DERIVATION OF THE INITIAL LUMINOSITY FUNCTION AND THE PAST RATE OF STAR FORMATION

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The initial luminosity function $\psi(M_v)$ was introduced by Salpeter [1]. He assumed uniform formation of stars and derived the initial luminosity function from the observed main-sequence luminosity function and the life time of a star of magnitude M_v on the main sequence. Recently van den Bergh [2] considered the depletion of the interstellar gas by star formation. He found that at a constant rate of star formation the gas in the solar vicinity will be exhausted about 7×10^8 years from now.

It would seem very probable that the rate of star formation will depend on the amount of gas available, i.e. the gas density, and we will assume that the rate of star formation varies with the n-th power of the gas density. Direct evidence is available in the distribution of OB stars perpendicular to the galactic plane, which is narrower than the distribution of the gas. The gas has a dispersion of 140 psc while 184 southern cepheids observed by Walraven, Muller and Oosterhoff [3] have a dispersion of 65 psc. The difference in distribution may be explained if the rate of star formation varies with the square of the gas density, i.e. if $n = 2$. So :

$$f(t)\ \psi(M_v) = \text{const} \times (\text{gas density})^2, \tag{1}$$

if $f(t)\ \psi(M_v)$ is the number of stars formed per unit interval of time, at time t. It is assumed that the luminosity function of formation, or initial luminosity function $\psi(M_v)$ does not vary with time. The function $f(t)$ is the rate of star formation. It does not apply to Population II objects.

The integrations are started at $t = 0$ with 100 % gas. At first the values of $\psi(M_v)$ and the constant in equation (1) are estimated. Then numerical integration of equation (1) is carried out up to the present. The percentage of gas at present and the main sequence luminosity function may be calculated. These should agree with observational data and if not, other values of $\psi(M_v)$ and the constant are tried until a fit is obtained. The functions $f(t)$ and $\psi(M_v)$ are uniquely determined in this process. The ejection of gas by evolving stars was taken into account. The following simplified picture for the evolution of a star was assumed in the calculations. The star remains at the same absolute magnitude it was formed on the main sequence until about 7 % of its hydrogen is burned into helium. It then loses all its mass in excess of 0.7 solar masses and becomes a white dwarf. The giant stage has therefore been neglected. Stars with $M_v = +3.5$ are taken to have a life time on the main sequence equal to the age of the Galaxy. The life times for brighter stars were computed on the assumption of homology. The luminosity functions and the gas density refer to a cylinder perpendicular to the galactic plane near the sun, so as to take into account the different distribution in z for stars of different absolute magnitude. It is assumed that no gas or stars leave or enter this cylinder. The mass density near the sun is 55 solar masses/psc^2. The gas density at present was taken as 11 solar masses/psc^2, or 20 %.

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The results of the integrations are given below. Also given in some cases are results for the case n = 0, i.e. a constant rate of star formation.

(A) The initial luminosity function $\psi(M_v)$ is given in Table 1 and shown in Figure 1. Also given are the initial luminosity function for the case n = 0 and the observed main sequence luminosity function $\varphi(M_v)$. All luminosity functions are identical for M_v below + 4 because

Table 1

M_V	$\psi(M_V)$		$\varphi(M_V)$
	n = 2	n = 0	
− 5	.05	.01	.000 005
− 4	.14	.03	.000 034
− 3	.27	.06	.000 13
− 2	.48	.11	.000 55
− 1	.87	.20	.002 5
0	1.27	.29	.010
1	1.35	.34	.032
2	1.36	.39	.106
3	1.11	.54	.354
4	1.13	1.13	1.13
5	2.09	2.09	2.09

TABLE 1. Initial luminosity functions $\psi(M_V)$ for n = 2 and n = 0, and the observed main sequence luminosity function $\varphi(M_V)$, all in a cylinder of 1 psc² perpendicular to the galactic plane.

these stars have not had time to evolve off the main sequence as yet. Actually, $\psi(M_v)$ gives the total number of stars ever formed in a cylinder of 1 psc² in the solar neighbourhood. The initial luminosity function for the case n = 2 shows a slight maximum around $M_v = +1$. This may be due to our assumption of homology of stars of different mass not being valid. Hoyle [4] has just shown that rather large deviations from homology do occur. Taking his results roughly into account the luminosity functions $\psi(M_v)$ should be lowered by about half the difference between the cases n = 2 and n = 0 in Figure 1. For the case n = 2, the distribution of luminosities in the range $M_v = +1.5$ to $+5.5$, which is very sensitive to the value of n chosen, agrees nicely with the distribution in open clusters.

(B) The rate of star formation and the percentage of gas as a function of time are given in Table 2 and shown in Figures 2 and 3. The unit of time used is the age of the Galaxy, 5 to

Table 2

t	f(t)	% gas
0	5.5	100
0.1	2.4	66
0.2	1.5	52
0.3	1.00	43
0.4	.74	37
0.5	.57	32
0.6	.45	29
0.7	.37	26
0.8	.31	24
0.9	.26	22
1	.22	20

TABLE 2. The rate of star formation f(t) and the percentage of gas. t = 1 is the present.

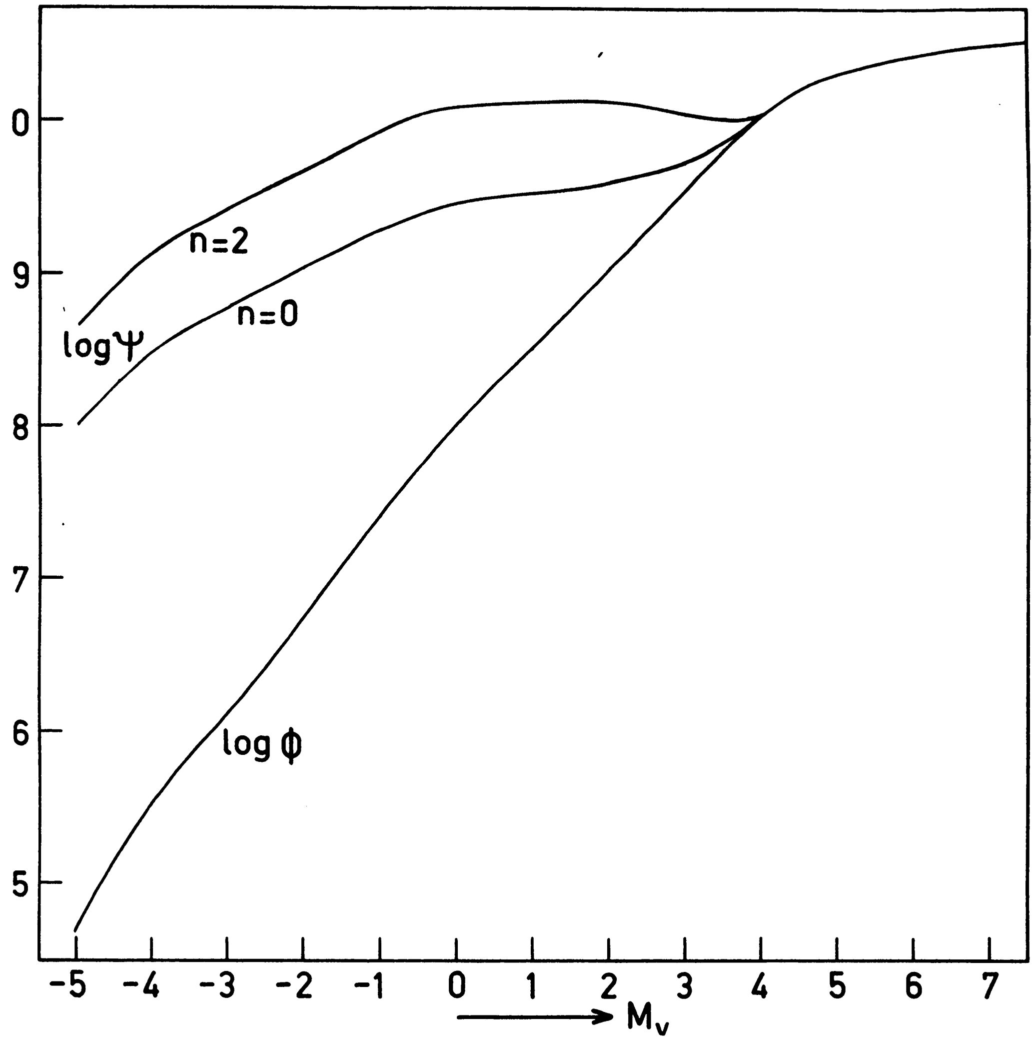

FIGURE 1. Initial luminosity functions $\psi(M_V)$ for n = 2 and n = 0, and the observed main sequence luminosity function $\varphi(M_V)$.

10×10^9 years. The present rate of star formation is $4\frac{1}{2}$ times slower than the average rate (which is unity). The average age of all stars fainter than $M_v = +3.5$ is 0.75 of the age of the Galaxy.

(C) The total mass of gas which has been used up to the present in the formation of stars is 67 solar masses per psc^2, or 122 % of the total mass density. The rate of gas consumption at present is 1.5 to 3.0 solar masses per psc^2 per 10^9 years, depending on the age of the Galaxy.

(D) The total mass of gas ejected by evolving stars is 23 solar masses per psc^2 or 42 % of the total mass density. The rate of ejection at present is 0.6 to 1.1 solar masses per psc^2 per 10^9 years.

(E) The predicted number of white dwarfs is 6.4 per psc^2. (The total number of stars may be around 100 per psc^2.) It is expected from their age distribution and the theory of cooling at constant radius (Schwarzschild [5]), that only 1.5 white dwarfs per psc^2 will be brighter

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than $M_v = 14.0$. This agrees rather well with the observations, the statistics of which are very poor however.

(F) The present abundance of helium in the interstellar gas may be around 34 %, the value derived by Mathis [6] for the Orion nebula. This abundance can be explained if the average efficiency E with which original hydrogen is ejected in the form of helium by evolving stars is 40 %. The corresponding variation of helium abundance Y(t) of the interstellar gas with time is shown in Figure 4. If we take the age of the Galaxy to be 8×10^9 years, the helium abundance 5×10^9 years ago, when the sun was formed, is found to be 21 %. This is in excellent agreement with a solar model computed by Schwarzschild, Howard and Härm [7] who found a value of about 20 %. Also shown in Figure 4 is the run of Y(t) for the case of constant rate of star formation (n = 0). Even with 100 % efficiency of conversion of hydrogen into helium, it is impossible to account for the present helium abundance.

In regions of the galactic system where the total mass density is, say, 10 times the density near the sun, the rate of star formation in the beginning—and therefore the rate of gas consumption—will have been 100 times faster. As a result the percentage of gas will decrease much

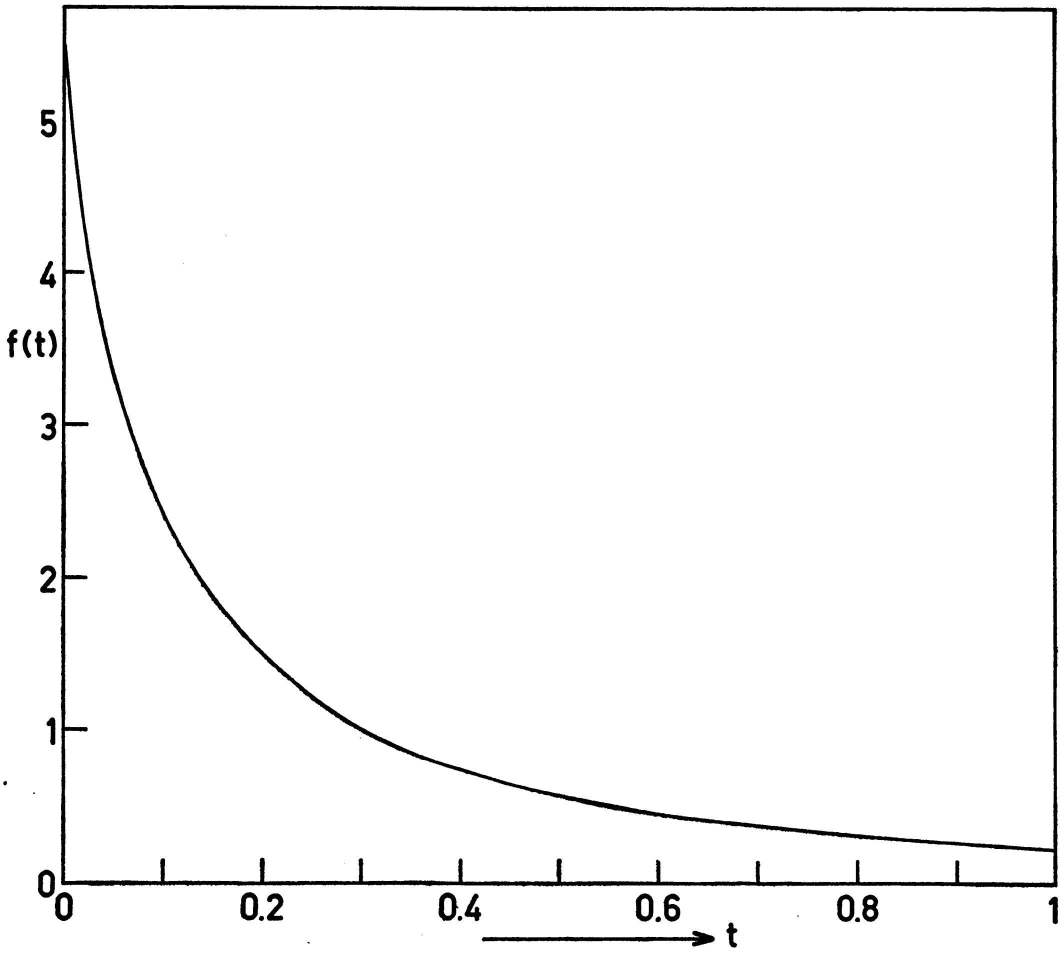

FIGURE 2. The rate of star formation f(t). t = 1 is the present.

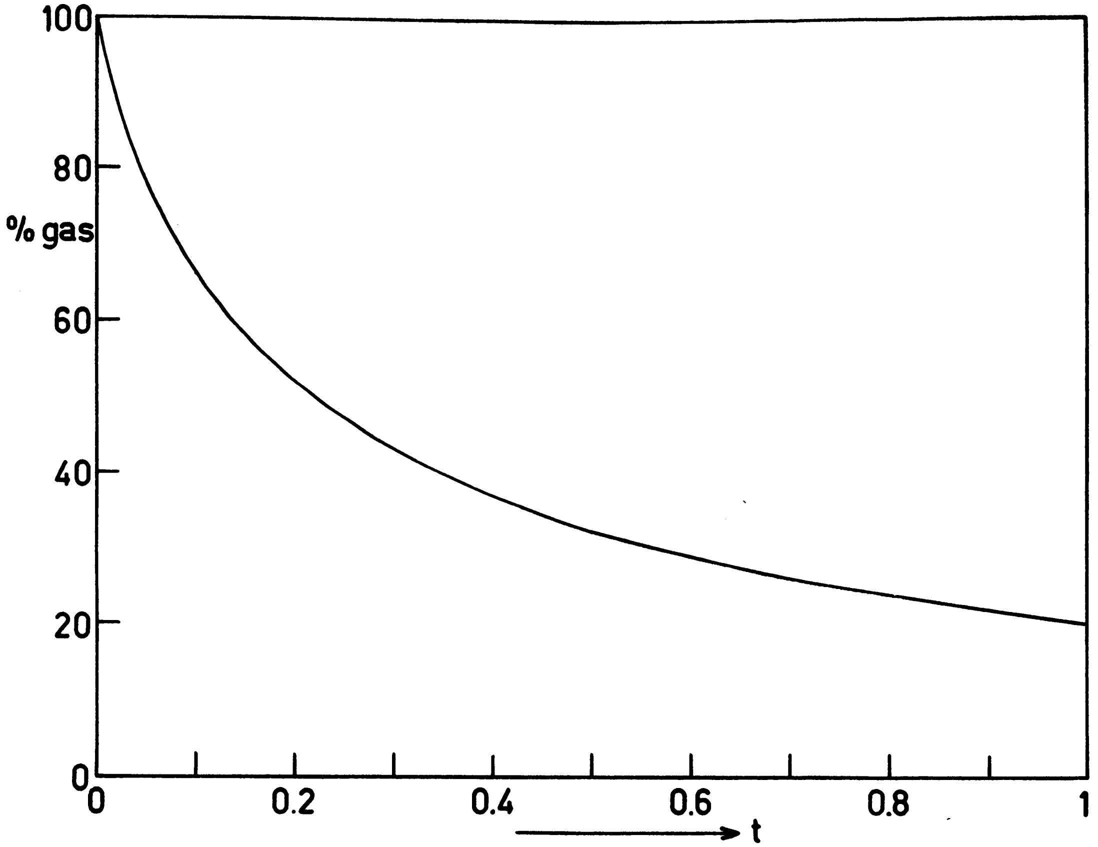

FIGURE 3. The percentage of gas in the solar neighbourhood as a function of time. t = 1 is the present.

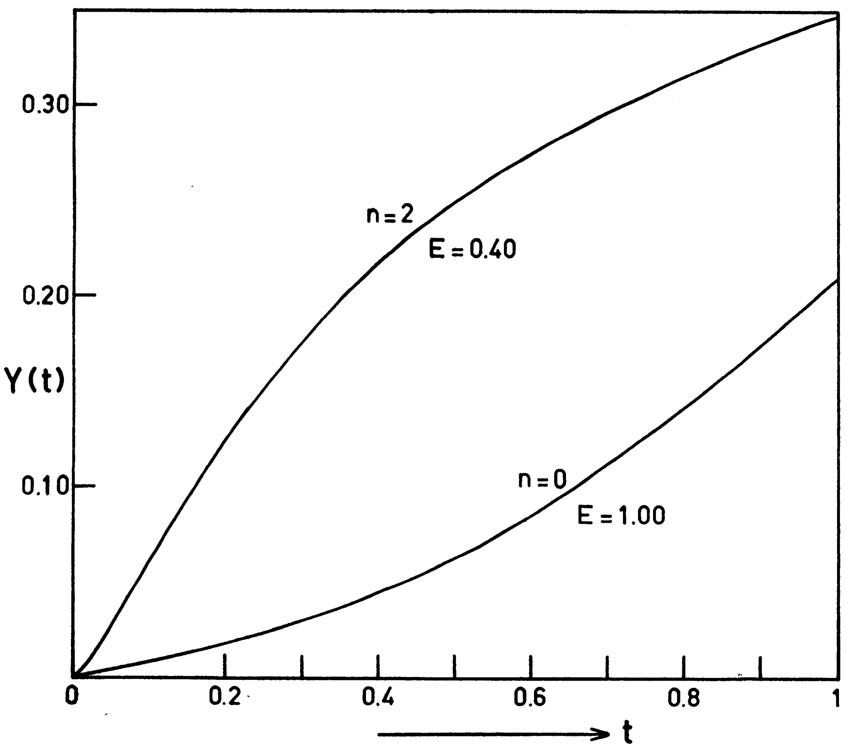

FIGURE 4. The helium abundance in the interstellar gas as a function of time for n = 2 and n = 0.

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faster than near the sun. Actually, calculations show that the gas density at present should be of the same order throughout the system, in qualitative agreement with the distribution of neutral hydrogen observed at 21 cm wavelength.

We may extend these considerations to galaxies and expect that the giant ellipticals, which have a high mass density, will contain a small percentage of gas. Systems with low mass density, like the Small Magellanic Cloud, should have a high percentage of gas.

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Discussion

Salpeter : I want to report on a calculation which was mathematically very similar to Dr. Schmidt's, but with a somewhat different philosophy : I wanted to be able to treat the galaxy as a whole; in particular to see if one can use the *same* laws on rates of star formation in the Halo Population II as well as Population I, in spite of the fact that the Halo stars seemed to have formed so rapidly.

In order to be able to treat the galaxy as a whole in a simple manner I assumed $n = 1$ in Dr. Schmidt's notation, i.e. rate of star formation per unit volume proportional to the first power of density. I also carried out the calculations in the opposite order. i.e. I used for the birthrate function the observed main-sequence luminosity function in very young clusters, as compiled by Sandage and van den Bergh.

I quote only one result of my calculations : If we estimate that about 20 percent of the mass of the Galaxy is in the form of the high-velocity Halo Population II stars, one can calculate the time required to form this amount of stars. With the assumption $n = 1$, the time to form the first 20 percent of stars is gratifyingly short, about 400 million years (4×10^8), or possibly even as short as 200 million years. If n is larger than unity, which was made plausible by Dr. Schmidt, these times will be somewhat longer.