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Numerical Effects on Wave Propagation in Atmospheric Models

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Abstract. Ray tracing techniques have been used to investigate numerical effects on the propagation of acoustic waves in a non-hydrostatic dynamical core discretised using an Arakawa C-grid horizontal staggering of variables (Arakawa & Lamb 1977) and a Charney-Phillips vertical staggering of variables (Charney & Phillips 1953) with a semi-implicit timestepping scheme. It is found that the space discretisation places limits on resolvable wavenumbers and redirects the group velocity of waves towards the vertical. Wave amplitudes grow exponentially with height due to the decrease in the background density, which can cause instabilities in whole-atmosphere models. However, the inclusion of molecular viscosity and diffusion acts to damp the exponential growth of waves above about 150 km. This study aims to demonstrate the extent to which numerical wave propagation causes instabilities at high altitudes in atmosphere models, and how processes that damp the waves can improve these model's stability.

Keywords. atmospheric effects, diffusion, methods: numerical, waves.

1. Introduction

There is a current need for whole-atmosphere models to be developed as tools for space weather forecasting. The Met Office Unified Model dynamical core (Wood *et al.* 2014) is to be extended into a whole-atmosphere model, but initial attempts to raise the top boundary of this model give rise to instabilities, due to a combination of numerical effects and missing physical processes (Griffin & Thuburn 2018). The intended outcome of this study is to determine the effect of numerical wave propagation on the dynamical core, and to consider how the model's stability might be improved.

For propagation in a two-dimensional vertical cross section of the atmosphere, a wave packet with wavevector $\boldsymbol{k} = (k, m)$ and frequency ω approximately satisfies the local dispersion relation $\omega = \Omega(k, m; x, z, t)$, where the dependence on position $\boldsymbol{x} = (x, z)$ and time t can arise through variations in the background wind, temperature or stratification. In this work, any dependence on x or t is neglected. The wave packet then propagates along a space-time trajectory (a ray) at the group velocity $\boldsymbol{c}_g$ that depends on $\boldsymbol{k}$ and temperature T and the wave vector also evolves according to the ray tracing equations:

$$\frac{D_{\boldsymbol{c}_g}}{Dt}(\boldsymbol{x}) = \boldsymbol{c}_g(\boldsymbol{k}, T) = \boldsymbol{\nabla}_{\boldsymbol{k}}\Omega, \qquad \frac{D_{\boldsymbol{c}_g}}{Dt}(\boldsymbol{k}) = -\boldsymbol{\nabla}_{\boldsymbol{x}}\Omega, \tag{1.1}$$

(Lighthill 1978). The derivative operator $D_{\boldsymbol{c}_g}/Dt = \partial/\partial t + \boldsymbol{c}_g \cdot \nabla_{\boldsymbol{x}}$ represents the rate of change of a variable with time at a position moving with the group velocity $\boldsymbol{c}_g$. These equations are solved using a forward Euler timestep to describe the evolution of the position and wavevector of a wave packet. The wave energy conservation law:

$$\frac{D_{\boldsymbol{c}_g}}{Dt}(\log W) + \boldsymbol{\nabla}_{\boldsymbol{x}} \cdot \boldsymbol{c}_g + \frac{2}{\tau} = 0, \tag{1.2}$$

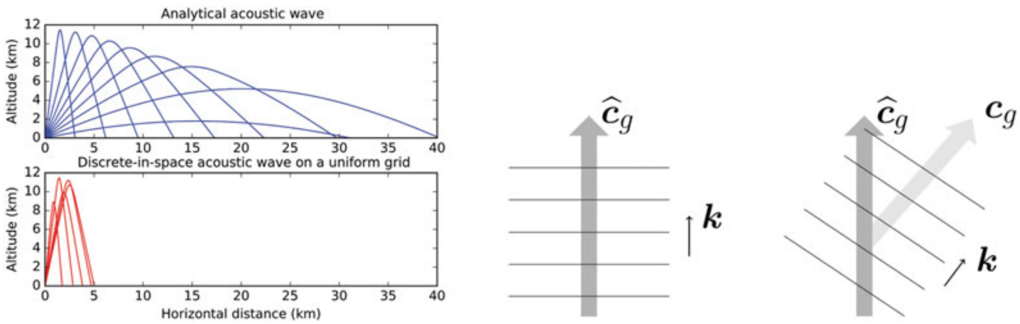

Figure 1. left: A comparison of ray plots of acoustic waves with a total wavenumber of $4 \times 10^{-5}\,\mathrm{rad\,m^{-1}}$ initiated with a range of wavevector angles measured from the horizontal, from 0 to $\pi/2$ (above: the analytical wave equation, and below: the spatially-discrete wave equation with $\Delta x = 100\,\mathrm{km}$, $\Delta z = 1\,\mathrm{km}$).Right: Illustrations of the directions of analytical group velocities $\boldsymbol{c}_g$ and spatially-discrete group velocities $\widehat{\boldsymbol{c}}_g$ of acoustic waves for varying wavevectors $\boldsymbol{k}$ (left: $k = 0$: the wavevector, analytical and numerical group velocities go in the same direction, and right: $k = \pi/\Delta x$: the largest k that can be considered for the wave to be resolved: $\boldsymbol{c}_g$ goes in the same direction as $\boldsymbol{k}$, but $\widehat{\boldsymbol{c}}_g$ is still vertical).

(Lighthill 1978) describes the evolution of the rate of growth of wave amplitudes, where W represents the wave energy per unit volume, and τ represents the timescale of any processes that act to damp the waves.

From the Navier-Stokes governing equations, a wave equation, and a dispersion relation $\widehat{\omega} = \widehat{\Omega}(\widehat{k}, \widehat{m}; x, z)$ (where the 'hats' denote the effective wavenumbers and frequency as seen by the continuous dispersion relation) and group velocity $\boldsymbol{c}_g$ can be found. The 'hat' variables and their derivatives change depending on whether the continuous equations or the spatially- and/ or temporally-discrete equations are being considered. However, the same ray tracing equations are used in all cases, so the analytical and numerical wave propagation can be directly compared.

2. Overview

The main effects of the space discretisation on wave propagation are twofold. Firstly, the coarse horizontal resolution ($\Delta x \sim 100\,\mathrm{km}$) compared with the finer vertical grid spacing ($\Delta z \sim 10\,\mathrm{km}$) means that waves are less well resolved in the horizontal direction than in the vertical. This prevents waves from propagating far horizontally, as demonstrated by Figure 1.

Secondly: there is a limit on horizontal wavenumbers k that may be used for waves to be resolved: $k < \pi/\Delta x$. For $k \sim \pi/\Delta x$, where the wavevector angle is low, the analytical group velocity points in the same direction, but the spatially-discrete group velocity points straight up, as seen in Figure 2. This causes excessive amounts of wave energy to be channelled upwards into the thermosphere in atmospheric models.

The time discretisation has the effect of slowing wave propagation, but maintaining the trajectory of the wave packet, as shown by Figure 3. This effect is much greater on high frequency acoustic waves than on gravity waves.

The wave amplitude grows exponentially with height due to the decrease in the background density, and varies with temperature. Molecular viscosity is a real physical process that reduces the growth of wave amplitudes in the thermosphere above $\sim 130\,\mathrm{km}$, as shown by Figure 4. Off-centering the semi-implicit timesteps can also artificially damp wave amplitudes throughout the whole atmosphere.

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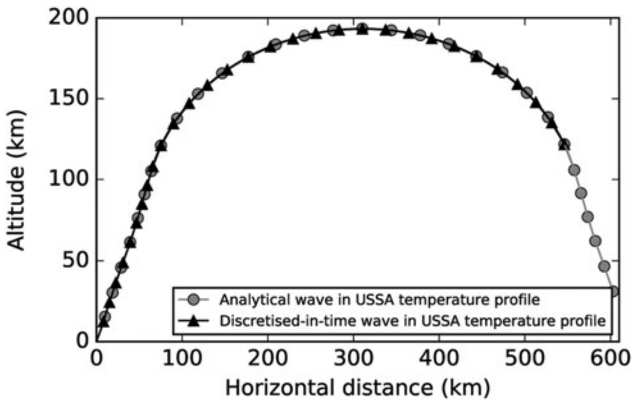

Figure 2. Ray plot of acoustic waves with a total wavenumber of 1×10^{-3} rad m^{-1}, initiated at an angle 0.3π using either the analytical wave equation or the temporally-discrete wave equation with a centred-in-time timestep of 3 seconds. The dots represent one-minute intervals and the simulation was run for 30 minutes.

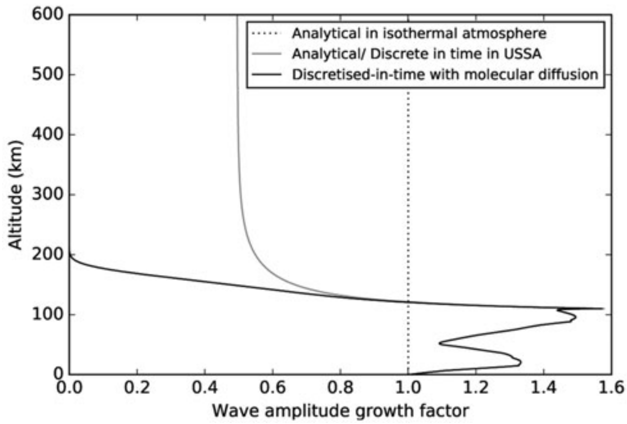

Figure 3. A comparison of the wave energy at each height for a vertically propagating acoustic wave with a total wavenumber of 1×10^{-4} rad m^{-1} for the cases with and without molecular viscosity and diffusion.

The improvement of the Met Office Unified Model dynamical core's stability with the inclusion of molecular viscosity and diffusion has been confirmed by experiments with a one-dimensional column version of the dynamical core. A small amount of off-centering is required in order to extend the top boundary up from 100km to the molecularly diffused region above 150km, whereupon molecular viscosity acts to regulate wave growth, keeping the model stable up to altitudes of 600km at the top of the thermosphere.

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