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A Fixed Point Theorem and the Existence of a Haar Measure for Hypergroups Satisfying Conditions Related to Amenability

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Abstract. In this paper we present a fixed point property for amenable hypergroups that is analogous to Rickert's fixed point theorem for semigroups. It equates the existence of a left invariant mean on the space of weakly right uniformly continuous functions to the existence of a fixed point for any action of the hypergroup. Using this fixed point property, certain hypergroups are shown to have a left Haar measure.

1 Introduction

Hypergroups arise as generalizations of the measure algebra of a locally compact group wherein the product of two points is a probability measure rather than a single point. The formalization of hypergroups was introduced in the 1970s by Jewett [6], Dunkl [3], and Spector [12]. Actions of hypergroups have been considered in [10,13]. Amenable hypergroups have been considered in [1,7,11,14]. As with groups, there are connections between invariant means on function spaces and fixed points of actions of the hypergroup.

It is a longstanding problem whether or not every hypergroup has a left Haar measure. It is known that if a hypergroup is compact, discrete, or abelian, then it does admit a Haar measure [3,6,12].

Every locally compact group G admits a left Haar measure λ_G (see *e.g.,* [4, §15]). This Haar measure can be viewed as a positive, linear, (not necessarily bounded), left translation invariant functional on $C_C(G)$. It follows from the existence of this functional that for positive $f \in C_C(G)$ and positive $\mu, \nu \in M(G)$,

$$\mu * f(x) \le \nu * f(x) \quad \forall x \in G \Rightarrow \|\mu\| \le \|\nu\|. \tag{1.1}$$

A similar statement can be made for amenable groups and positive, continuous, bounded functions with non-zero mean value (see Remark 4.1). In [8, §2.32], Paterson describes a similar notion (the Translate Property) for considering the translations of sets by group elements rather than the translation of functions by measures. One might expect this statement to be true more generally, but there are examples of bounded functions on non-amenable groups for that (1.1) does not hold. In [9],

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Rosenblatt describes supramenable groups which are defined in terms of the translate property including showing that the free group on two generators is not supramenable.

The main purpose of this paper is to show that for amenable hypergroups, the existence of a Haar measure is equivalent to this positivity property of translations (1.1). In Section 2, we provide the neccessary preliminaries. In Section 3, we prove a fixed point theorem for the action of an amenable hypergroup. In the final section, we use property (1.1) to prove the existence of a measure that is invariant for all translations of f. We build on this and use the fixed point theorem to prove the final result.

2 Background and Definitions

Definition 2.1 A hypergroup, H, is a non-empty locally compact Hausdorff topological space that satisfies the following conditions (see [2] for more details on hypergroups).

(i) There is a binary operation $*$, called convolution, on the vector space of bounded Radon measures turning it into an algebra.

(ii) For $x, y \in H$, the convolution of the two point measures $\delta_x, \delta_y \in M(H)$ is a probability measure, and $\operatorname{supp}(\delta_x * \delta_y)$ is compact.

(iii) The map $(x, y) \mapsto \delta_x * \delta_y$ from $H \times H$ to the compactly supported probability measures on H is continuous.

(iv) The map $H \times H \ni (x, y) \mapsto \operatorname{supp}(\delta_x * \delta_y)$ is continuous with respect to the Michael topology on the space of compact subsets of H.

(v) There is a unique element $e \in H$ such that for every $x \in H$, $\delta_x * \delta_e = \delta_e * \delta_x = \delta_x$.

(vi) There exists a homeomorphism $\check{}: H \to H$ such that for all $x \in H$, $\check{\check{x}} = x$, which can be extended to $M(H)$ via $\check{\mu}(A) = \mu(\{x \in H : \check{x} \in A\})$ and such that $(\mu * \nu)\check{} = \check{\nu} * \check{\mu}$.

(vii) For $x, y \in H$, $e \in \operatorname{supp}(\delta_x * \delta_y)$ if and only if $y = \check{x}$.

Definition 2.2 We denote the continuous and bounded complex-valued functions on H by $C(H)$. For the compactly supported continuous functions we use $C_C(H)$. We use $C(H)^+, C_C(H)^+$ to denote the functions in those spaces that take only non-negative real values. We denote the complex space of bounded regular Borel measures on H by $M(H)$, those that are compactly supported measures by $M_C(H)$, and those that are real valued measures by $M(H, \mathbb{R})$.

Definition 2.3 For $x \in H, f \in C(H)$ we denote the *left translation of f by x* as $L_x f \in C(H)$ defined by

$$L_x f(y) = f(\check{x} * y) = \int f(t) d(\delta_{\check{x}} * \delta_y)(t).$$

One should be careful with the notation $f(\check{x} * y)$, because $\check{x} * y$ is a probability measure and not generally a point in H.

In addition, we can extend this to left translation by measures in $M_C(H)$,

$$L_\mu f(y) = f(\check{\mu} * \delta_y) = \int f(t) d(\check{\mu} * \delta_y)(t).$$

Similarly, we define right translation of $f \in C_C(H)$ by $\mu \in M_C(H)$ as

$$R_\mu f(y) = f(\delta_y * \check{\mu}) = \int f(t) d(\delta_y * \check{\mu})(t).$$

Definition 2.4 A left Haar measure for H is a non-zero regular Borel measure (with values in $[0, \infty]$), λ that is left-invariant in the sense that for any $f \in C_C(H)$, we have that $\lambda(L_x f) = \lambda(f)$ for all $x \in H$.

Remark 2.5 It remains an open question whether every hypergroup admits a left Haar measure. If H does admit a left Haar measure λ, however, it is unique up to a scalar multiple [6]. For hypergroups with a left Haar measure we are able to define the standard $L^p(H)$ function spaces.

Definition 2.6 We say that a continuous function $f \in C(H)$ is *right uniformly continuous* (resp. *weakly right uniformly continuous*) if the map

$$H \ni x \mapsto L_x f$$

is continuous in norm (weakly). We denote the collection of right uniformly continuous functions (resp. weakly right uniformly continuous) on H by $UCB_r(H)$ (resp. $WUCB_r(H)$).

Remark 2.7 Skantharajah [11] showed that for hypergroups with left Haar measure, $UCB_r(H) = L^1(H) * L^\infty(H)$.

3 Fixed Point Property

Let H be a hypergroup. Let E be a Hausdorff locally convex vector space and let $K \subset E$ be a compact, convex subset. Suppose that there is a separately continuous mapping $\cdot : H \times K \to K$. Then for $x, y \in H$ and $\xi \in K$ the weak integral

$$\int_H (t \cdot \xi) d(\delta_x * \delta_y)(t)$$

exists uniquely in K.

Definition 3.1 A *separately continuous* (resp. *jointly continuous*) *action* of H on K is a separately continuous [jointly continuous] mapping $\cdot : H \times K \to K$ such that

(i) $e \cdot \xi = \xi$ for all $\xi \in K$;
(ii) $x \cdot (y \cdot \xi) = \int_H (t \cdot \xi) d(\delta_x * \delta_y)(t)$.

Furthermore, the action is called *affine* if, for each $x \in H$ $\xi \mapsto x \cdot \xi$ is affine.

The following theorem is similar to a result of Rickert for amenable semigroups as presented in [8].

Theorem 3.2 *Suppose that $UCB_r(H)$ has a left invariant mean m. Then for each jointly continuous affine action of H on some K, a compact convex subset of a Hausdorff locally convex vector space, there is a point $\xi_0 \in K$ such that $x \cdot \xi_0 = \xi_0$ for all*

$x \in H$. Furthermore, the result holds if $UCB_r(H)$ is replaced by $WUCB_r(H)$ and simultaneously jointly continuous is replaced by separately continuous.

Proof Suppose that there is a hypergroup action of H on K. We denote the set of affine functions on K by $\mathrm{Aff}(K)$. It is clear that for each point $\xi \in K$, evaluation at ξ is a mean on $\mathrm{Aff}(K)$. Indeed, K can be identified with the collection of all means on $\mathrm{Aff}(K)$ and this identification is an affine homeomorphism. (see for instance [8, Proposition 2.20]).

Given this identification, we see that the existence of a fixed point in K is equivalent to the existence of a mean on $\mathrm{Aff}(K)$ that is invariant under the action of H.

Additionally, the action of H on K can be used to induce an action of H on $\mathrm{Aff}(K)$ by $x \odot \phi(\xi) = \phi(\check{x} \cdot \xi)$.

Suppose that $\phi \in \mathrm{Aff}(K)$ and $\xi_1 \in K$. Let $\widehat{\xi_1}: \mathrm{Aff}(K) \to C(H)$ be defined by $\widehat{\xi_1}(\phi)(x) = \phi(x \cdot \xi_1)$. It is clear that $\widehat{\xi_1}(\phi)$ is a continuous function provided that that action of H is (separately) continuous. We further claim that if the action is separately continuous, then $\widehat{\xi_1}(\phi)$ is in $WUCB_r(H)$, and if it is jointly continuous, then $\widehat{\xi_1}(\phi)$ is in $UCB_r(H)$.

Furthermore, left translation commutes with $\widehat{\xi_1}$ in the following sense: for $x \in H, \phi \in \mathrm{Aff}(K)$

$$\begin{aligned} L_x \widehat{\xi_1}(\phi)(y) = \widehat{\xi_1}(\phi)(\check{x} * y) &= \int \phi(t \cdot \xi_1)\, d(\check{x} * y)(t) \\ &= \phi\Big(\int t \cdot \xi_1\, d(\check{x} * y)(t) \Big), && \text{since } \phi \text{ is affine} \\ &= \phi\big(\check{x} \cdot (y \cdot \xi_1)\big) && \text{by (ii).} \end{aligned}$$

Now, to show the first claim, consider some mean F on $C(H)$ and some net $x_\alpha \to x$ in H. Then $F \circ \widehat{\xi_1}: \mathrm{Aff}(K) \to \mathbb{C}$ is a mean on $\mathrm{Aff}(K)$. It follows then, that there is some $\xi_2 \in K$ such that $F \circ \widehat{\xi_1}(\phi) = \phi(\xi_2)$.

Therefore,

$$\begin{aligned} \langle F, L_{\check{x}_\alpha} \widehat{\xi_1}(\phi) \rangle = \langle F \circ \widehat{\xi_1}, \check{x}_\alpha \odot \phi \rangle = \check{x}_\alpha \odot \phi(\xi_2) = \phi(x_\alpha \cdot \xi_2) \\ \to \phi(x \cdot \xi_2) = \langle F, L_{\check{x}} \widehat{\xi_1}(\phi) \rangle \end{aligned}$$

From this we conclude that for the same holds any $F \in C(H)^*$. That is, that $\widehat{\xi_1}(\phi)$ is in $WUCB_r(H)$. Now since there is a left invariant mean M on $WUCB_r(H)$ it follows that $M \circ \widehat{\xi_1}$ is a mean on $\mathrm{Aff}(K)$ that corresponds to evaluation at some point $\xi_0 \in K$. It is apparent that ξ_0 is a fixed point of the action of H.

For the second claim, observe that $\|\check{x_\alpha} \odot \phi - \check{x} \odot \phi\| \to 0$ as $x_\alpha \to x$ since the action is jointly continuous. Furthermore, since $\widehat{\xi}$ is contractive for each $\xi \in K$, $\|L_{\check{x}_\alpha} \widehat{\xi_1}(\phi) - L_{\check{x}} \widehat{\xi_1}(\phi)\| \to 0$ as $x_\alpha \to x$. But this shows that $\widehat{\xi_1}(\phi)$ is in $UCB_r(H)$ as required. By a similar argument as above, the mean on $UCB_r(H)$ generates a fixed point in K. ∎

4 Existence of a Left Haar Measure

The existence of a Haar measure on an arbitrary hypergroup is still an open question. In this section we present an approach motivated in part by a result of Izzo [5], which

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uses the Markov–Kakutani fixed point theorem to prove the existence of a Haar measure on abelian groups. We use the fixed point Theorem 3.2 from the previous section to show that amenable hypergroups that satisfy an additional positivity property of translations have a left Haar measure. This property is related to amenability in the following sense.

Remark 4.1 Every locally compact group G admits a left Haar measure. Some of the key properties of the Haar measure are as follows. For any $f, g \in C_C(G)$ and $\mu \in M(G)$, we have

$$f(x) \le g(x) \forall x \in G \Rightarrow \int_G f(x)\, d\lambda(x) \le \int_G g(x)\, d\lambda(x), \text{ and}$$
$$\int_G (\mu * f)(x)\, d\lambda(x) = \mu(H) \int_G f(x)\, d\lambda(x).$$

Hence, for every non-zero $f \in C_C(G)^+$ and $\mu, \nu \in M(G)^+$ if $\mu * f \le \nu * f$, then $\|\mu\| \le \|\nu\|$.

Relatedly, if G is amenable with left invariant mean $m \in C(G)^*$, then for every $f \in C(G)^+$ with $m(f) > 0$ and $\mu, \nu \in M(G)^+$ if $\mu * f \le \nu * f$, then $\|\mu\| \le \|\nu\|$.

Definition 4.2 Let H be a hypergroup. Fix some non-zero $f \in C_C(H)^+$. We say that H has the *positivity property of translations of f* if for every $\mu, \nu \in M^+(H)$,

$$L_\mu f \le L_\nu f \Rightarrow \|\mu\| \le \|\nu\|. \tag{4.1}$$

Lemma 4.3 *Suppose that H is a hypergroup with the positivity property of translations of some non-zero $f \in C_C(H)^+$.*

Then there is a positive, linear functional Γ from the real-vector space $C_C(H, \mathbb{R})$ to $\mathbb{R}$ such that $\Gamma(f) = 1$ and $\Gamma(L_\rho f) = \rho(H)$ for $\rho \in M_C(H, \mathbb{R})$.

Proof Let $V_f := \{L_\mu f : \mu \in M_C(H, \mathbb{R})\}$. Since $M_C(H, \mathbb{R})$ is a vector space, it follows that V_f is also a vector space. Define the linear functional Γ_f on V_f by

$$\Gamma_f(L_\mu f) = \mu(H).$$

Claim: Γ_f is well defined.

If $L_\mu f = L_\nu f$, then consider Jordan decompositions $\mu = \mu_+ - \mu_-$ and $\nu = \nu_+ - \nu_-$. Then $L_{(\mu + \mu_- + \nu_-)} f = L_{(\nu + \nu_- + \mu_-)} f$, and these are both positive with equal norm by property (4.1), hence $\mu(H) = \nu(H)$. Therefore, the map Γ_f is well defined.

Claim: Γ_f is positive.

By a similar argument, if $0 \le L_\nu f$, then $L_{\nu_-} f \le L_{\nu_+} f$, hence $\nu_-(H) \le \nu_+(H)$ and so $\nu(H) = \nu_+(H) - \nu_-(H)$ is positive.

By [2, Lemma 1.2.22] for any $g \in C_C(H, \mathbb{R})$ there is a $\mu \in M(H, \mathbb{R})_C^+$ such that $g \le L_\mu f$. Hence we can apply M. Riesz's extension theorem to extend Γ_f to a positive linear functional on all of $C_C(H, \mathbb{R})$. ∎

The above lemma shows that the collection of positive linear functionals on $C_C(H)$ sending all translations of the function f by elements of H to 1 is non-empty. By applying the fixed point theorem to this collection, we prove the existence of a non-zero positive linear functional on $C_C(H)$ that is fixed under the action of translation by elements of the hypergroup. This non-zero positive linear functional is precisely a Haar measure.

Corollary 4.4 *Suppose that H has the positivity property of translations of some non-zero $f \in C_C(H)^+$ as above. Then the collection K of all positive linear functionals Λ on $C_C(H)$ satisfying*

$$\Lambda(L_\mu f) = \mu(H) \text{ for every } \mu \in M_C(H)$$

is non-empty.

Proof It suffices to use the complexification of Γ from Lemma 4.3. ∎

Lemma 4.5 *For fixed $f \in C_C(H)^+$ with $f \not\equiv 0$, define $\|\cdot\|_f : C_C(H) \to [0,\infty)$ by*

$$\|g\|_f := \inf\{\|\mu\| : \mu \in M(H)^+_C, L_\mu f \geq |g|\}.$$

This is a norm on $C_C(H)$.

Proof In the notation of Jewett's [6, §4.3] proof of the existence of a subinvariant measure, $\|g\|_f = [|g|, f]$. He proves positive homogeneity, subadditivity, and positivity when considering positive functions g. It is clear that $[|g|, f]$ is indeed a norm on $C_C(H)$. ∎

Theorem 4.6 *Suppose H has the positivity property of translations of some non-zero $f \in C_C(H)^+$. Suppose also that $WUCB_r(H)$ has a left invariant mean. Then H has a left Haar measure.*

Proof Consider the vector space $C_C(H)$ with norm $\|\cdot\|_f$. By the Banach–Alaoglu theorem, the unit ball of $(C_C(H), \|\cdot\|_f)^*$ is compact in the weak-star topology.

Let K be the collection of all positive linear functionals Λ on $C_C(H)$ satisfying:

$$\Lambda(L_\mu f) = \mu(H) \text{ for every } \mu \in M_C(H).$$

By Corollary 4.4, K is not empty. Since, for $\Lambda \in K$, $|\Lambda(g)| \leq \Lambda(|g|) \leq \|g\|_f$, it follows that K is contained in the unit ball of $(C_C(H), \|\cdot\|_f)^*$. Therefore, K is relatively compact.

Indeed, if there is a convergent net in $(C_C(H), \|\cdot\|_f)^*$ $\Lambda_\alpha \to \Lambda$ where each $\Lambda_\alpha \in K$, then Λ is in K and so K is closed. Therefore K is compact.

To see that K is convex, suppose that $\Lambda_1, \Lambda_2 \in K$ and $0 \leq c \leq 1$. Then

$$(c\Lambda_1 + (1-c)\Lambda_2)(L_\mu f) = c\mu(H) + (1-c)\mu(H) = \mu(H),$$

so $(c\Lambda_1 + (1-c)\Lambda_2)$ is an element of K.

It now suffices to show that left translation is an action of H on K and that this action is separately continuous. From this we can apply Theorem 3.2 to find a point in K that is fixed under left translation.

For $x \in H$, $g \in C_C(H)$, and $\Lambda \in K$ we define $x \cdot \Lambda(g) = \Lambda(L_{\check{x}} g)$. Then $e \cdot \Lambda = \Lambda$ and $x \cdot \Lambda(L_\mu f) = \Lambda(L_{(\delta_{\check{x}} * \mu)} f) = 1$, so K is closed under the action of H. Furthermore,

$$\begin{aligned} x \cdot (y \cdot \Lambda)(g) &= y \cdot \Lambda(L_{\check{x}} g) = \Lambda(L_{\check{y}} L_{\check{x}} g) = \Lambda(L_{\delta_{\check{y}} * \delta_{\check{x}}} g) \\ &= \int \Lambda(L_{\check{t}} g) d(\delta_x * \delta_y)(t)) = \int t \cdot \Lambda(g) d(\delta_x * \delta_y)(t). \end{aligned}$$

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To see that this action is separately continuous, consider a net $x_\alpha \to x \in H$. Eventually x_α will stay within a compact neighbourhood N of x. Then for $\Lambda \in K$ and $g \in C_C(H)$ it suffices to consider the restriction $\Lambda|_{\check{N}*\mathrm{supp}(g)}$ of Λ, which is a finite measure on the compact set $\check{N} * \mathrm{supp}(g) := \cup\{\mathrm{supp}(\delta_s * \delta_t) : \check{s} \in N, t \in \mathrm{supp}(g)\}$. This allows us to consider the right translate of g by this measure (which is a continuous function), and we get

$$\begin{aligned} x_\alpha \cdot \Lambda(g) = \Lambda(L_{\check{x}_\alpha} g) = \Lambda|_{\check{N}*\mathrm{supp}(g)}(L_{\check{x}_\alpha} g) &= \big(\delta_{x_\alpha} * \Lambda|_{\check{N}*\mathrm{supp}(g)}\big)(g) \\ &= \delta_{x_\alpha}\big(R_{\check{\Lambda}|_{\check{N}*\mathrm{supp}(g)}} g\big) \to \delta_x\big(R_{\check{\Lambda}|_{\check{N}*\mathrm{supp}(g)}} g\big) = x \cdot \Lambda(g). \end{aligned}$$

Now suppose there is a net $\Lambda_\beta \to \Lambda \in K$. So for $x \in H$ and $f \in C_C(H)$,

$$x \cdot \Lambda_\beta(f) = \Lambda_\beta(L_{\check{x}} f) \to \Lambda(L_{\check{x}} f) = x \cdot \Lambda(f).$$

Therefore the action is separately continuous.

By Theorem 3.2 there is a fixed point Λ_0 in K under the action of H. This fixed point is a positive linear functional on $C_C(H)$ satisfying

$$\Lambda_0(g) = \Lambda_0(L_x g) \quad \forall g \in C_C(H), x \in H.$$

It is non-zero, since $\Lambda_0(f) = 1$ so Λ_0 is a left Haar measure for H. ∎

Corollary 4.7 *Let H be a hypergroup with a left translation invariant mean on $WUCB_r(H)$. The following are equivalent.*

(i) *There is a left Haar measure on H.*
(ii) *The property of positivity of translations holds for every $f \in C_C(H)^+$.*
(iii) *The property of positivity of translations holds for some nonzero $f \in C_C(H)^+$.*

Proof The proof that (i) implies (ii) is contained in Remark 4.1 for groups, and it applies verbatim to hypergroups. That (ii) implies (iii) is obvious. That (iii) implies (i) is the content of Theorem 4.6. ∎

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