Neutrino Transport in a Type II Supernova Environment

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Abstract

We discuss the full problem of neutrino transport in the context of a Type II supernova environment. We describe the "standard model" of Type II supernova, which involves the core collapse and core bounce of an $8-20M_\odot$ star. Although the shock produced in the standard model is supposed to expel the outer layers of the star, this does not happen in the best numerical collapse calculations without considerable numerical tweaking, if at all. In this model, neutrino transport plays an important role. We describe the weakness of the best transport approximation currently used in collapse calculations, the flux-limited diffusion approximation. We investigate the effect of a full transport approach by constructing two different types of model atmospheres, with conditions chosen to closely approximate conditions found in full collapse calculations. The first model is to construct equilibrium plane-parallel hydrostatic neutrino atmospheres. The second is to solve the equations of neutrino hydrodynamics for a spherical shell of matter, with a piston and a source of neutrinos at the lower boundary. We describe in detail the neutrino interactions which are important in the supernova problem. We present the equations of neutrino hydrodynamics and neutrino hydrostatics which are solved in the two models, and emphasize the differences between photon transport and neutrino transport. We describe briefly a "toy" model, the neutrino Eddington atmosphere, which highlights the differences between photon and neutrino atmospheres. We present the results of our model atmosphere calculations for a few selected cases of interest. Finally, we summarize alternatives and extensions to the standard model which are of current interest and may hold the answer to the Type II supernova problem. However, the delicate balance between competing effects suggest that a rigorous calculation of neutrino transport will be required to obtain a definitive answer.

Introduction and Motivation

The cause of Type II supernovae is still something of a mystery. A "standard" model has emerged over the years which has the potential of explaining all of the observed properties of Type II supernova. Unfortunately, the numerical models which handle the complex physics of the problem most correctly also fail to produce any explosion at all without considerable fiddling (Wilson 1980, Bowers and Wilson 1982, Arnett 1983, Woosley this conference; for a excellent non-technical summary, see Bethe and Brown 1985; a more technical review is found in Trimble 1982, 1983). Neutrino transport plays a crucial role in this standard model.

The standard scenario begins with an $8-20M_\odot$ (the exact mass range is somewhat uncertain) star at the end of nuclear burning. The star becomes unstable due to nuclear dissociation and electron capture and begins to collapse. The collapse of inner $\sim 1/2$ of the $\sim 1.2M_\odot$ core proceeds homologously (Goldreich and Weber 1980), with velocity $v(r) \propto r$. As the star collapses, the neutrino losses from the star before neutrino trapping (see below) determine both the eventual lepton number per baryon Y_ℓ and the entropy per baryon s of the core, thereby determining the size and composition of the core:

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$M_{core} \approx M_{Chandrasekhar} \propto Y_\ell^2$. Larger homologous cores act as more efficient "pistons" at bounce, and mean that the mantle is correspondingly smaller. Larger cores and smaller mantles both imply that the shock forming at the edge of the core at bounce will have an easier time propagating through the mantle. (However, a smaller *iron* core is better for shock propagation, since the shock has to expend less energy dissociating the iron as it travels outward. The homologous region is smaller than the iron core of the more massive supernova progenitors.)

As the density rises above $\rho \sim 10^{11}$ gm cm^{-3} in the collapsing core, the "optical" depth to neutrino transport exceeds unity, and the neutrinos become trapped. After trapping, neutrinos are kept in equilibrium and play little role in subsequent events before bounce, beside contributing the pressure of a degenerate Fermi gas. At nuclear densities ($\rho \sim 10^{14}$ gm cm^{-3}) the homologously collapsing core bounces as a unit, and a shock is formed at its outermost edge. The shock propagates outwards along the decreasing density gradient until it reaches the "neutrinosphere". At this point, the energy the shock has lost by dissociating the heavy nuclei in the mantle has taken an significant toll, and the shock cannot survive (in the best numerical models, e. g. Wilson 1980, Bowers and Wilson 1982, Arnett 1983) the additional loss of energy to neutrino radiation. The shock dies, and there is no explosion. If neutrino transport is left out, an explosion is relatively easy to obtain, primarily because no lepton number is lost and the core is larger.

The best collapse codes currently use the flux-limited diffusion approximation to the transport equation to numerically calculate the solution of the transport equation for neutrinos. This approximation, although designed to be valid in both the thick (neutrino optical depth $\tau \gg 1$) and thin ($\tau \ll 1$) limits, breaks down in the neutrinosphere region where $\tau \sim 1$. The neutrinosphere region is large for energies of interest (figure 1), so the region where flux limited diffusion breaks down is also large. (At any one radius, the range of energies over which it breaks down is small.) However, changing the transport to numerically solve the exact transport equation in full collapse codes is not possible, since they would become prohibitively expensive to run. In order to investigate neutrino transport with greater accuracy, it is necessary to give up some of the generality of the full collapse codes.

The physics of stellar collapse is extremely complex. Typical densities of interest range from $10^8 < \rho < 10^{14}$ gm cm^{-3}, and typical energies are on the order of a few MeV. The star takes on the order of one second to go from instability through bounce to the propagation of an outward moving shock. The equations of neutrino hydrodynamics — hydrodynamics coupled to neutrino transport — must be solved to fully understand this problem.

In this paper we will summarize two attempts to study neutrino transport in physical situations similar to those found in supernova collapse. In the first approach, we solve the plane-parallel, *static* neutrino atmosphere problem. This approach is particularly useful in the study of the physical differences between photon transport and neutrino transport. In particular, there are *four* conserved fluxes, the energy flux $\mathcal{F}$, the electron lepton number flux $\mathcal{N}_e$, the μ lepton number flux $\mathcal{N}_\mu$, and the τ lepton number flux $\mathcal{N}_\tau$ ($\mathcal{N}_\tau$ and $\mathcal{N}_\mu$ are identically zero in supernova collapse). The second approach models the supernova mantle (the region above the homologous core) as a spherical shell of matter, with a piston at the bottom. The piston is driven into the shell at the bottom, and the equations of neutrino hydrodynamics are solved to study the fate of the shock.

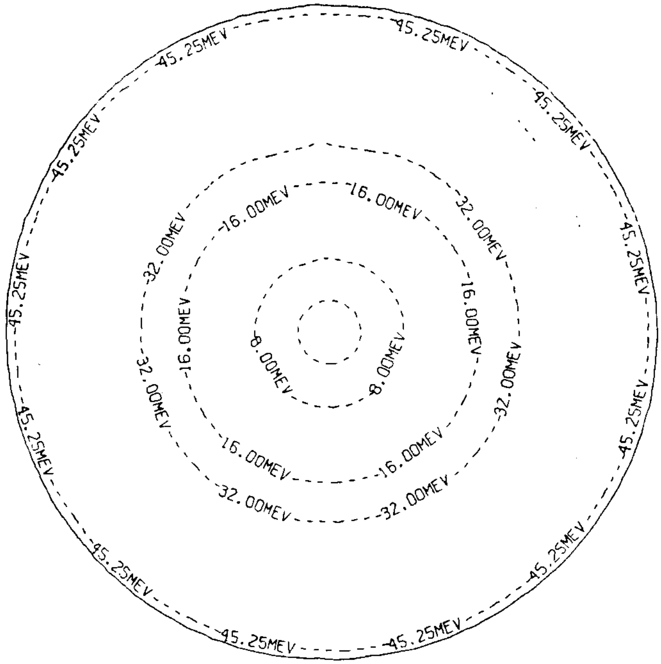

Figure 1: This graph shows schematically the inner 5×10^7 cm of a collapse calculation by Arnett (1985). The dashed lines indicate approximately where the neutrino optical depth τ_ν falls below unity for the indicated energy (the innermost dashed line is the decoupling radius of 4 MeV neutrinos). Note that the region over which neutrinos of interesting energies 1 $MeV < \epsilon <$ 50 MeV decouple is quite large. A "neutrinosphere" is by no means as definite as a photosphere is.

Basic Physics

Microphysics: Neutrino interactions

Figure 2 shows the neutrino interactions important in supernova collapse. They are divided into three types. Scattering reactions do not result in any lepton number exchange between the gas and the neutrinos, although energy may be exchanged. Absorption-emission processes result in the destruction or creation of a neutrino or antineutrino. Pair production-absorption mechanisms are the only ones which can produce μ and τ neutrinos in these physical conditions.

The important *scattering* reactions are:

1. neutrino scattering from nucleons ($n, p + \nu_{e,\mu,\tau} \longleftrightarrow n, p + \nu_{e,\mu,\tau}$). This occurs only via Z^0 meson exchange (neutral currents), and hence affects electron, μ, and τ neutrinos equally. Since nucleons are much more massive than typical neutrino energies, little energy exchange occurs by this reaction;

2. neutrino scattering from nuclei ($A + \nu_{e,\mu,\tau} \longleftrightarrow A + \nu_{e,\mu,\tau}$). This also occurs only via the neutral current. This scattering process is *coherent*, and the cross section is $\propto A^2$, where A is the atomic number of the nucleus in question;

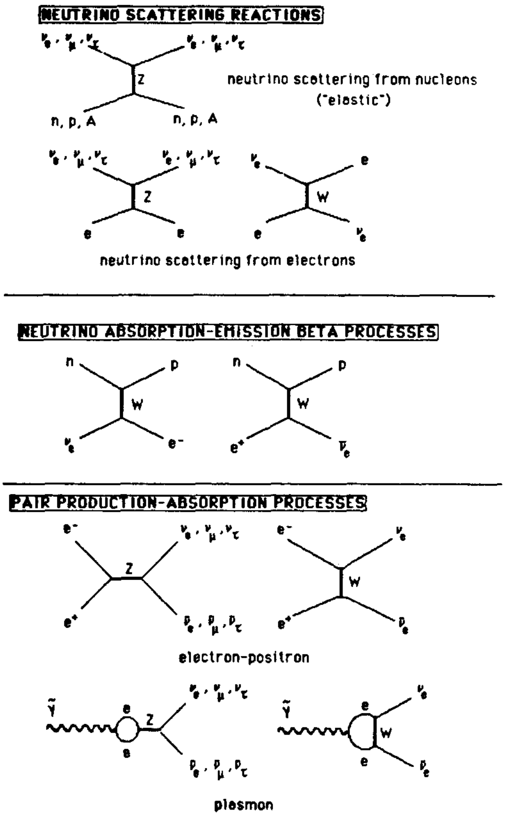

Figure 2: The neutrino interactions which are important in Type II supernova collapse. These interactions are subdivided into 1. scattering reactions, where neutrinos may interchange energy but no lepton number with the gas; 2. absorption-emission reactions, where electron neutrinos exchange both energy and lepton number with the gas; 3. pair production-absorption mechanisms, which again exchange energy but no net lepton number between neutrinos and the gas. The pair production-absorption reactions are the only production mechanisms for μ and τ neutrinos.

3. neutrino scattering from electrons ($e + \nu_{e,\mu,\tau} \longleftrightarrow e + \nu_{e,\mu,\tau}$). This occurs via the neutral current for μ and τ neutrinos, but scattering of electron neutrinos by electrons also has a charged current component (exchange of $W^{\pm}$ mesons). The electron mass is much less than typical neutrino energies, so this reaction is potentially useful for energy equilibration between neutrinos and the gas.

The absorption-emission beta mechanisms are the only ones by which lepton number can be gained or lost by the gas. The only important ones are:

1. electron neutrino emission-absorption ($n + \nu_e \longleftrightarrow p + e^-$), and
2. electron antineutrino emission-absorption ($n \longleftrightarrow p + e^- + \bar{\nu}_e$).

These reactions cause both chemical and energy equilibration of the gas and the neutrinos. Similar reactions do not occur for μ and τ neutrinos and antineutrinos because of the absence of muons and tauons in the gas, due to their large rest masses.

The *pair production-absorption* reactions of importance are

1. electron-positron pair annihilation-creation ($e^- + e^+ \longleftrightarrow \nu_{e,\mu,\tau} + \bar{\nu}_{e,\mu,\tau}$), and

2. conversion of plasmons (collective states of the electron-photon gas) into neutrino-antineutrino pairs and *vice versa* (*plasmon* $\longleftrightarrow \nu_{e,\mu,\tau} + \bar{\nu}_{e,\mu,\tau}$).

These reactions are neutral current reactions for μ and τ neutrinos, with a charged current contribution for electron neutrinos. They exchange energy, but no net lepton number, between neutrinos and the gas. These are the *only* production mechanisms available for μ and τ neutrinos. The plasmon reaction is of minor importance compared to the electron-positron reaction.

Macrophysics: Neutrino Transport Equations

The basic equations of neutrino hydrodynamics in spherical geometry, correct to order u/c where u is the fluid velocity, are as follows (see Schinder and Shapiro 1982a,b, 1983 for details): The neutrino transport equation *in an inertial frame* is given by

$$\frac{1}{c}\frac{\partial f}{\partial t} + \frac{(1-\mu^2)}{r}\frac{\partial f}{\partial \mu} + \mu\frac{\partial f}{\partial r} = \Gamma - \Lambda f. \tag{1}$$

Here $f(\epsilon, \mu, r)$ is the neutrino distribution function for any species of neutrino (see Mihalas 1978 for the similar equation for photon transport), $\Gamma(\epsilon, \mu, r)$ is the neutrino emissivity due to all sources, including scattering, $\Lambda(\epsilon, \mu, r)$ is the total neutrino absorptivity, corrected for "stimulated absorption", ϵ is the neutrino energy, and μ is the cosine of the angle made by the neutrino momentum with the outward normal. The transport equation in the comoving frame is quite complicated, and may be found (for photon transport) in Mihalas (1978). Similar equations may be written for each type of neutrino and antineutrino with the appropriate emissivity and absorptivity. Baryon conservation requires

$$\frac{\partial n_B}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}r^2 n_B u = 0, \tag{2}$$

where $n_B \equiv \sum_A (An_A) + n_p + n_n$. Electron lepton number conservation requires

$$\begin{aligned}\frac{\partial}{\partial t}(n_{e^-} - n_{e^+}) &+ \frac{1}{r^2}\frac{\partial}{\partial r}[(n_{e^-} - n_{e^+})ur^2] \\ &= -\int \frac{\epsilon^2 d\epsilon}{(hc)^3}\Bigg(\frac{\partial(f-\bar{f})}{\partial t} \\ &+ c\left[\frac{(1-\mu^2)}{r}\frac{\partial(f-\bar{f})}{\partial \mu} + \mu\frac{\partial(f-\bar{f})}{\partial r}\right]\Bigg)2\pi d\mu \\ &= -2\pi c\int \frac{\epsilon^2 d\epsilon}{(hc)^3}\left(\Gamma - \Lambda f + \bar{\Lambda}\bar{f} - \bar{\Gamma}\right)d\mu,\end{aligned} \tag{3}$$

the latter equality following from the neutrino transport equation (1). Here n_{e^-} and n_{e^+} are the number densities of electrons and positrons, respectively. Muon and tauon lepton number are automatically conserved, because these neutrinos are only produced in pairs. Conservation of momentum requires

$$\frac{\partial \rho u}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}r^2\rho u^2 + \frac{\partial P}{\partial r} + \rho\frac{\partial \phi}{\partial r} = -\sum_{i=e,\mu,\tau}\int \frac{\epsilon^3 d\epsilon}{(hc)^3}c\mu(\Gamma_i - \Lambda_i f_i + \bar{\Gamma}_i - \bar{\Lambda}_i\bar{f}_i)2\pi d\mu, \tag{4}$$

where use has been made of the neutrino transport equation to write the right hand side of the equation. Here ϕ is the gravitational potential, ρ is the mass-energy density

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(which includes the energy of the photon gas), and P is the matter plus photon pressure. Conservation of energy requires

$$\frac{\partial}{\partial t}\left(\rho\varepsilon + \frac{1}{2}\rho u^2 + \rho c^2\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho u\left(\epsilon + \frac{u^2}{2} + c^2\right) + Pur^2\right) + u\rho\frac{\partial \phi}{\partial r}$$
$$= -\sum_{i=e,\mu,\tau} \int \frac{\epsilon^3 d\epsilon}{(hc)^3} 2\pi c d\mu(\Gamma_i - \Lambda_i f_i + \bar{\Gamma}_i - \bar{\Lambda}_i \bar{f}_i), \tag{5}$$

where ε is the internal energy per unit mass of the gas.

In many supernova collapse models, the neutrino transport equation (1) is replaced by the flux limited diffusion approximation

$$\frac{dF}{dt} = \nabla \cdot D\nabla F(r,\nu) + \left(\frac{dF}{dt}\right)_{cap} + \left(\frac{dF}{dt}\right)_{comp} + \left(\frac{dF}{dt}\right)_{scat} + \left(\frac{dF}{dt}\right)_{ea}$$
$$+ \left(\frac{dF}{dt}\right)_{rad} + \left(\frac{dF}{dt}\right)_{pair} \tag{6}$$

(Bowers and Wilson 1982). Here D is the diffusion coefficient, F is the neutrino energy spectrum $\propto \epsilon^3 f$, and the dF/dt terms on the right hand side represent electron capture on nuclei, "field compression" (the effect that volume changes in gas elements have on the neutrinos inside of them), scattering, absorption and emission, pair production, and radiation acceleration. The diffusion coefficient D is written

$$D = \frac{\lambda c}{3 + \lambda \left|\frac{d\ln F}{dr}\right| \xi}.$$

Here λ is the mean free path of the neutrino species, and ξ is a dimensionless fine tuning function. The intent is to have equation (6) go to the proper result in both the free streaming ($\tau_\nu \ll 1$) and diffusion ($\tau_\nu \gg 1$) limits. Equation (6) goes to

$$\frac{dF}{dt} = \nabla \cdot \frac{\lambda c}{3}\nabla F + \ldots$$

in the diffusion limit, and to

$$\frac{dF}{dt} = \nabla \cdot \hat{\mathbf{r}} cF + \ldots$$

in the free streaming limit.

Plane-Parallel, Hydrostatic Atmospheres

In the plane-parallel hydrostatic case, equation (1) is replaced by

$$\mu\frac{\partial f}{\partial z} = \Gamma - \Lambda f, \tag{7}$$

where z is the height above the base of the atmosphere. The lepton number equation (3) is replaced by the requirement that lepton number flux be conserved,

$$\frac{\partial \mathcal{N}_{e,\mu,\tau}}{\partial z} = 2\pi c \int \frac{\epsilon^2 d\epsilon}{(hc)^3}\left(\Gamma_{e,\mu,\tau} - \Lambda_{e,\mu,\tau} f_{e,\mu,\tau} + \bar{\Lambda}_{e,\mu,\tau}\bar{f}_{e,\mu,\tau} - \bar{\Gamma}_{e,\mu,\tau}\right) d\mu$$

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$$= 2\pi c \frac{\partial}{\partial z} \int \frac{\epsilon^2 d\epsilon}{(hc)^3} \mu d\mu (f_{e,\mu,\tau} - \bar{f}_{e,\mu,\tau}) \tag{8}$$
$$= 0,$$

the momentum equation (4) by the requirement of hydrostatic equilibrium

$$\frac{\partial P}{\partial z} + \rho g = -\int \frac{\epsilon^3 d\epsilon}{(hc)^3} c\mu \sum_{i=e,\mu,\tau} (\Gamma_i - \Lambda_i f_i + \bar{\Gamma}_i - \bar{\Lambda}_i \bar{f}_i) 2\pi d\mu, \tag{9}$$

and the energy equation (5) by conservation of energy flux

$$\begin{aligned} \frac{\partial \mathcal{F}}{\partial z} &= \sum_{i=e,\mu,\tau} \int \frac{\epsilon^3 d\epsilon}{(hc)^3} 2\pi c d\mu (\Gamma_i - \Lambda_i f_i + \bar{\Gamma}_i - \bar{\Lambda}_i \bar{f}_i) \\ &= 2\pi c \frac{\partial}{\partial z} \sum_{i=e,\mu,\tau} \int \frac{\epsilon^3 d\epsilon}{(hc)^3} \mu d\mu (f_i + \bar{f}_i) \\ &= 0. \end{aligned} \tag{10}$$

Comparison of photon vs. neutrino transport

The differences between neutrino transport and photon transport are many (figure 3). Neutrinos carry lepton number (corresponding to the "flavor" of neutrino, μ, τ, or electron); the total lepton number of the star cannot change except by neutrino radiation. Both neutrinos and antineutrinos of all three flavors are important. Emissivities and absorptivities are very energy dependent ($\propto \epsilon^2$ at least, where ϵ is the energy of an individual neutrino). There are no constant cross-sections similar to Thomson scattering in photon transport. Neutrinos are Fermi-Dirac particles, not bosons like photons. Because of this fact, "stimulated absorption" (or "blocking") rather than stimulated emission occurs. Scattering of neutrinos by nucleons and nuclei, while energy dependent, is similar to Thomson scattering in that the energy of the neutrino is conserved. Unlike photon-electron scattering, neutrino-electron scattering can serve as an energy equilibration mechanism, since the neutrino energy can change by a large fraction of the initial energy. Some of these facts exclude the use of standard numerical techniques used in photon transport.

The differences in the above equations from those used in photon transport reflect the physical differences between photons and neutrinos and the differences in the physical regimes where neutrino and photon transport are important. For example, we have to use baryon conservation (equation (2)) instead of mass conservation, because changes in nuclear species means mass is no longer conserved (for instance, neutrons are $1.3 MeV$ heavier than protons). An extra equation is necessary to conserve lepton number. The energy "source" term (the right hand side of equation (5)) contains contributions from the six different neutrinos and antineutrinos. These differences are reflected in the plane-parallel hydrostatic limit by the requirement that four fluxes, energy, electron lepton number, muon lepton number, and tauon lepton number, be conserved instead of one.

Toy Model: The Neutrino Eddington Atmosphere

Before presenting the results of our numerical calculations, we will briefly describe a simple analytic plane-parallel neutrino atmosphere, the neutrino Eddington atmosphere. We assume for convenience that only electron type neutrinos exist in this atmosphere, with equal numbers of neutrinos and antineutrinos, so that $f = \bar{f}$, (or in other words,

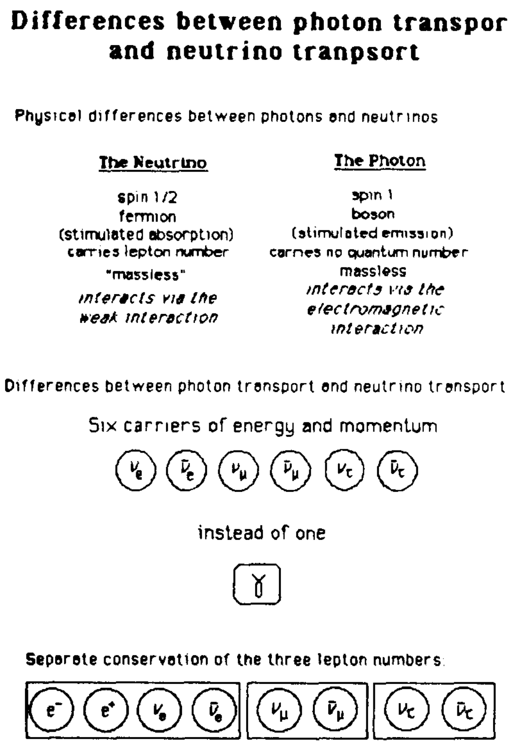

Figure 3: The differences between photon transport and neutrino transport are summarized.

the electron neutrino chemical potential is identically zero). We will also assume that the diffusion approximation

$$f = {}^{eq}f + \frac{\mu}{\Lambda} \frac{\partial\, {}^{eq}f}{\partial m}, \tag{11}$$

where ${}^{eq}f$ is the Fermi distribution at the local temperature T and zero chemical potential, is valid *throughout* the atmosphere. We also assume that the neutrino opacity may simply be written $\Lambda = \Lambda_o(\epsilon/\epsilon_o)^2$, where Λ_o and ϵ_o are constants. We also define the Rosseland mean opacity Λ_R by performing the appropriate integral over the diffusion equation (11) to get

$$\mathcal{F} = \frac{c}{3\Lambda_R} \frac{\partial U_\nu}{\partial m}, \tag{12}$$

where $U_\nu = (7/8)aT^4$ is the local energy density of neutrinos and antineutrinos.Then, with these assumptions, we can find the temperature T as a function of the column mass density m (details can be found in Schinder and Shapiro 1982a). The column mass density m (gm cm^{-2}) is defined so that $m = 0$ at the surface and m increases with depth. We find that $T(m)$ may be written in terms of the flux $\mathcal{F}$ according to

$$T^2 = \frac{9(hc)^3}{2\pi^3 c} \frac{\Lambda_o}{\epsilon_o^2} \mathcal{F} m + T_s^2, \tag{13a}$$

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where

$$T_s = \left[\frac{30(hc)^3}{7\pi^5 c}\mathcal{F}\right]^{\frac{1}{4}} \tag{13b}$$

is the temperature at the surface. We can further define the Rosseland mean optical depth τ_R,

$$\tau_R(m) \equiv \int_0^m \Lambda_R dm'. \tag{14}$$

With this definition, we may solve for τ_R in terms of m, and rewrite equation (12) in the more familiar form

$$T^4 = \frac{3}{2}T_s^4\left(\tau_R + \frac{2}{3}\right). \tag{15}$$

However, in this case, because of the energy dependence of the absorptivity Λ, τ_R itself is an integral function of $T(m)$. To get the actual matter profile, we will make the simple assumption that

$$P = (1 + Y_e)\frac{\rho kT}{m_p} + \frac{15}{24}aT^4. \tag{16}$$

Here Y_e is the total number of electrons *plus* positrons per baryon. The factor of $15aT^4/24$ represents the pressure of photons and neutrinos. This expression for P is not valid if electrons are degenerate, but is roughly valid after the shock has passed and electron degeneracy decreases. Now the equation of hydrostatic equilibrium requires that $P = gm + P_o$, where P_o is the pressure at the surface. We choose $P_o = 15aT_s^4/24$. Then we may find $\rho(m)$ by

$$\begin{aligned}\rho(m) &= \frac{m_p}{kT(m)}\left(P(m) - \frac{15}{24}aT^4(m)\right)\\ &= \frac{m_p}{k\sqrt{\frac{9(hc)^3}{2\pi^3 c}\frac{\Lambda_o}{\epsilon_o^2}\mathcal{F}m + T_s^2}}\left\{gm + P_o - \frac{15}{24}a\left(\frac{9(hc)^3}{2\pi^3 c}\frac{\Lambda_o}{\epsilon_o^2}\mathcal{F}m + T_s^2\right)^2\right\}.\end{aligned} \tag{17}$$

Note the difference between photon and neutrino atmospheres. In the photon case T^4 is proportional to $\mathcal{F}m$ when $\Lambda_R = \Lambda_o$ is constant. In the neutrino case, where

$$\Lambda_R = \frac{7\pi^5}{5}\Lambda_o\left(\frac{T}{\epsilon_o}\right)^2,$$

we find $T^2 \propto \mathcal{F}m$. The Eddington model atmosphere is shown in figure 4 where it is compared with more detailed model calculation.

Numerical Results

Plane parallel hydrostatic atmospheres

Equations (7-10) were numerically differenced and solved by a method described in detail in Schinder and Shapiro (1982a). Basically, we adopted the complete linearization scheme of Mihalas (1978). We implicitly differenced the neutrino transport equation (7), and solved it by linearization, iterating until convergence was achieved, and holding the values of gas variables (T, ρ, etc.) fixed. We then linearized the full set of equations (7-10) and solved them for new values of the gas variables. This procedure was continued until the

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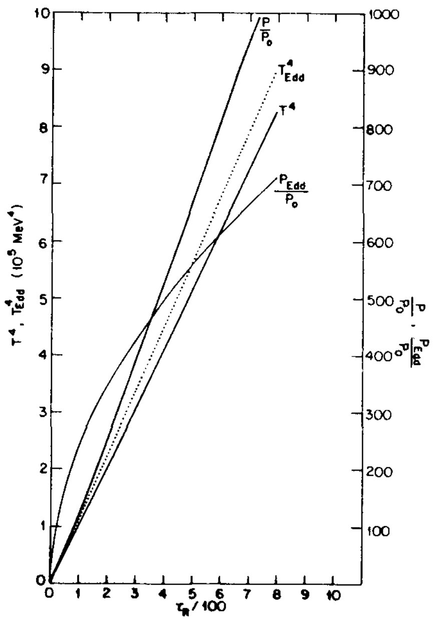

Figure 4: The quantity T^4 and the pressure P, calculated in a plane parallel atmosphere model as discussed below, is compared with the Eddington approximation developed above. The quantities T^4_{Edd} and P_{Edd} are the values predicted by the Eddington approximation. The pressures P and P_{Edd} are normalized to P_o which here has a value $P_o = 1.3 \times 10^{29}$ dyn cm^{-2}. The values of $\mathcal{F}$, $\mathcal{N}$, and g (defined below) are the same as those given in figure 7.

equilibrium solution was attained. This method requires the solution of $2 \times I \times J \times K$ dimensional matrix equations, where I is the number of spatial gridpoints (gridpoints in z), J is the number of energy gridpoints, and K is the number of angular gridpoints, by the Newton-Raphson method. The factor of 2 arises from the fact that there are reactions (the pair production-absorption reactions) which couple neutrinos and antineutrinos of the same flavor together. It is therefore necessary to solve the neutrino and antineutrino transport equations *simultaneously*. In the calculations we will describe below, $I = 40$, $J = 10$, and $K = 6$.

Hydrostatic atmospheres were modeled upon the collapse calculations of Arnett (1977) and Wilson (1980) by choosing values of the free parameters g (the gravitational acceleration), $\mathcal{F}$ (the energy flux), and $\mathcal{N}_e$ (the electron lepton number flux) similar to those they found just after bounce. We assumed that the atmosphere is plane parallel, consisting of neutrons, protons, electrons and positrons *only* (no nuclei), and that protons and neutrons are non-relativistic and non-degenerate. Electrons and positrons are assumed to be relativistic and of arbitrary degeneracy. Convection is not included. We assumed that the neutrino diffusion approximation may be used at the base of the atmosphere as a boundary condition.

Figure 5 and 6 show the results of one such calculation done with electron neutrinos and antineutrinos only (no μ or τ neutrinos). Figure 5 shows the specific energy flux F_ϵ (proportional to the first angular moment of the intensity $(I(\epsilon,\mu) = (\epsilon^3/(hc)^2)f(\epsilon,\mu))$ for the base of the atmosphere (the dotted lines) and the surface of the atmosphere (solid

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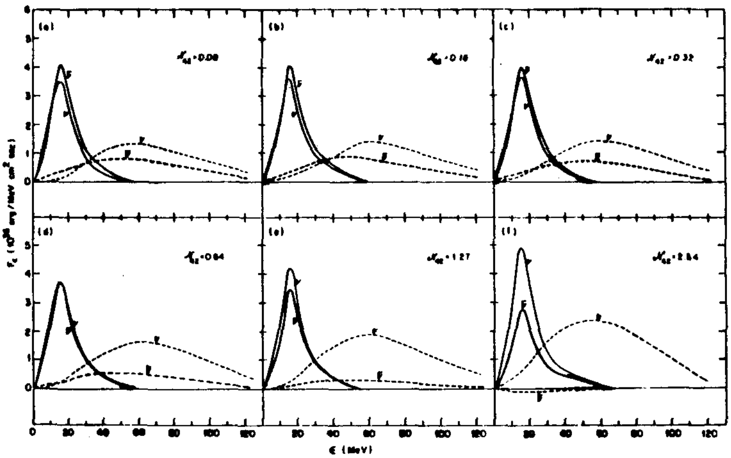

Figure 5: The specific energy flux $F_\epsilon = \int_{-1}^{1} \mu\, d\mu \epsilon^3 f$ for electron neutrinos and antineutrinos as a function of the neutrino (antineutrino) energy ϵ for six representative cases. The values of $\mathcal{F}$ and g have been held fixed at $\mathcal{F} = 1.58 \times 10^{38}$ erg cm^{-2} s^{-1} and $g = 2.36 \times 10^{12}$ cm^2 s^{-1}, respectively, while $\mathcal{N}_e$ increases by factors of 2 as indicated. The solid lines show values of the emergent flux at the surface, while the dashed lines are the values at the base of the atmosphere.

lines). The energy flux $\mathcal{F}$ is held constant at a value of 1.58×10^{38} ergs cm^{-2} sec^{-1}, so the sum of the areas under the neutrino and antineutrino curves is a constant. The gravitational acceleration g was held constant at a value of 2×10^{12} cm^2 sec^{-1}, corresponding to $M \approx 0.9 M_\odot$, $r = 7.1 \times 10^6$ cm. Accordingly, the luminosity $L = 4\pi r^2 \mathcal{F} = 1.0 \times 10^{53}$ erg cm^{-2} sec^{-1} The electron lepton number flux $\mathcal{N}_e$ was allowed to vary from a small value to a large value by factors of 2 as indicated. Notice that as the number flux increases, neutrinos increasingly predominate over antineutrinos. At the highest value of the number flux, neutrinos completely predominate (notice that at the base, antineutrinos actually carry a small *negative* flux). Figure 6 shows the density ρ, temperature T, and number of electrons per baryon Y_e. Notice that there is a density inversion for small values of $\mathcal{N}_e$ (the production of electron positron pairs in the high T bottom layers of the atmosphere is substantial), which disappears as the number flux increases. This is because the pairs are generating a substantial amount of pressure support.

Figure 7 shows the results of a calculation in which μ and τ neutrinos are included. For convenience, all of the number fluxes are set equal but small ($\mathcal{N}_i \ll (\mathcal{F}_i/\langle\epsilon_i\rangle)$): $\mathcal{N}_e = \mathcal{N}_\mu = \mathcal{N}_\tau = 7.925 \times 10^{40}$ cm^{-2} sec^{-1}. The energy flux $\mathcal{F} = 2.631 \times 10^{37}$ erg cm^{-2} sec^{-1}, and the gravitational acceleration $g = 6.14 \times 10^{11}$cm sec^{-2}. The spectra of μ and τ neutrinos are identical, since they interact identically with the gas. Here the sum of the areas under the electron, μ, and τ neutrino and antineutrino spectra is a constant. Notice that μ and τ neutrinos carry a large fraction of the energy, due to the importance of thermal pair annihilation.

Hydrodynamic Atmospheres

We will now briefly turn to the results of the hydrodynamic model. In this model, we take a spherical shell of gas, and by means of a piston at the bottom, we induce a shock. An initial neutrino spectrum is introduced at the base of the shell (the Fermi distribution at the local temperature and neutrino chemical potential is used). The gas consists of non-relativistic neutrons, protons, and two representative nuclei (helium and iron), and

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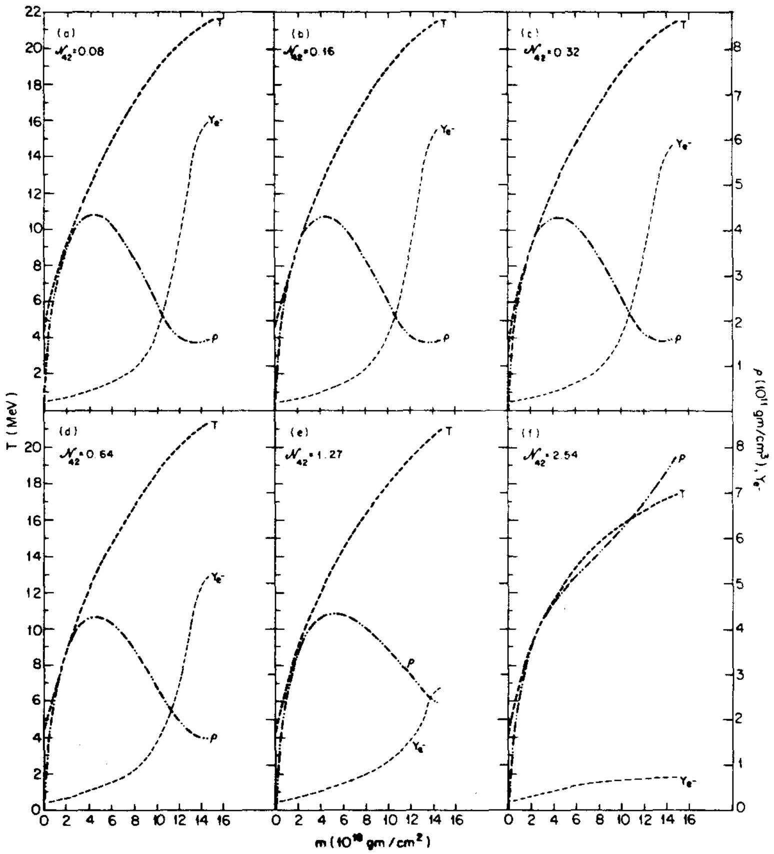

Figure 6: The temperature T, mass-energy density ρ, and number of electrons per baryon Y_{e^-} as functions of m for the cases shown in figure 5.

extreme relativistic electrons and positrons of arbitrary degeneracy. The equations of neutrino transport are differenced implicitly in the coordinate frame, and the equations of hydrodynamics are solved in the comoving frame using the Lax-Wendroff method. Transformations between the two frames are, of course, necessary to solve the problem correctly. It is difficult to solve the neutrino transport equation in the comoving frame because of the presence of the shock.

We present a single case here (a search of parameter space is currently underway). Figure 8 shows the model initially, just after the piston has started outward. In figure 8a, the velocity u is everywhere inward (except for the piston itself), and the density profile is decreasing outward. This initial configuration roughly mimics the outer atmosphere of an imploding core just prior to the passage of the outward shock. At $t = 0$ neutrino interaction processes are turned on everywhere in the shell. At the base of the shell, an outward flux of neutrinos, satisfying a (Fermi) distribution of neutrinos at the local temperature and neutrino chemical potential is introduced (the temperature T at the base of the atmosphere is at this time $7.6 MeV$, and the neutrino chemical potential $\mu_\nu = \mu_e + \mu_p - \mu_n = 83.7\ MeV$).

For $t > 0$ the coupled neutrino transport-hydrodynamic equations (1) - (5) are solved simultaneously for the shell. We assume that the shell sits in a constant gravitational

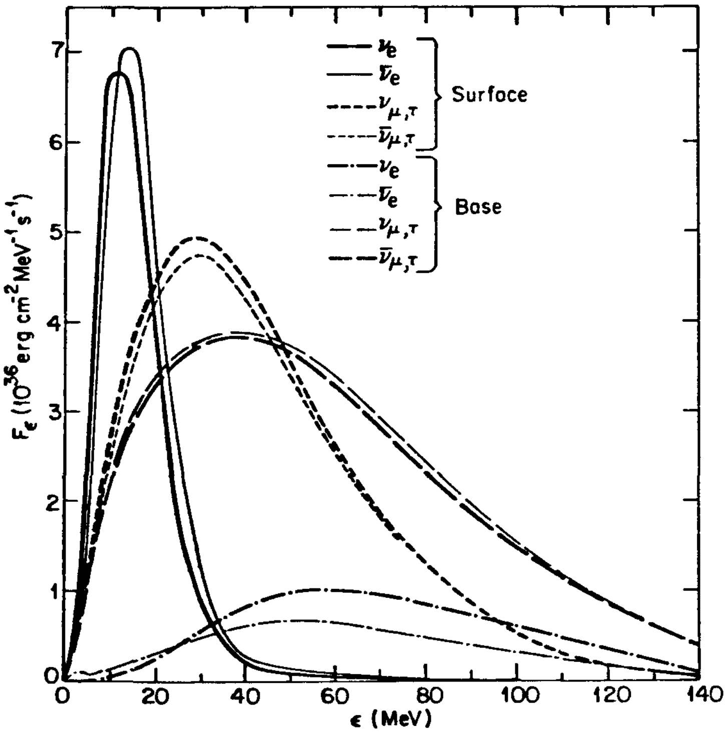

Figure 7: The specific energy flux F_ϵ for the three flavors of neutrinos and antineutrinos as a function of the neutrino (antineutrino) energy ϵ. The values of F_ϵ at the surface and at the base of the atmosphere for the various neutrino types are as indicated. The total energy flux $\mathcal{F} = 1.35 \times 10^{39}$ erg cm^{-2}s^{-1}, the lepton number fluxes $\mathcal{N}_e = \mathcal{N}_\mu = \mathcal{N}_\tau = 7.925 \times 10^{40}$ cm^{-2} s^{-1}, and the gravitational acceleration $g = 7.50 \times 10^{12}$ cm^2 s^{-1}.

potential corresponding to a constant core mass $M = 1.4M_\odot$; its self-gravitation is also included. The neutrino transport equation (1) is differenced implicitly and solved by complete linearization in the distribution function f, using the current values of gas variables. This value of f is then used in equations (2)-(5) to find the new values of the gas variables. This example was calculated with $I = 40$, $J = 10$, and $K = 7$.

In figure 8b, we show the quantities

$$\Delta_E = 2\pi c \int \frac{\epsilon^3 d\epsilon}{(hc)^3} d\mu (\Gamma - \Lambda f + \bar{\Gamma} - \bar{\Lambda}\bar{f}) \tag{18}$$

which quantifies the energy exchanged between the gas and neutrinos ($\Delta_E > 0$ implies energy flow from neutrinos to the gas), and

$$\Delta_N = 2\pi c \int \frac{\epsilon^2 d\epsilon}{(hc)^3} d\mu \,(\Gamma - \Lambda f + \bar{\Lambda}\bar{f} - \bar{\Gamma}) \tag{19}$$

which quantifies the exchange of lepton number between the neutrinos and the gas ($\Delta_N > 0$ implies lepton number flow from neutrinos to the gas).

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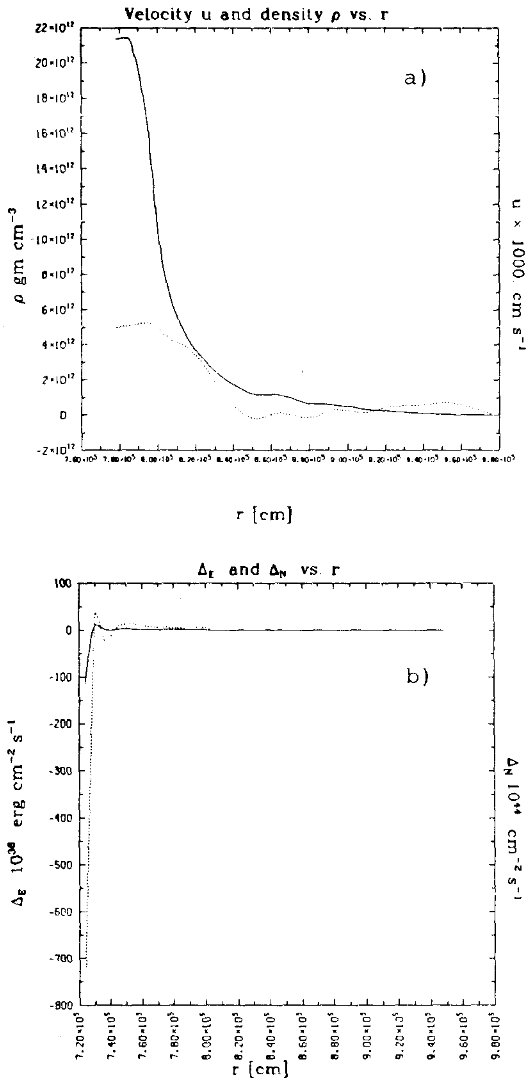

Figure 8: A snapshot of a spherical shell of matter shortly after the piston at the inner surface has begun to travel outward. A neutrino source at the local temperature and neutrino chemical potential is also located at the inner surface. Figure 8a shows the profile of the density ρ (the solid line) and the velocity u (the dotted line) vs. radius r. Figure 8b shows the quantities Δ_E (equation 18, the solid line) and Δ_N (equation 19, the dotted line).

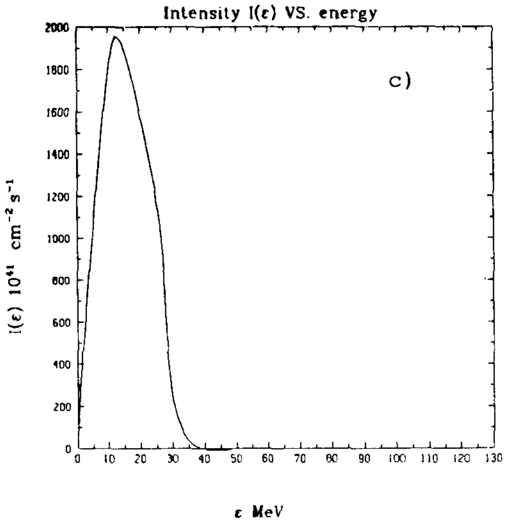

Figure 8c shows the emergent neutrino flux (F_ϵ at the outer edge of the shell)

Figure 9 shows the same case at a later time. Here the initial shock is just about to die. Notice in figure 9b that neutrinos are doing just what one would expect; they take energy from behind the shock ($\Delta_E < 0$) and transfer it to the gas in front of the shock ($\Delta_E > 0$). We have not yet found a single case, in an iron dominated atmosphere,

where the shock survives and may reasonably be expected to produce an explosion. The principle reason for this is the extraction of energy from the shock due to the dissociation of iron into helium.

Computer Requirements

The hydrostatic results presented above were done on the Cray 1A at the National Center for Atmospheric Research (NCAR). Each production run ($I = 40$, $J = 10, K = 6$) took roughly 15 CPU minutes. The 750,000 word memory was filled. The hydrodynamic code is currently running on the FPS-164 array processor of the University of Chicago's Astrophysics Computation Facility. Each production run ($I = 40$, $J = 10$, $K = 7$) takes about 24 CPU hours and about 4 megabytes of memory. The accuracy of the hydrostatic calculations was a few percent, and the final hydrodynamic calculations should have a similar accuracy.

Type II Supernovae: Alternative Explanations

We will briefly describe here some other possibilities, which may eventually prove more fruitful than the standard scenario for explaining Type II supernovae. Wilson (1983) has found that when carried to times much later than such calculations have been carried before, the shock is reenergized as some of the neutrinos diffusing out from the core deposit their energy in the shock region. The shock then proceeds to move out and cause an explosion. However, this mechanism may be important only for the more massive end of the supernova mass spectrum $\sim 20M_\odot$, since changes in the nuclear equation of state may allow smaller stars to explode (Bethe and Brown 1985). For larger stars at least, this neutrino reheating mechanism may save the standard model.

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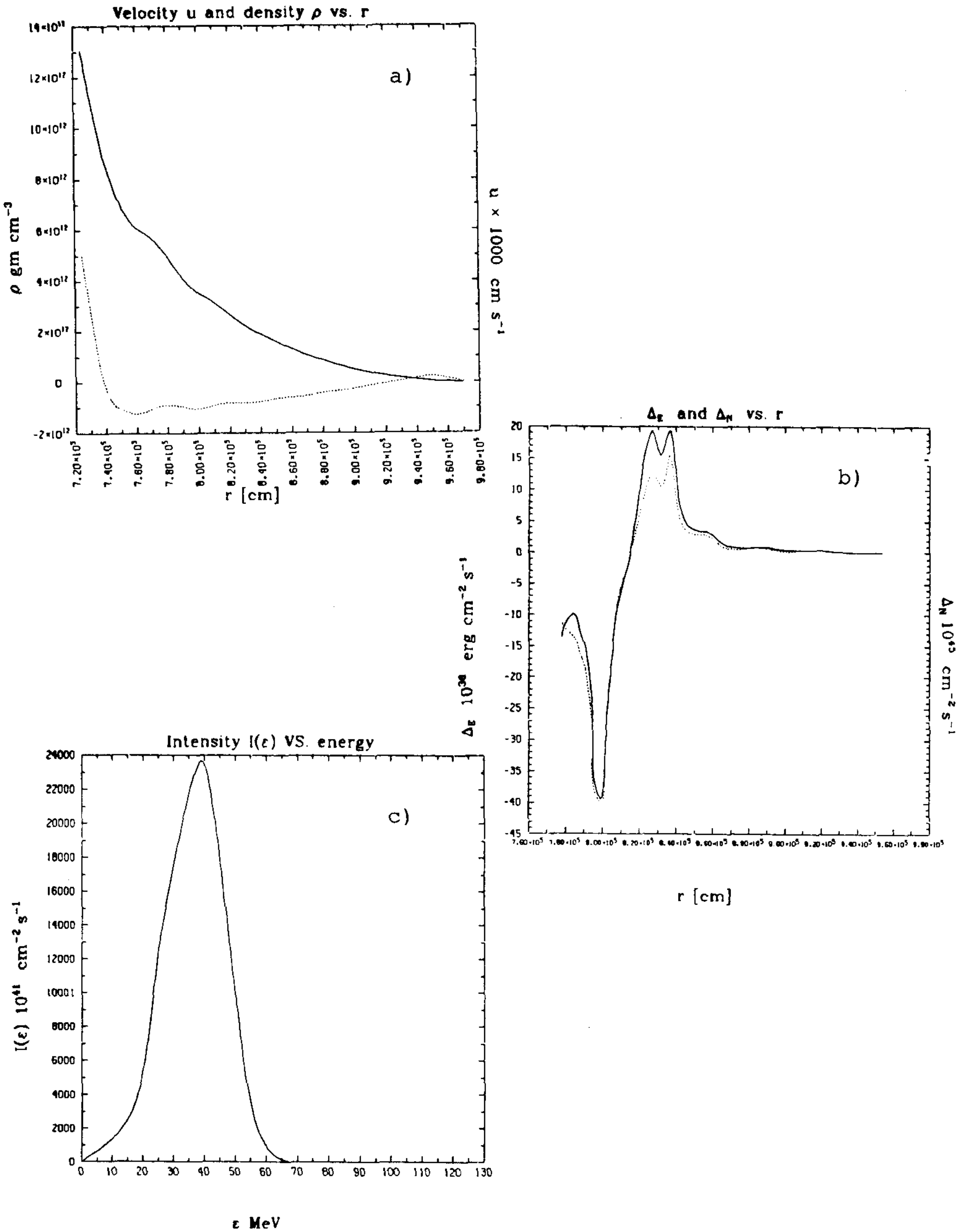

Figure 9: A snapshot of the same shell of matter at a later time $t = 8.04 \times 10^{-6}$sec. Here the shock has ceased its outward propagation. Figure 9a shows the density ρ (the solid line) and velocity u (the dotted line) as functions of r.

Figure 9b shows the quantities Δ_E (the solid line) and Δ_N (the dotted line) at time $t = 8.04 \times 10^{-6}$sec.

Figure 9c shows the emergent neutrino flux at the surface at time $t = 8.04 \times 10^{-6}$sec.

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Another possibility, first raised by Epstein (1979) and extended by Colgate and Petschek (1980), is that neutrinos are *physically* transported by large scale convective movement of matter in the star. The idea here is that instead of having to diffuse out, the neutrinos are carried out of the interior of the star and are released in outer layers, where their energy can help expel the outer layers. The mechanism is very simple: since the matter deep in the interior is very lepton rich because neutrinos are trapped, and the matter on top of it is relatively lepton poor, a Rayleigh-Taylor instability is driven by the lepton gradient and a rapid, large scale overturn of the core occurs. Wilson (1980) included convection by essentially mixing length theory and found little difference in his calculations. Smarr, Wilson, Barton, and Bowers (1981) performed a two dimensional hydrodynamic calculation, including neutrino transport in the diffusion approximation, which show that only the outer portion of the core undergoes convective overturn. Lattimer and Mazurek (1981) have pointed out that entropy gradients can compete with the lepton gradients and stabilize the star. They expect only the shocked layers surrounding the core to overturn and question whether or not a stellar explosion due to convective overturn is possible.

An intriguing possibility is suggested by the recent calculations of Salpeter and Shapiro (1981) who considered the perturbative role of *photon* transport in the outermost layers of the neutrino atmosphere following shock heating. They showed that the photon luminosity was less than the neutrino luminosity by the *square root* of the ratio of the photon to neutrino opacity ($L_\nu/L_\gamma \propto \sqrt{\kappa_\gamma/\kappa_\nu}$). This ratio is typically very large, $\kappa_\gamma/\kappa_\nu \sim 10^{16}$. However, they noted, the photon *Eddington* luminosity is less than the neutrino Eddington luminosity by a *full factor* of this ratio (i.e. $L_{E_\nu}/L_{E_\gamma} \propto \kappa_\gamma/\kappa_\nu \sim 10^{16}$). This suggests that even when the neutrino luminosity is sub-Eddington, the photon luminosity can be super-Eddington. Thus photons, in principle, may drive an appreciable mass flux in the outer layers, aiding the ejection of mantle material. Because their calculations dealt with a coupled photon- neutrino *hydrostatic* atmosphere, they could not pursue this dynamical issue in detail. The corresponding hydrodynamic problem is currently under investigation (Shapiro, Wasserman, and Duncan 1985).

Other possibilities are more exotic. If neutrinos have mass, they can oscillate between the three flavors. However, Wolfenstein (1979) has shown that neutrino oscillations are severly suppressed in stellar collapse. It is also possible that neutrinos are given mass by Majorana mass terms (Gelmini and Roncadelli 1981), which could radically affect the physics of stellar core collapse (Kolb, Tubbs, and Dicus 1982).

Conclusions

In this paper, we have discussed the crucial role which neutrino transport plays in the standard model of Type II supernovae. We described the major neutrino interactions which occur in physical conditions typical of stellar collapse. We presented the equations of neutrino hydrodynamics (and their plane-parallel, static limits), and pointed out the differences between neutrino hydrodynamics and photon hydrodynamics. We presented the results of two different attempts to model neutrino transport in physical conditions typical of stellar collapse, first by constructing plane-parallel static "neutrino atmospheres", and secondly by investigating the neutrino hydrodynamics of a spherical, piston driven shell of matter. Finally we briefly described alternatives or extensions to the standard model which may hold the key to an eventual resolution of the Type II supernova problem. *In all cases, the delicate balance between competing effects (e.g. inward gravity vs. outward pressure; shock energising vs. shock dissipation, etc.) suggests that a rigorous calculation of neutrino transport will be required to obtain a definitive answer.*

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DISCUSSION

Shu: I wanted to comment that very similar problems to the one you have are encountered in the accretion shocks of protostar theory. The experience there is that a good compromise between computational precision and computational speed is to use variable Eddington factors. The advantage to such a procedure is that it is relatively simple to effect iteration corrections if you are unsatisfied with the initial guess for the Eddington factor. Another comment is that in any transport scheme, one you have the source and sink terms from a full calculation, you can ray trace to check how good (or bad) your transport scheme was. My understanding is that Mayle has done this for Wilson's code, and found that the flux-limited diffusion scheme is not too bad.

Schinder: I wasn't aware of Mayle's check. I'll certainly have to try ray tracing the results of my code. Back when I was first starting the plane-parallel calculations I tried variable Eddington factors and couldn't get them to work; I've been reluctant to try them ever since.

Owacki: Is the "bridging" law for flux-limited neutrino transport derived from first principles, or is it just a convenient parameterization that fits in the right limits?

Schinder: I think it's just a parameterization, but I'm no expert on flux-limited diffusion. I mentioned it because it's the most commonly used transport approximation in full collapse codes. Perhaps someone else knows.

Mihalas: It's just a parameterization.

Klein: The extreme balance that occurs between physical processes that make the critical difference whether or not a star explodes clearly makes it imperative to know the neutrino opacity to a high degree of accuracy. To what accuracy do we know the neutrino opacities and is the outcome of explosion vs. non-explosion sensitive to the uncertainty within the error bars of the neutrino opacity?

Schinder: If you believe the Weinberg-Salam theory of the weak interaction, then you can just sit down and calculate whatever opacities you need. $\sin^2 \theta_W$ is pinned down very closely nowadays, so there's no room here for changing the opacities greatly. There are certain opacities (neutrino-neutrino scattering, for instance) which are usually left out or fudged because they are just too difficult to handle numerically. Neutrino-neutrino scattering is in principle as important as electron-neutrino scattering. If Dave Tubbs is here, perhaps he could comment; he did some work to check Wilson's treatment of electron scattering in his collapse code.

Tubbs: We used Monte Carlo techniques to "calibrate" Wilson's scheme of neutrino energy redistribution due to neutrino-electron scattering. Jim used a Fokker-Planck equation. His initial results differed sometimes substantially from the Monte Carlo, but he and Bowers felt that parameters in the Fokker-Planck equation could be adjusted to give satisfactory agreement with the transport results. One, of course, must be cautious in calibrating such equations in physical regimes where they are not meant to work.

Schinder: If you don't believe the Weinberg-Salam theory, then things can in principle change radically. Dave Tubbs did some work on Majorana neutrinos; Dave, do you have any comments?

Tubbs: Back of the envelope calculations (done by Dicus, Kolb, and myself), using a Majorana model for neutrinos (where lepton number is *not* conserved), indicated the possibility of substantial changes in the entropy structure of the collapsing core. I believe Wilson and Arnett independently and in somewhat *ad hoc* fashions included these effects in some of their calculations and saw only a minor effect on the supernova outcome. Wilson's calculations may have predated his delayed-explosion model.

Icke: Since we haven't got a session on the radiation hydrodynamics of the early Universe, unfortunately, maybe it's fair to ask you: what part of this transport code is applicable to early cosmological epochs, e.g. can I regard the early Universe as one of your stationary stars turned inside out, with the proper boundary conditions?

Schinder: Unfortunately, I don't know much about neutrinos in the early universe. As I understand, they decouple early on before any structure has formed in the universe. I doubt that my calculations would be of much use.

Szentgyorgyi: How sensitive are the dynamics to the number of neutrino types, i.e. can one set limits on the number of neutrino types via supernova collapse calculations?

Schinder: I don't think so. During collapse, the only neutrinos around are electron type neutrinos. After bounce, when the shock heats up the material surrounding the core other types occur, but since it is difficult to get an explosion with just three types, adding more types probably won't help, and I can't see how a meaningful limit can be found.

Pethick: Do you have any comment on the relationship of your calculations for an idealized situation with the results of detailed collapse calculations?

Schinder: I try to make the physical conditions in my models resemble closely those found in actual collapse calculations. There is always the possibility that my idealizations will cause something to occur that won't in an actual collapse calculation. One of the purposes of this work is to use it to calibrate a simpler neutrino transport approximation which can then be used in a realistic collapse calculation.

Blandford: Would you care to comment on the prospects of observing these neutrinos directly and perhaps comparing theory with observation?

Schinder: I understand that the experiments designed to detect proton decay have detected "cosmic" neutrinos, so it's in principle possible. One of the things I intend to do with my current code is to put in some of the less important opacities, such as neutrino absorption by nuclei, which should occur only if the neutrino is above an energy threshold, and see if there are discernible features in the spectrum. If these features are found, there may be as useful to neutrino observers as absorption/emission lines in photon spectra are.

Van Riper: At the beginning of your talk you suggested that a correct treatment of the transport may allow the prompt shock to survive. Do you actually know what this correct method is?

Schinder: No, I don't. It's just always intrigued me that the shock always seems to die in the "neutrinosphere"; it just can't survive the extra loss of energy to neutrinos. The other point is that only a small portion of the energy stored in neutrinos just after bounce is necessary to cause the observed explosion. These facts lead me to suspect that perhaps a better treatment of the transport might allow the shock to survive with just enough energy to cause the explosion.

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