DIAGNOSTIC OF ASTROPHYSICAL PLASMA IN NEIGHBORHOOD OF NEUTRON STARS

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If in the neighborhood of neutron stars exist clouds of hydrogen atoms, they are the natural astronomical object for realization of the model of hydrogen atom in a strong magnetic field $\sim 10^8$ T.

When the charged particle of velocity $\vec{v}$ moves in the homogeneous magnetic field $\vec{H}$ in the coordinate system fixed on the particle, there appears an electrical field $\vec{E} = \left(\vec{v} \times \vec{H}\right)/c$, where c is the velocity of light and $\hbar = m = e = 1$.

So, our problem of motion of the hydrogen atom in a strong magnetic field is equivalent of consideration of hydrogen atom in crossed electric $\vec{E}$ and magnetic field $\vec{H}$. The Hamiltonian $\mathcal{H}$ has the form

$$\mathcal{H} = \mathcal{H}_0 + V_1 + V_2,$$

where

$$\mathcal{H}_0 = -\frac{1}{2}\Delta - \frac{1}{r}, \quad V_1 = \vec{E}\cdot\vec{r} + \frac{1}{2c}\vec{H}\cdot\vec{L}, \quad V_2 = \frac{1}{8c^2}\left(\vec{H}\times\vec{r}\right)^2,$$

and $\vec{L} = \vec{r}\times\vec{p}$ is an angular momentum. The eigenvalue problem was first investigated in first order perturbation theory (neglecting the term V_2) (see Born, Pauli and Zimmerman et al.). The basic energy term is $\mathcal{E}_0 = -1/n^2$.

The first correction to the energy has the form $\mathcal{E}_1 = \omega q$, where $q = n' + n''$ and $n', n'' = -j, -j+1, \cdots, j, \quad j = (n-1)/2$, while ω is the modulus of the vector $\vec{\omega} = \vec{H}/(2c) - 3n\vec{E}/2$.

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The quadratic, in the field intensities, correction $\mathcal{E}_2$ to the energy is the sum of the second order correction from V_1 and the first order correction from V_2. The problem is solved through the separation of the variables in elliptic cylindrical coordinates on a sphere in four-dimensional space (Solov'ev and Braun). Introducing the operators $\vec{I}_1 = \left(\vec{L} + \vec{A}\right)/2$, $\vec{I}_2 = \left(\vec{L} - \vec{A}\right)/2$, where $\vec{A}$ is the Runge-Lenz vector $\vec{A} = \vec{p} \times \vec{L} - \vec{r}/r$, and $\gamma = 3ncE/H$. The second-order correction in energy has the form

$$\mathcal{E}_2 = \frac{n^4E^2}{16}\left[3q^2 - 17n^2 - 19 - \frac{6}{1+\gamma^2}\left(n^2 - 3q^2 - 1\right)\right] + \frac{n^2H^2}{16c^2}\left(3n^2 + 1 - q^2 + \lambda\right).$$

Here λ is the eigenvalue of the operator $\Lambda = b(I_{1\alpha} - I_{2\alpha})^2 - 16I_{1\beta}I_{2\beta}$, in which $b = \gamma^2 - 1 - 2/\left(1+\gamma^2\right)$, and $I_{i\alpha}$ is the component of $\vec{I}_i$ along the vector $\vec{\omega}_i$, while $I_{i\beta}$ is the component of $\vec{I}_i$ in the direction lying in the $(\vec{\omega}_1, \vec{\omega}_2)$ plane and orthogonal to the vector $\vec{\omega}_i$.

The eigenvalues λ cannot be computed analytically but the problem reduces to the solution of the difference equation

$$\left\{\left[(n-q)^2 - (k-1)^2\right]\left[(n+q)^2 - (k-1)^2\right]\right\}^{1/2} C_{k-2} + (bk^2 - \lambda)C_k + \left\{\left[(n-q)^2 - (k+1)^2\right]\left[(n+q)^2 - (k+1)^2\right]\right\}^{1/2} C_{k+2} = 0,$$

where $n' - n'' = k$ while C_k are the coefficients in the expansion of the correct zeroth-order functions in terms of the basics functions.

Contrarry to the first-order correction which is a linear function of principal quantum number n, the second-order correction in energy is proportional to n^6E^2, and also to n^4H^2/c^2. In the case of a strong magnetic field (neutron star) and big n (Rydberg atom) the last term is not negligible. The presence of the electrical field leads to a spectral modification, by interpreting which we can, in principle, determine the velocity v and, after that, the temperature of the plasma.

References

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