ranging in the level of difficulty: from easy computations of $\zeta(24)$ and standard calculus results, like Mertens's theorem that the Cauchy product C of an absolutely convergent series A and a conditionally convergent series B converges and $C = AB$, to research problems like the criterion for a point to be a zero of the Riemann zeta function. This criterion is a result by a certain unsung mathematician Ashoke Kumar Mustafy—one of the hundreds in the book—who discussed mathematics with the boy Ranjan Roy and believed that it could lead to a proof of the Riemann hypothesis. Other exercises are complemented by colourful historical vignettes, for instance the one discussing Lusin's conjecture about almost everywhere convergence of the Fourier series of square integrable functions. There are even purely geometric exercises such as the one asking for the circumradius of a cyclic quadrilateral (Parameshvara's formula). Readers who work through the cornucopia of exercises will definitely have their money's worth.

I have found a number of minor typos and omissions, which is inevitable in a book of this enormous scope. Most of them are harmless and do not affect its readability: misspelling of Holmboe, wrong birth years for Roger Cotes (1682-1716) and Pierre de Fermat (1601-1665), incorrect dates of the English Civil War (1642-1651) or a missing reference to Faulhaber's work in the bibliography. The ones I noticed with mathematical import are the missing exponent a_j in the statement of Dyson's conjecture on p. 485, Vol. 1 and, ironically, the wrong sign of the term $\frac{1}{4}(\pi - \theta)^2$ in giving the correct version of Landen's incorrect formula on p. 424, Vol. 1. The number of mathematical descendants of Karl Weierstrass (1815-1897) needs almost daily updating. According to Mathematics Genealogy Project it is 38,568 but here we find 16,585—obviously from the time of the 2011 publication.

This is a superb book and one of the reasons for that is Roy's writing which is engaging and never dry. I recommend it highly to anyone with an interest in history of mathematics, analysis and number theory. Roy's book should really be on the shelf of every lover of mathematics.

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MARTIN LUKAREVSKI
Department of Mathematics and Statistics,
University “Goce Delcev”, Stip, North Macedonia
e-mail: *martin.lukarevski@ugd.edu.mk*

Geometry Transformed by James R. King, pp 282, £91.50 (paper), ISBN 978-1-47046-307-6, American Mathematical Society (2021)

This interesting and nicely written text offers an approach to Euclidean geometry that, although it has some historical precedent, is rarely discussed in the current textbook literature. It is an axiomatic approach, which in itself is not uncommon (see, for example, *Axiomatic Geometry* by Lee) but the axioms are somewhat nonstandard and focus on geometric transformations rather than more traditional axiom subjects like the Euclidean parallel postulate or triangle congruence criteria.

As an undergraduate more than a half-century ago, I took a course on geometric transformations, the text for which was a book by Ewald called *Geometry: An*

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Introduction, which I didn't much like: I found it hard to read, and I seem to recall that I thought that some of Ewald's proofs were incorrect. (It must have seemed that way to the professor, too, because I remember that in class lectures he added an axiom to those in the text.) One thing about the book that I did like, however, was its emphasis on geometric transformations, which I thought was a fascinating blend of geometry and algebra. Ewald's book even went so far as to approach the subject from an axiomatic viewpoint; the author indicated that this was influenced by the work of Bachmann in the 1950s. Apparently Bachmann's work was preceded by other work, decades earlier, by people like Thomsen and Schmidt.

In the years since then, a number of textbooks on the subject of geometric transformations have been published. Many of them did not approach them axiomatically but instead developed (or assumed) the basic facts about Euclidean geometry, and then introduced transformations, defining each of the basic transformations geometrically. Books along these lines include Martin's *Transformation Geometry*, *Transformational Plane Geometry* by Umble and Han, *Continuous Symmetry* by Barker and Howe, and *Symmetries* by Johnson. There are also books that study, at least in part, geometric transformations in non-Euclidean geometry, such as the book by Lee mentioned above and Greenberg's *Euclidean and Non-Euclidean Geometries: A Historical Approach*. All of these books use the underlying geometry to prove facts about the transformations; none of them write down axioms for the transformations themselves. The closest that I have seen to this latter point of view is the book by Lee mentioned in the first paragraph above; after spending an entire book discussing Euclidean (and hyperbolic) geometry from an axiomatic point of view, Lee devotes a short appendix to explaining how the SAS congruence axiom in his set of axioms can be replaced by an equivalent one that provides for the existence of reflections.

Over the years, also, the subject of geometric transformations has percolated down (at least in the United States) to secondary school mathematics education. Geometric transformations now make up part of what is known here as the Common Core syllabus for high school geometry. At the same time, however, there has been an unfortunate trend away from proof at that level. While the high school geometry course that I took around 1965 routinely required students to read and produce "two column" proofs, the students in my upper-level geometry courses at Iowa State University consistently tell me that they did few if any proofs in their high school courses.

The book now under review returns us to Ewald's approach to geometric transformations, though it is much better written, more accessible to students, and more fun to read. It is largely Euclidean in coverage; although passing reference to non-Euclidean geometry is made, no systematic development of that subject is made.

The text begins with a survey chapter, explaining the role of rigid motions in formulating the notion of congruence. (Rigid motions, as defined here, are not synonymous with isometries; the latter are distance-preserving bijections, but the former are also assumed to preserve angles. Later it is proved that this last condition actually follows as a theorem.) Very few facts about the Euclidean plane are used in this chapter except the notion of distance and angle. This paucity of information is remedied in the next chapter, which introduces axioms for the plane. The author's selection of axioms is in the spirit of Birkhoff rather than Hilbert: there are, initially, axioms for incidence and plane separation, and also ruler and protractor axioms that allow for measurement of distance and angles in terms of real numbers. Following these four, axioms related to geometric transformations are stated: the reflection axiom asserts that given any line, there is a non-identity rigid motion that fixes every point on that line; the dilation axiom essentially states that a dilation preserves angle measure and scales all distances by the stretching factor.

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Chapters 3 through 6 begin to explore the consequences of these axioms, with the dilation axiom not being used much yet. These chapters develop both the basic properties of isometries (e.g., that any one can be written as the product of at most three reflections, and that any isometry is a rigid motion) and also the most basic facts of Euclidean geometry (e.g., the standard congruence criteria, the base angles of an isosceles triangle are equal, the sum of the angles of a triangle is 180 degrees). Some less basic material (e.g., Fagnano's Problem) is also included. A lot of the results in these chapters are actually theorems of what is often called neutral geometry—i.e., that body of geometry which does not assume any parallel postulate at all, either Euclidean or non-Euclidean. The author mentions neutral geometry but does not dwell on it. For example, although he proves in chapter 4 (and again in chapter 7) that the sum of the angles of a triangle is 180 degrees, he then proves in chapter 6 the weaker result that an exterior angle of a triangle is greater than either remote interior angle, using a proof that does not rely on the dilation axiom. In doing so, the author points out that he is deliberately using a proof that is valid in neutral geometry, "a point that will be of interest to some readers."

The next three chapters discuss aspects of Euclidean geometry in which the dilation axiom plays a more prominent role. Chapter 7 explains how this axiom can be used to prove the Euclidean parallel postulate (sometimes called Playfair's postulate—that through a point not on a line, there is exactly one line parallel to the given line, which is equivalent to Euclid's Fifth Postulate). Chapter 8 explores the concept of similarity, for which the dilation axiom is tailor-made, and chapter 9 addresses the subject of area in Euclidean geometry, proceeding from polygonal figures to more subtle ones like the circle.

In chapter 10, the artistic aspects of rigid motions (wallpaper and frieze groups) are discussed, after some background material discussing how rigid motions multiply together. This is a lengthy chapter and, I thought, perhaps the most technical one in the book.

The final chapter of the book introduces coordinates. The author discusses not only traditional affine and Cartesian coordinates but also barycentric coordinates, another topic not often covered in geometry textbooks.

The transformation-theoretic axiomatic approach taken in this text is interesting and, because it is somewhat nonstandard, this book should make an excellent reference for people interested in the foundations of geometry and/or the ways to teach it. But the book itself was not written primarily as a reference but as a textbook. The student reader is always kept in mind: the writing is clear and detailed, the author takes pains to motivate new ideas as they arise, and on occasion some result is looked at from different perspectives and given different proofs.

There are a number of end-of-chapter exercises, including student projects. The projects tend not to be theoretical but rather of the do-it-yourself variety, requiring the student to engage in cutting, pasting and folding (or using a computer). Instructors who like to assign proof problems, or problems calling for the construction of examples or counter-examples, may wish there was a somewhat bigger selection available. Solutions are not provided in the text, and there is, to my knowledge, no solutions manual for faculty.

For any person considering using this book as a text, one aspect of this material must be kept in mind. A rigorous, axiomatic approach to Euclidean geometry, of any kind, has one inherent pedagogical drawback: to do things properly (as the author of this book does), considerable time must be spent on proving "obvious" results for which many students may have trouble seeing the need for rigorous proof. For example, early on in this book, the author proves the following fact: given a line

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containing distinct points *A* and *B*, and a point *C* not on the line, then there is exactly one other point *D* such that angles *CAB* and *BAD* are equal. Of course, learning why it is necessary to prove such results is an important part of the education of a mathematics student, but it still takes time and can, for some, get a bit boring. I think the author here should be commended for taking the time and trouble to do things right, but instructors using this book may wish to think about how well a rigorous approach like this might play out in the classroom.

The author ameliorates this problem to some extent by not feeling the need to prove everything, and occasionally one finds places in the book where a proof has been deliberately omitted because it is overly technical or subtle. On such occasions, however, the author makes clear that this omission is noted, rather than glossed over. And, despite the fact that a lot of time is spent on rather elementary aspects of geometry, proved rigorously and axiomatically, the author also manages to include a reasonable amount of what might be considered more advanced material, including, for example, Ceva's Theorem.

I have a few minor quibbles. It might have been nice if there had been an introductory chapter discussing the axiomatic method in general, and perhaps comparing it with the less-than-rigorous approach taken by Euclid. Such an introductory chapter might also have introduced the concept of a model, which could have enhanced the author's brief comment that all the axioms but the Dilation Axiom are valid in neutral geometry (and hence in any model of hyperbolic geometry).

It would also have been nice if the author had indicated what parts of each chapter could be omitted. I suspect that even if an instructor concentrates on the first eight chapters, there is still more material here than can be conveniently discussed in a semester course. Particularly since the back cover of the book stresses its flexibility for different syllabi, a good discussion or table of chapter dependence, and some indication of just what syllabi are being referred to, would have been useful. And finally, the Index to the book could be improved; Ceva's Theorem, for example, does not appear under "Ceva" or "Theorem" but under "Triangle".

But these are, as I said, relatively minor quibbles. Overall the book is a valuable one; it is always nice to see a new approach to an old subject, particularly when the material is handled as deftly as it is here. Instructors teaching geometry, or just interested in the subject for their own pleasure, should definitely look at this book.

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MARK HUNACEK
Iowa State University,
Ames, IA 50011, USA
e-mail: *mhunacek@iastate.edu*

Basic statistics with R by Stephen C. Loftus, pp 283, £57.95 (paper), ISBN 978-0-12820-788-8, Academic Press/Elsevier (2021)

This book aims to introduce the basics of statistical inference to a non-specialist readership, presumably that addressed by the headline that Loftus quotes from *The New York Times*: "For Today's Graduate, Just One Word: Statistics". Mathematical theory is cut to a minimum; its place is taken by diagrams and calculations on real data sets, using the open-source language R. Chapters on the statistical concepts are interspersed with instructions on using R, and there are a few exercises in each chapter, with answers at the end. The book is, therefore, suitable for self-study by anyone who wants a hands-on acquaintance with analysing data.

The touch is quite light, at least until the later chapters on testing and confidence intervals for two parameters where there is a risk of the text becoming a somewhat indigestible list of formulae. The explanations are largely clear, if conventional