KINEMATIC DYNAMO IN TURBULENT CIRCUMSTELLAR DISKS

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Abstract. Many circumstellar disks associated with objects ranging from protoplanetary nebulae to accretion disks around compact stars allow for the generation of magnetic fields by an $\alpha\omega$ dynamo. We have applied kinematic dynamo formalism to geometrically thin accretion disks. We calculate, in the framework of an adiabatic approximation, the normal mode solutions for dynamos operating in disks around compact stars. We then describe the criteria for a viable dynamo in protoplanetary nebulae, and discuss the particular features that make accretion disk dynamos different from planetary, stellar, and galactic dynamos.

Key words: accretion disks – protoplanetary disks – $\alpha\omega$ dynamos

1. Circumstellar Accretion Disks

Geometrically thin, optically thick, turbulent accretion disks are believed to surround many stars. Some of these disks form around compact objects such as white dwarfs, neutron stars, and black holes. Those disks are located deep in the gravitational well of the central stars, where viscous stress is large and the disk's temperature is high. As a result of this high temperature, the disks have a very high degree of ionization and are ideal places for the an $\alpha\omega$ dynamo to operate. On the other hand, protostellar/protoplanetary disks or nebulae are much more extended objects, thus cool and weakly ionized. In those disks the ability of the dynamo to operate depends crucially on the strength of nonthermal ionization processes such as cosmic rays and radioactive isotopes that must produce ionization levels high enough to couple magnetic field to the gas.

Most accretion disks are considered in the framework originally proposed by Shakura and Sunyaev (1973), in which the turbulent viscous stress $\mathcal{T}_{\phi r} = \omega\rho\nu = \omega\rho l_0 v_0 = \alpha_{ss}\rho C_s^2$. Here ω is Keplerian angular velocity, $\nu = l_0 v_0$ is kinematic turbulent viscosity, ρ is the gas density, C_s is the sound speed, and l_0 and v_0 are turbulent mixing length and turbulent velocity respectively. The constant $\alpha_{ss} = (l_0/h)(v_0/C_s) < 1$ is the dimensionless strength of turbulent viscosity and h is the disk's half-thickness. Shakura *et al.* (1978) suggested that irrespective of the unknown source of turbulence, $v_0 \approx \omega l_0$, so $l_0/h \approx v_0/C_s = M_t$, where M_t is the turbulent Mach number. Accepting Shakura *et al's* prescription for v_0 yields $\alpha_{ss} = M_t^2$. In the simplest turbulent dynamo theory the magnetic turbulent diffusivity, $\eta_{\rm turb}$, is usually assumed to be equal to $l_0 v_0$, thus $\eta_{\rm turb}$ is numerically equal to ν and can be expressed as $\eta_{\rm turb} = \alpha_{ss} h^2\omega$. In an accretion disk the turbulence is strongly affected by rotation. Under such conditions, the radial dependence of the helicity of turbulence ("α-effect") is given by $\alpha \approx l_0 v_0/h = \alpha_{ss} h\omega$ (Ruzmaikin *et al* 1988). We can use this simplified description of the disk's turbulence to solve the equation governing the evolution of a large-scale dynamo magnetic field generated in a disk.

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2. Adiabatic Approximation Technique

Solving the induction equation for the large-scale magnetic field **B** for turbulent disk models presents us with many cumbersome mathematical difficulties. First, difficulties arise in handling the boundary condition at infinity in the cylindrical disk geometry. Close to the disk faces, the potential that describes the magnetic field outside the disk interior decays exponentially with distance from the disk surface. On the other hand, far away from a disk, the magnetic field generated by currents in the disk should resemble a multipole and decay as $r^{-(n+1)}$, where $n \geq 2$. Numerical methods have been developed to overcome these difficulties (Stepinski and Levy 1988, Elstner *et al* 1990), however, they are prohibitively inefficient when applied to accretion disks where the ratio of the radial to vertical dimensions is typically 0.01 to 0.001. Instead, we consider only the approximate, "thin disk" boundary conditions: toroidal field B and vertical derivative of poloidal vector potential A vanish at the disk surfaces. Moreover, we observe that in a thin disk the magnetic field diffuses much more rapidly to the vertical boundary of the disk than along the radius. Thus, an asymptotic method, first brought to dynamo theory by Ruzmaikin *et al* (1985) in the galactic dynamo context, called an adiabatic approximation, can be applied to separate effectively the radial and vertical dependence of a generated field. Stepinski and Levy (1991; hereafter SL91) formulated the general and rigorous algorithm based on an idea of adiabatic approximation, which can be directly applied to solving the dynamo equation in the context of an accretion disk. In this algorithm the solutions to the induction equation are sought in the form $B(r,z) = Q(r)b(r,z)$ and $A(r,z) = Q(r)a(r,z)$. Functions b and a are the solutions of the dynamo equations found by neglecting the radial derivatives of the magnetic field. They constitute a so-called "local" solution, depending on the radial coordinate r only parametrically through a radially varying coefficient, the effective dynamo number D_{eff}, given by

$$D_{\text{eff}} = \frac{3}{2}\frac{\alpha\omega h^3}{(\eta + \eta_{\text{turb}})^2}, \tag{1}$$

where η is resistive magnetic diffusivity. Solving the "local" dynamo problem yields $\gamma(r)$, customarily called the local rate of magnetic field exponential growth. This is misleading inasmuch as the magnetic field does not grow at different rates at different radial locations. Instead, within a radial localization of the normal mode it grows at the same "global" rate Γ. The radial structure and the global growth rate Γ of the magnetic field are determined by the solution of the radial equation

$$\lambda^2(\eta + \eta_{\text{turb}})\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\right)Q + (\gamma(r) - \Gamma)Q = 0, \tag{2}$$

subject to the boundary conditions at the inner and outer disk edges. The constant λ is the ratio of vertical to horizontal length scales in the disk and the local growth rate $\gamma(r)$ is used as a "potential" function. Adiabatic approximation thus provides a way to obtain critical dynamo numbers and radial, as well as vertical, distributions of magnetic field normal modes for an accretion disk dynamo described by appropriate α, ω, h, η, and η_{turb}.

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3. Accretion Disk Dynamos

Accretion disks around compact stars are hot; their typical temperatures T are about $10^4 - 10^6$ K depending on the mass of the central star and the location in the disk. Resistive magnetic diffusivity $\eta = c^2(4\pi\sigma)^{-1} \approx 10^{12}T^{-3/2}$ cm^2/sec is much smaller than a turbulent magnetic diffusivity $\eta_{\rm turb}$, which thus totally dominates the total magnetic diffusivity. It is important to note that for a disk's magnetic field to persist long enough in comparison to the disk's viscous time, it must be contemporaneously regenerated by a dynamo mechanism because the characteristic diffusion time $t_{\rm diff} = h^2/\eta_{\rm turb}$ is typically very short in comparison with the disk's lifetime. Putting $\eta = 0$ and substituting the appropriate expressions for $\eta_{\rm turb}$ and α in eq.(1) we find that $D_{\rm eff} = 1.5\alpha_{\rm ss}{}^{-1}$, independent of r and determined exclusively by the strength of turbulence. It was shown in SL91 that, in the first approximation, magnetic fields (a normal mode or any linear combination of them) can be maintained only in those parts of the disk where $D_{\rm eff}$ exceeds a certain critical value $D_{\rm crit}$, which was calculated to be about 12. Thus, we conclude that for an $\alpha\omega$ dynamo to operate successfully in the highly ionized accretion disk the condition $\alpha_{ss} < 0.125$, or equivalently $M_{\rm t} < 0.35$, must be met. This condition is very likely to be satisfied because we think that turbulence in accretion disks is significantly subsonic, therefore a dynamo is very likely to maintain the magnetic field throughout the entire radial extent of such a disk.

The "local" approximation yields the criteria for the *existence* of dynamo generated fields in a disk. The adiabatic approximation can provide the radial structure and global growth rate Γ of magnetic normal modes. It is accomplished by solving eq.(2) with local growth rate $\gamma(r) = (\gamma_0/\alpha_{\rm ss})\omega$ (for details see SL91). It is interesting to note that γ is a decreasing function of r; even so $D_{\rm eff}$ is constant along the radius. Only in an ideal steady state, when $\gamma_0 = 0$ and no timescale is relevant, does $D_{\rm eff} = $ const translate into $\gamma = $ const $= 0$.

As an example, consider a disk with $\alpha_{\rm ss} = 0.05$. It was calculated in SL91 that in such a disk there are 48 overcritical normal modes. The most important finding of SL91 calculations is that the fastest growing mode does not dictate the overall disk's magnetic field because it is confined to only about the inner 5% of the disk, beyond which it becomes evanescent. The second mode, growing approximately three times slower than the leading mode, become evanescent beyond the inner 8% of the disk's radial extent. Thus we expect that the second mode actually describes the disk's overall field in the ring between 5% and 8% of the disk's radial extent. Other, progressively less overcritical, modes are confined between the center and some progressively larger, cutoff radii. The least overcritical mode, approximately steady state solution, spans the entire disk. We conclude that, within the kinematic theory, the overall structure of magnetic field generated by an accretion disk dynamo consists of a superposition of a large number of overcritical modes, each dominating the total magnetic field in a particular, rather small region of a disk. We expect that such magnetic field structure is a general attribute of thin disk dynamos because it arises from the radial variation of the local growth rate, γ. The radial variation of γ in highly ionized disks is a direct consequence of radial variability of characteristic diffusion time, which in a thin disk has only a local meaning and must be calculated using the local values of the disk's half-thickness h and turbulent diffusivity η_{turb}, themselves both functions of radial coordinate. This further suggests that we may not be able to describe the magnetic field in an accretion disk by means of a single state evolving on one well-defined timescale. Instead, it is likely that the system is better described by a set of many quasi-localized states each evolving on a different timescale.

4. Protoplanetary Nebula Dynamos

Technically speaking, protostellar/protoplanetary nebulae are also accretion disks. In fact, according to present theories, most of the matter that would ultimately form the star must pass through such a disk in order to shed the excess angular momentum it carries. The distinction we are introducing here is based on the degree to which different disks are ionized. Whereas accretion disks around compact stars are very well ionized, protoplanetary nebulae, as a rule, are only very slightly ionized. There are no regions of the protoplanetary nebula where the temperature is high enough to cause thermal ionization of hydrogen, the main gas constituent. However, in the innermost parts of the nebula the temperature may exceed 1000 K, enough for thermal ionization of potassium. This would yield the degree of ionization x of about 10^{-7}. In the rest of the nebula the significant levels of electrical conductivity require nonthermal ionization that is provided mostly by cosmic rays and additionally by *in situ* radioactive isotopes ^{26}Al and ^{40}K. Those nonthermal sources can typically provide a degree of ionization up to about $10^{-11} - 10^{-12}$. As a result, typical resistive magnetic diffusivity, η, is comparable to, or in some nebular regions even larger than, turbulent magnetic diffusivity η_{turb}. Therefore when calculating the criteria for the existence of dynamo generated magnetic fields for those nebulae, one cannot assume that turbulent diffusivity dominates magnetic dissipation. Instead, the full total diffusivity $\eta + \eta_{\mathrm{turb}}$ must be used in eq.(1) to calculate the radial dependence of D_{eff}. This requires determination of the radial dependence of η from the ionization state of the nebula, which was done by Stepinski (1992). Typically, $D_{\mathrm{eff}} > D_{\mathrm{crit}}$ in the innermost parts of the nebula, where potassium is thermally ionized, and in the outer parts, where surface density is too low to screen cosmic rays (which are a dominant nonthermal ionization source) from penetrating into the midplane regions where the bulk of the nebular gas resides. Thus, nebular regions located between the maximum thermal radius and minimum nonthermal radius define the gap in which $D_{\mathrm{eff}} < D_{\mathrm{crit}}$ and magnetic field cannot be maintained. The width and the location of this gap depend very much on the nebula evolutionary stage, but its existence is a robust feature since it reflects what we presently think are the basic physical realities of protostellar/protoplanetary nebulae.

We have not attempted to use an adiabatic approximation to find the radial structure of normal modes for nebular dynamos. Nevertheless, the qualitative character of those modes can be easily deduced from radial behavior of D_{eff}, which is large and equal to about $1.5\alpha_{\mathrm{ss}}{}^{-1}$ in the innermost and outer regions of the nebula, and has a deep "well" of much smaller values in the intermediate parts of the nebula. Thus we expect leading normal modes to be localized in two distinctive nebular regions: one in the inner nebula and the other in the outer nebula. Again, as in the case of accretion disk dynamos, the overall structure of the generated field is likely to consist of many quasi-localized states, each evolving on a different timescale.

5. Some Remarks on Disk Dynamos

Faced with the lack of any *direct* observations of magnetic fields in accretion disks, our goals in studying them are different from those for galactic as well as stellar and planetary dynamos. One clearly identified reason for studying magnetic fields in disks is the desire to understand the magnetic transport of angular momentum – potentially the leading mechanism of disk evolution. In addition, we would like to connect our theory to some observed features, such as jets and flares, that *morphologically* point to the existence of magnetic fields in those disks. Disk dynamos cannot be investigated numerically using the methods developed for galactic dynamos because they are limited to handle disks with λ not larger than 0.1, far short of $\lambda = 0.01 - 0.001$, which we typically encounter in accretion disks. We either have to study them using an asymptotic methods like an adiabatic approximation, or develop new numerical schemes that could handle the peculiarities of thin disk geometry.

Thin accretion disks, with masses small in comparison to the central mass, are characterized by the Keplerian differential rotation, regardless of the strength of the stress. This is because the disk, unlike the star or the planet, is an open system – matter enters it, loses angular momentum, and accretes on the central object. Changing the strength and the character of the stress would change the structure of the disk and the rate of accretion, but not the profile of differential rotation. Therefore, magnetic fields generated by disk dynamos, although they would influence the character of turbulence and add very significantly to the overall stress, are unable to change the profile of differential rotation, removing one nonlinearity from an otherwise very nonlinear and difficult problem.

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