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Hardy spaces of exact forms on domains

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We consider Hardy spaces $\mathcal{H}^1_{z,d}(\Omega, \wedge^k)$ of exact $k-$forms supported in strongly Lipschitz domains Ω of $N-$dimensional Euclidean space $\mathbb{R}^N$ and aim to give their atomic decompositions, characterize their dual spaces, and establish "div-curl" type theorems on domains Ω.

When $k = 2$ and $\Omega = \mathbb{R}^3$, $\mathcal{H}^1_{z,d}(\Omega, \wedge^k)$ reduces to a divergence-free Hardy space on $\mathbb{R}^3$. We exhibit a divergence-free atomic decomposition of the space and give a "div-curl" type theorem on $\mathbb{R}^3$. We also investigate some applications of the "div-curl" type theorem to coercivity properties of some polyconvex quadratic forms which come originally from the linearization of polyconvex variational integrals studied in nonlinear elasticity on $\mathbb{R}^3$. When Ω is the whole space $\mathbb{R}^N$, the upper half-space $\mathbb{R}^N_+$, a special Lipschitz domain or a bounded strongly Lipschitz domain in $\mathbb{R}^N$, we prove atomic decompositions of $\mathcal{H}^1_{z,d}(\Omega, \wedge^k)$ as sums of exact atoms with supports in Ω. These results follow from tent space arguments along with a reproducing identity. We then use those decompositions to characterize dual spaces of $\mathcal{H}^1_{z,d}(\Omega, \wedge^k)$. In addition we establish "div-curl" type theorems on Ω with applications to coercivity.

The content of the thesis is roughly as follows. The first chapter contains preliminary material on Hardy spaces, BMO spaces, tent spaces and Sobolev spaces. The only original feature of the chapter seem to be an observation that the curl operator is surjective from the Sobolev space $H^1_0(\Omega, \mathbb{R}^3)$ to a subspace of $L^2(\Omega, \mathbb{R}^3)$ when Ω is a smooth and simply-connected domain in $\mathbb{R}^3$.

The second chapter deals with estimates of Jacobian determinants on a bounded strongly Lipschitz domain Ω in $\mathbb{R}^2$. The corresponding estimate on $\mathbb{R}^2$ is due to Coifman, Lions, Meyer and Semmes [**1**]. Also in this chapter, we give a decomposition theorem of $\mathcal{H}^1_z(\Omega)$ into "Jacobian" quantities. A similar decomposition of $\mathcal{H}^1(\mathbb{R}^N)$ into "div-curl" quantities was obtained in [**1**, Theorem III.2].

In Chapter III, we consider the three-dimensional divergence-free Hardy space. We prove a divergence-free atomic decomposition of the space: any function in the space

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can be decomposed into a sum of divergence-free atoms. Using the decomposition we characterize its dual: the BMO-type space. Applying the duality relationship between the divergence-free Hardy space and the BMO-type space we establish a "div-curl" type theorem, which is used to prove some coercivity properties of certain polyconvex quadratic forms. The divergence-free Hardy space on $\mathbb{R}^N$ was studied by Gilbert, Hogan and Lakey in [**2**]. They gave its atomic decomposition by using a divergence-free wavelet decomposition of the divergence-free $L^2(\mathbb{R}^N, \mathbb{R}^N)$ space [**3**]. The idea we used here is different from that in [**2**] and is valid for cases of domains and forms. Our proof relays on a reproducing identity and techniques in harmonic analysis, including atomic decompositions of tent spaces, Whitney decompositions and reflection maps.

The atomic decomposition and the "div-curl" type theorem in Chapter III are generalized to Hardy spaces $\mathcal{H}^1_d(\mathbb{R}^N, \wedge^k)$ in Chapter IV. In this chapter, we also prove a decomposition theorem of $\mathcal{H}^1_d(\mathbb{R}^N, \wedge^k)$ into "$du \wedge dv$" quantities, which is an extension of [**1**, Theorem III.2].

Chapter V is perhaps the heart of the thesis. In this chapter, we first consider a divergence-free Hardy space on the upper-half space $\mathbb{R}^N_+$ and prove a divergence-free atomic decomposition of the space, for this we need to use the even and odd functions. Then we study the Hardy space $\mathcal{H}^1_{z,d}(\Omega, \wedge^k)$ when Ω is $\mathbb{R}^N_+$ or a special Lipschitz domain. We give its atomic decomposition and characterize its dual space. The key technique is to apply a reflection map defined on the domain Ω. We also establish a "div-curl" type theorem on Ω.

Finally in Chapter VI, we deal with the atomic decomposition and the duality of $\mathcal{H}^1_{z,d}(\Omega, \wedge^k)$ when Ω is a bounded strongly Lipschitz domain.

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