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Families of Particle-Like Fractals with Differing Shapes and Boundary Fractal Dimensions

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The boundary fractal dimension, Db, is related to the roughness or tortuosity of the surface of the particle, and is becoming more widely used to quantify the degree of ruggedness of irregular objects [1]. This paper describes a computer generated family of fractals [2], and the publicdomain tool running under Windows [*] used to make them [3]. These fractals are variations of the Koch Snowflake – the geometrical "monster", known for over 100 years, that has finite area and infinite perimeter. The Koch Snowflake is the arch-typical fractal for those interested in using fractal dimensions (and related measurements) to describe shapes of objects such as microscopic particles. The Koch family described here has Db continuously variable from Db = 1.0 to Db ~ 2.0, and shapes varying continuously from 'spiky' to 'clubby' or cauliflower-like. These fractals are available on the web as GIF images[4].

The Koch snowflake is generated as in Figure 1, with a = b = 1/3. Variations of both shape and Db are varied by changing a and b. If a is close to ½, spikes-on-spikes are drawn rather than triangles-on-triangles, resulting in shapes like in Figure 2. These shapes are fuzzy looking when rendered sufficiently fine, although if Db is too close to 2, that is if the curve is too space filling, thenadjacent spikes will touch. In cases like this, the fractal must be rendered at sufficiently low resolution to preserve its tortuosity (Figure 3), and step lengths smaller than the straight line segments seen here must be avoided when measuring Db, as smaller steps are not appropriate, since the fractal is not rendered at these step lengths. The real fractal has no such straight segments without side branches.

A useful representation of the family is Figure 4, with Db on the abscissa, and a, the lengthof the side (Figure 1), which serves as a shape parameter, on the ordinate. The Koch snowflake is represented by the dot at Db = 1.26, **a** = 1/3. The fractals vary from the triangle, D=1.0, to those with spiky or 'cauliflowery' sides at higher Db.

There are limits on the range of the shape factor, for any given D. At low D, the shape factor can cover almost the full range of **a**, while for high D, **a** needs to be near 0.5 for the fractal to avoid mathematical self intersection. There are also restrictions due to the pixel size when rendered as a digital image [4], that can be compensated for somewhat by appropriately restricting the number of steps or smallness of the added line segments.

These fractals may be useful for estimating Db by visually comparing them with particle images. They are also useful for generating test images for evaluating image analysis algorithms that measure Db. Rather than testing a method with just a few fractals of differing D, the accuracy, precision, linearity, can be tested with finely spaced values of D, and with different shapes having the same values of D [3,4].

Note that projections (shadows) of particle outlines (especially if they are not clusters) are limited in how rough they can be, because of cracks which cannot be seen edge-on for example. This family of fractals includes values of Db = 1.0 – 1.2 – the useful range for most particle projections. Test fractals in the literature usually have Db greater than 1.2.

References:

[1] M. Allen et al, Powder Technology 84 (1995) 1-14.

[2] D. Stoyan & H. Stoyan, FRACTALS, RANDOM SHAPES AND POINT FIELDS, John Wiley & Sons, 1994.

[3] Software available at www.nist.gov/lispix.

[4] D. Bright, "Particle-Like Fractal Images for Testing Algorithms …" (this proceedings).

* Certain commercial equipment, instruments, or materials are identified in this report to specify adequately the experimental procedure. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

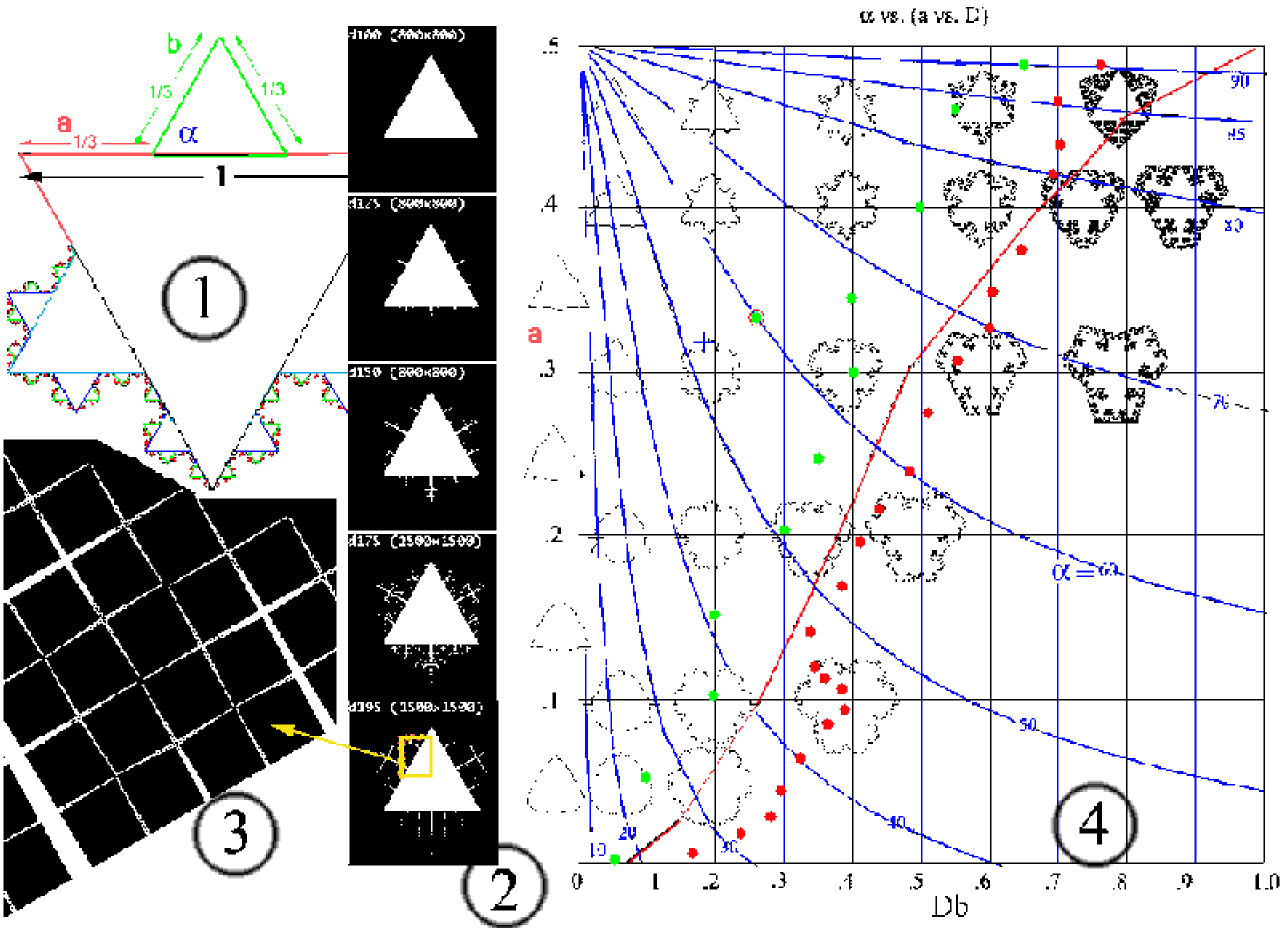

FIG. 1) Diagram of constructing Koch snowflake. Top half – 1st stage, parameters a, b, α. Bottom: first five stages of construction in color.

FIG. 2) Five examples of von Koch family, shape parameter a=.49, different Db labeled at top. (Top: Db = 0.0, bottom: Db = 1.95). Correspond to five locations along top of chart in Fig. 4.

FIG. 3) Blow up of yellow box in Fig. 2, showing coarse rendering resolution (only 4 steps) to avoid touching of spikes. Some touch anyway.

FIG. 4) Koch family D-a parameter space. Koch Snowflake at green dot with red circle. Red line – nominal useful limit – figures to left may be useful, figures to right mathematically self intersect. Red dots – mathematical self intersection limits found experimentally using rendering routine. Green dots – self intersection limits when rendered as digital image – found from inspection and blobbing of images.

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