MACHINE CALCULATIONS OF EVOLUTION AT CONSTANT MASS

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This communication is concerned with recent computations made with the aid of the IBM 704. The problem was set up in collaboration with C. B. Haselgrove, using a method already described in detail [1]. Facilities for making the computations were generously provided by the IBM Corporation.

It is important to know how far the zero-age main sequence is dependent on the initial composition of a group of stars, particularly with reference to the use of the main sequence for

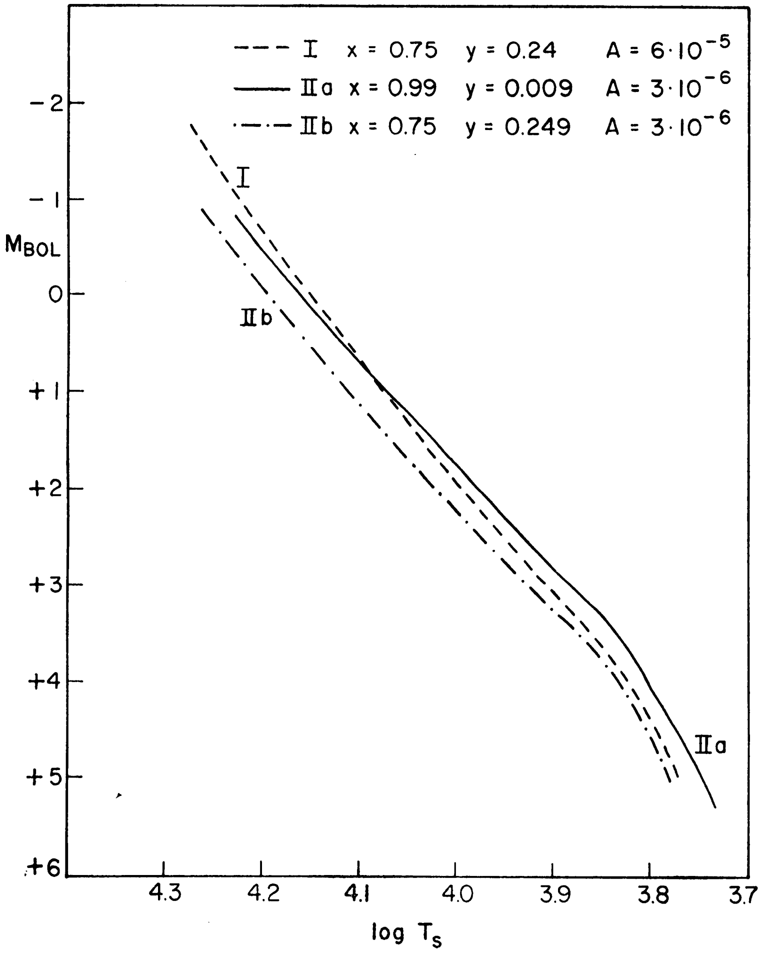

FIGURE 1. The computed zero-age main sequence for stars of various composition. Case I is for Population I stars; Case II a and II b for Population II stars and shows the effect of variation of helium, Y, to hydrogen, X, abundance.

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calibrating various types of distance indicator. The relevant portion of the main sequence in this connection ranges from absolute magnitude + 5 to about 0.0. It is shown in Figure 1 for the following three cases :

(I)	$X = 0.75$,	$Y = 0.24$,	$A = 6 \times 10^{-5}$,
(II a)	$X = 0.99$,	$Y = 0.009$,	$A = 3 \times 10^{-6}$,
(II b)	$X = 0.75$,	$Y = 0.249$,	$A = 3 \times 10^{-6}$,

where X is the hydrogen concentration by mass, Y the helium concentration by mass, and A is the abundance of metal atoms by number compared to hydrogen. Case (I) corresponds to a typical Population Type I star. The metal abundance in (II a) and (II b) is appropriate to Population Type II stars. The helium concentration in Type II stars is unknown, so that the two cases (II a) and (II b) have been chosen to provide the opposite extremes of very low and very high helium concentration.

It is seen that the maximum variation between the Type I main sequence and the two extremes for Type II amounts to about 0.4 mag., the variation being reckoned at a fixed surface temperature. Somewhat surprisingly, the Type II sequence, case (II), lies above the Type I sequence at the faint end. *

Turning now to evolution away from the zero-age main sequence, an important datum in the theory of stellar evolution is the time required for a star to move away to the right of the main sequence. This evolution time is close to, but somewhat less than, the total lifetime of a star. Results for a number of cases are given in Table 1.

Table 1

Evolution times for type I stars						
M	30.1	8.92	3.89	1.52	1.09	
m_{Bol}	− 8.1	− 4.9	− 1.6	+ 2.6	+ 4.5	
T	5.55	25.7	128	1390	9250	(7400)
K	4.06	3.20	1.82	1.00	1.65	
Effective surface temperature at zero age						
(°K)	44,104	27,979	18,342	8,641	6,219	
Convective core in terms of total mass						
	0.58	0.39	0.24	0	0	

M = mass in terms of the Sun,
m_{Bol} = absolute bolometric magnitude at zero age,
T = evolution time in 10^6 years,
K = (evolution time × luminosity)/(mass), the limit of K being chosen so that K = 1.00 when M = 1.52.

It is worth pointing out that the lifetimes of massive stars are considerably greater (by a factor of about 3) than might be expected from the lifetimes of the fainter stars. The reason is of course that the stars are not homologous; the fainter stars have no convective cores, whereas a massive star possesses a convective core that contains an appreciable fraction of the total mass.

* *Footnote added in proof:* Allowance for the formation of Be reduces these differences to less than 0.4 mag.

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Two values are given for the evolution time in the case of mass 1.09 ⊙. The lower value represents a reduction by 20 per cent, which probably somewhat exaggerates the effect of neutrino loss from the decay of B^8. That is to say, the lifetime should lie in the range between the two values given in the table.

The details of the evolutionary tracks, as these stars move away from the main sequence, are shown in Figure 2. The scale factors are of course different in the different cases. They

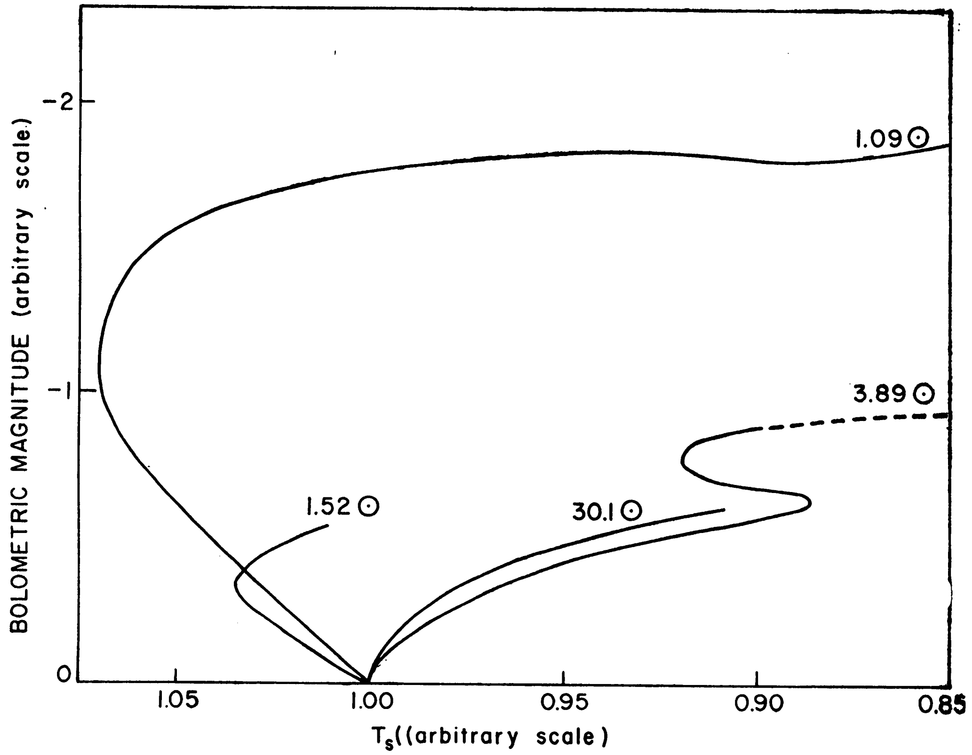

FIGURE 2. Evolutionary tracks for Population I stars of different masses, as they move away from the main sequence; both coordinates are normalized to the initial main sequence.

are immediately obtained from the values given in the above table. There are many comments that can be made on these tracks :

(a) Faint stars begin their evolution by moving upwards parallel to the main sequence and then turn away to the right. More luminous stars move immediately away to the right.

(b) Except in the case of the smallest mass (1.09 ⊙), the evolution reaches a stage where there is a very rapid movement to the right. The evolutionary tracks have been discontinued in the figure at this stage, except for 3.89 ⊙, which was followed along the dotted track still farther to the right beyond the part shown in the figure. The time required for this latter part of the evolution turned out to be less than 1 per cent of the time spent on the earlier section of the track.

In clusters of limited population it is unlikely that stars will be found along these extremely rapidly moving sections of the evolutionary track. This feature provides a natural explanation of the Hertzsprung gap that separates the main sequence from the giants and supergiants.

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(c) The oscillation in the track of 3.89 ⊙ has already been found by Haselgrove and Hoyle [2], and by Kushwaha [3]. This oscillation seems always to occur when the star in question possesses a convective core during the early phases of the evolution.

(d) During the part of its lifetime that a star spends near the main sequence, the luminosity increases by about 0.5 mag., except for faint stars where the rise is about 1.5 mag. The different modes of energy generation, carbon-nitrogen cycle and proton chain, play an important part in promoting this difference.

(e) The difference between 1.09 ⊙ and 1.52 ⊙ is much greater than was expected before the calculations were performed, a circumstance that seems to arise from opacity effects. The Keller-Meyerott opacity values yield a marked change of internal opacity as the mass decreases below 1.5 ⊙.

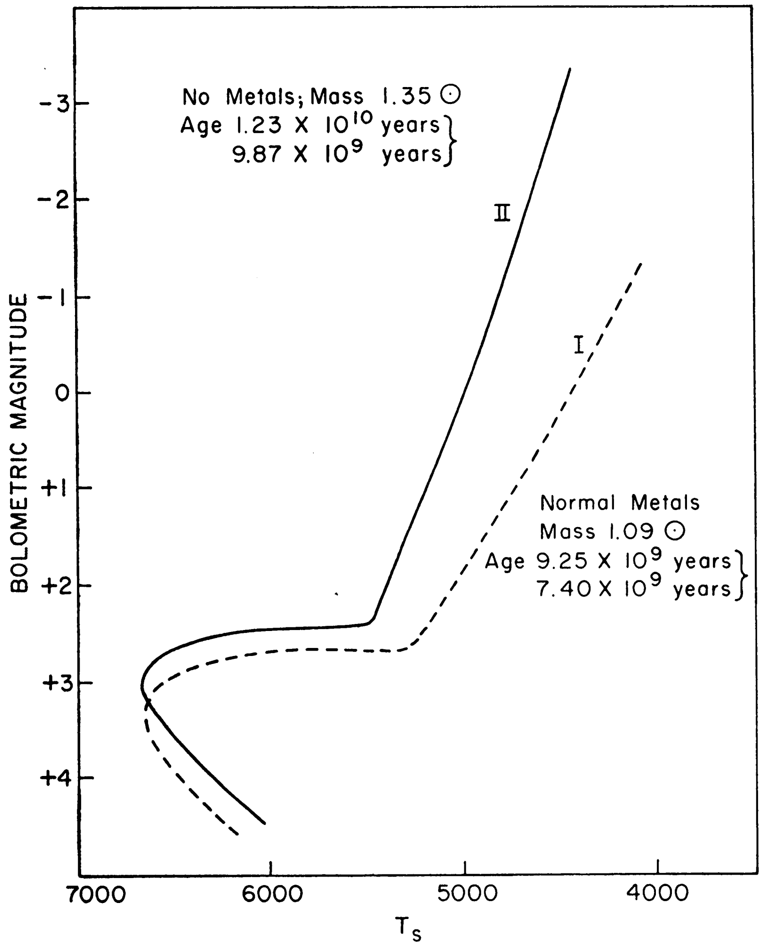

FIGURE 3. Computation of evolutionary tracks for Population II (upper solid curve), no metals, and for Population I, normal metals (lower dashed curve).

Turning now to a comparison of Type I and Type II giants Fig. 3 gives the calculated evolutionary tracks of stars that begin their evolution at about $+ 4^{m}.5$. The results fall near the color-magnitude arrays for the cluster M67, and for the globular clusters respectively. It is emphasized that bolometric absolute magnitudes are plotted in the figure. Allowance for bolometric corrections artificially increases the gap between the two sequences.

The lower section of Type II track was calculated with far greater precision than was obtained previously by Hoyle and Schwarzschild [4] and by Haselgrove and Hoyle [3], the latter authors using too high a value for the carbon-nitrogen reaction rate. The new results show a longer initial movement along the main sequence before the track moves away to the right. This increases the age estimate for the globular clusters considerably, the present value being in excess of 9×10^{9} years, even after full allowance has been made for neutrino loss through the production of B^{8}.

It scarcely seems possible to reduce the age by starting the evolution above $+ 4^{m}.5$,

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since the resulting track would then lie far too high in the diagram. Indeed the sweep to the right at $+ 2^m.5$ is about as high as observation will tolerate.

It is important to add that, although the evolutionary track for Type I appears to be lower than for Type II, the calculated age for Type I is less than for Type II. This arises because more hydrogen is burned at the corresponding stages of the track in the Type II case. This result emphasizes the danger of qualitative arguments which would suggest an opposite result.

Lastly, results are given in Table 2 for extremely massive stars with $X = 0.75$, $Y = 0.24$. It turns out that such stars are wholly convective and hence remain of uniform composition.

Table 2

Very massive, wholly convective stars

Mass ⊙	m_{Bol}	Effective surface Temperature (deg. K)	Radiation Pressure / Gravity
140.0	− 12.50	58,264	0.557
200.2	− 13.28	60,990	0.657
452.2	− 14.47	63,896	0.810
1034	− 15.55	65,376	0.929
2363	− 16.57	66,174	1.013
5405	− 17.53	66,699	1.066
12,350	− 18.47	67,104	1.103

For burning of 1/3 of the hydrogen, the lifetime is close to 10^6 years throughout the table. The last column refers to material at the photosphere. Evidently this ratio must be less than unity for the surface layers to be stable.

On the basis of these results it would seem as if stars with masses up to about 1000 ⊙ possess an equilibrium state at the value $X = 0.75$, although such an equilibrium state is probably dynamically unstable. Dynamical instability is associated with the generation of energy by nuclear processes, and will not arise until a massive condensing star has attained a structure that approximates to these equilibrium forms.

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