v, u \cos \beta, u \sin \beta).$$

It is easy to see that the tangent bundle of N is spanned by

$$Z_1 = -u \sin \beta \frac{\partial}{\partial x_7} + u \cos \beta \frac{\partial}{\partial x_8}, \quad Z_2 = -u \sin \alpha \frac{\partial}{\partial x_3} + u \cos \alpha \frac{\partial}{\partial x_4},$$

$$Z_3 = \frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2} + 2 \frac{\partial}{\partial x_6},$$

$$Z_4 = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \cos \alpha \frac{\partial}{\partial x_3} + \sin \alpha \frac{\partial}{\partial x_4} + \sqrt{5} \frac{\partial}{\partial x_5} + \cos \beta \frac{\partial}{\partial x_7} + \sin \beta \frac{\partial}{\partial x_8}.$$

Then, with respect to the Riemannian product structure F and usual metric tensor of $\mathbb{R}^8 = \mathbb{R}^4 \times \mathbb{R}^4$, $F(TN)$ becomes

$$F Z_1 = -Z_1, \quad F Z_2 = Z_2, \quad F Z_3 = \frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2} - 2 \frac{\partial}{\partial x_6},$$

$$Z_4 = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \cos \alpha \frac{\partial}{\partial x_3} + \sin \alpha \frac{\partial}{\partial x_4} - \sqrt{5} \frac{\partial}{\partial x_6} - \cos \beta \frac{\partial}{\partial x_7} - \sin \beta \frac{\partial}{\partial x_8}.$$

It is easily to check that

$$\cos^{-1}\left(\frac{g(FZ_3, Z_3)}{\|FZ_3\|.\|Z_3\|}\right) = \cos^{-1}\left(\frac{g(FZ_4, Z_4)}{\|FZ_4\|.\|Z_4\|}\right) = \cos^{-1}\left(-\frac{1}{3}\right).$$

Then N_T and N_θ can be taken as follows:

$$TN_T = \mathrm{Span}\{Z_1, Z_2\} \quad \text{and} \quad TN_\theta = \mathrm{Span}\{Z_3, Z_4\}.$$

Thus N_θ is a slant submanifold with slant angle $\theta = \cos^{-1}(-1/3)$. Furthermore, the metric tensor of $N = N_T \times_f N_\theta$ is given by

$$g_N = (6\,dv^2 + 9\,du^2) + u^2(d\alpha^2 + d\beta^2) = g_{N_\theta} + u^2 g_{N_T}.$$

Thus $N = N_\theta \times_{u^2} N_T$ is a warped product semi-slant submanifold of $\mathbb{R}^8$ with warping function $f = u$.

EXAMPLE 3.2. We consider the submanifold N in $\mathbb{R}^{10} = \mathbb{R}^4 \times \mathbb{R}^6$ given by

$$\varphi(u, v, \alpha) = \Bigg(\sqrt{3}u, \frac{2kv}{\sqrt{k^2+1}}, u\cos\alpha, -u\sin\alpha, -u\cos\alpha, -u\sin\alpha, -k\sin u, -k\sin v, k\cos u, k\cos v\Bigg),$$

where k is a constant which is not zero. We can easily see that the tangent bundle of N is spanned by vectors

$$\begin{aligned}
Z_1 &= \sqrt{3}\frac{\partial}{\partial x_1} + \cos\alpha\frac{\partial}{\partial x_3} - \sin\alpha\frac{\partial}{\partial x_4} - \cos\alpha\frac{\partial}{\partial x_5} - \sin\alpha\frac{\partial}{\partial x_6} \\
&\quad - k\cos u\frac{\partial}{\partial x_7} - k\sin u\frac{\partial}{\partial x_9}, \\
Z_2 &= \frac{2k}{\sqrt{k^2+1}}\frac{\partial}{\partial x_2} - k\cos v\frac{\partial}{\partial x_8} - k\sin v\frac{\partial}{\partial x_{10}}, \\
Z_3 &= -u\sin\alpha\frac{\partial}{\partial x_3} - u\cos\alpha\frac{\partial}{\partial x_4} + u\sin\alpha\frac{\partial}{\partial x_5} - u\cos\alpha\frac{\partial}{\partial x_6}.
\end{aligned}$$

Since FZ_3 is orthogonal TN and

$$\theta = \cos^{-1}\left(\frac{g(FZ_1, Z_1)}{\|Z_1\|.\|FZ_1\|}\right) = \cos^{-1}\left(\frac{g(FZ_2, Z_2)}{\|Z_2\|.\|FZ_2\|}\right) = \cos^{-1}\left(\frac{3-k^2}{5+k^2}\right),$$

$N_\perp$ and N_θ can be taken as follows: $TN_\perp = \mathrm{Span}\{Z_3\}$ is an anti-invariant distribution and $TN_\theta = \mathrm{Span}\{Z_1, Z_2\}$ is a slant distribution. Here F and g denote the Riemannian product structure and usual metric tensor of $\mathbb{R}^{10} = \mathbb{R}^4 \times \mathbb{R}^6$, respectively. Furthermore, the metric tensor of $N = N_\theta \times_f N_\perp$ is given by

$$g_N = (5 + k^2)\,du^2 + \left(\frac{k^4 + 5k^2}{k^2+1}\right)dv^2 + 2u^2\,d\alpha^2 = g_{N_\theta} + 2u^2 g_{N_\perp}.$$

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Thus $N = N_\theta \times_{\sqrt{2}u} N_\perp$ is a warped product semi-slant submanifold with slant angle $\theta = \cos^{-1}((3-k^2)/(5+k^2))$ and warping function $f = \sqrt{2}u$.

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