DYNAMICS OF AN ARTIFICIAL SATELLITE IN AN EARTH-FIXED REFERENCE FRAME : EFFECTS OF POLAR MOTIONS

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ABSTRACT

In an Earth-fixed reference frame, polar motions (precession, lunisolar nutation, free nutation) introduce small apparent forces in the equations of motion of an Earth satellite. We discuss the possibilities (a) of integrating the orbit in an Earth-fixed frame when tracking data are used for geophysical applications, and (b) of determining from orbital data a set of unknown parameters describing the long-period wandering of the pole.

1. INTRODUCTION

In an inertial reference frame the effects of polar motions on the range data which allow orbit determination for artificial satellites are mainly kinematical, since the observing stations are bound to the Earth.
On the other hand, for geophysical applications it is easier to study the satellite motion with respect to an Earth-fixed reference frame because in this case (a) the geopotential does not depend on kinematics of Earth rotation, (b) the observational model is very simple because the stations have a fixed position (except for lunisolar tides), (c) if the satellite orbit is geosynchronous, the motion in a body-fixed frame is very slow and this fact improves the stability of the numerical integration of the orbit. However, in an Earth-fixed frame apparent forces arise due to the fact that the Earth rotates with the angular velocity

$$\vec{\Omega}(t) = \vec{\Omega}_o + \Delta\vec{\Omega}(t) \qquad (1)$$

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where $\vec{\Omega}_o$ is a constant vector directed along the z-axis fixed within the body ($\Omega_o = 7.29 \times 10^{-5}$ rad s^{-1}), and $\Delta\vec{\Omega}$ is a small time-dependent vector which contains the contributions of precession and lunisolar nutation, free nutation and periodic variations in the length of the day.
In the following we discuss the difficulties and the advantages of the modelling of all these effects by means of apparent forces.

2. APPARENT FORCES

The motion of the Earth's rotation axis in an inertial frame can be described by a Fourier series giving the components of the angular velocity vector

$$\dot{\theta} = -\Omega_o \sum \varepsilon_i \sin \nu_i$$
$$\dot{\phi} \sin\theta = \Omega_o \sum \varepsilon_i \cos \nu_i \qquad (2)$$

where θ, ϕ and ψ are the Euler angles, $\nu_i = n_i\, t + \nu_{io}$ is an angular variable with period $T_i = 2\pi/n_i$. In a body-fixed frame the same motion of the angular velocity vector can be described by a different Fourier series

$$\Omega_x = \Delta\Omega_x = -\Omega_o \sum \varepsilon_i \sin(\nu_i - \psi)$$
$$\Omega_y = \Delta\Omega_y = \Omega_o \sum \varepsilon_i \cos(\nu_i - \psi) \qquad (3)$$

The different frequencies correspond to different physical effects, $n_i = 0$ and $\nu_{io} = 0$ for lunisolar precession, $T_i \simeq 40000$ yr for planetary precession, T_i between 18.6 yr and a few days for lunisolar nutation.
The free nutation of the Earth appears in the body-fixed reference frame with periods $\tau_i = T_i/(1-T_i)$ days, of the order of 1 yr, hence in the inertial frame with T_i close to 1 day.
The acceleration due to apparent forces is

$$\vec{A} = \vec{P}\wedge\dot{\vec{\Omega}} + 2\dot{\vec{P}}\wedge\vec{\Omega} + \vec{\Omega}\wedge(\vec{P}\wedge\vec{\Omega}) \qquad (4)$$

where $\vec{P}$ is the satellite position vector. By alling $\vec{A}_o$ the acceleration due to $\vec{\Omega}_o$ and neglecting the terms proportional to $\Delta\Omega^2$, we get

$$\vec{A} = \vec{A}_o + \vec{P}\wedge\Delta\dot{\vec{\Omega}} + 2\dot{\vec{P}}\wedge\Delta\vec{\Omega} + 2(\vec{\Omega}_o\cdot\Delta\vec{\Omega})\vec{P} - (\Delta\vec{\Omega}\cdot\vec{P})\vec{\Omega}_o - (\vec{\Omega}_o\cdot\vec{P})\Delta\vec{\Omega} \qquad (5)$$

Therefore the frequencies of the apparent forces are linear combinations of the $n_i - \dot{\psi}$ with the mean motion n_o of the satellite.
Long-period effects arise from terms with long-period arguments $\nu_i - \psi$ (like those corresponding to Chandler wobble) and from terms with short-periodic arguments $\nu_i - \psi$ giving a beat with the orbital mean motion. This latter case can be easily illustrated by a geosynchronous satellite. We assume an equatorial circular orbit :

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$$\vec{P}(0) = (x_o,0,0) \ , \ \dot{\vec{P}}(0) = (0,0,0) \ , \ n_o^2 = \Omega_o^2 = \frac{GM_O}{x_o^3} \qquad (6)$$

By linearizing in the perturbative parameters ε_i, if the z-component of P(t) is written as

$$z(t) = \sum \varepsilon_i \zeta_i(t) \qquad (7)$$

we have

$$\ddot{\zeta}_i + \Omega_o^2 \zeta_i = x_o \Omega_o (\Omega_o + \dot{\psi} - n_i) \sin(\nu_i - \psi) \qquad (8)$$

The solutions of equation (8) show a beat with frequencies $(n_i - \dot{\psi} + \Omega_o)/2$ and $(n_i - \dot{\psi} - \Omega_o)/2$. For instance, for the lunisolar precession term $n_i = 0$ and $\Omega_o - \dot{\psi} = \dot{\phi}\cos\theta$, so that the z-component of $\vec{P}$ oscillates with nearly diurnal period and amplitude modulated with a period of about 26000 yr. The lunisolar nutation harmonics with periods longer than 2 days appear in a body-fixed frame as beats due to a forcing term in the second member of equation (8), with a period shorter than 2 days (because $\tau_i = T_i/(1-T_i)$ days).

In conclusion the orbit of a satellite can be integrated in a body-fixed frame by using the apparent forces given by equation (5), where $\Delta\vec{\Omega}$ and $\Delta\dot{\vec{\Omega}}$ must include nearly diurnal variations coming from precession and lunisolar nutation, in addition to the effects of the long-period wandering of the pole. In order to obtain $\Delta\vec{\Omega}$ and $\Delta\dot{\vec{\Omega}}$ the angular astronomical data must be differentiated twice. This can be easily done if the angular data are represented by a Fourier series (as in analytical theories) or by a polynomial fit. The differentiation does not necessarily provide a good fit to the actual behaviour of $\Delta\vec{\Omega}$ and $\Delta\dot{\vec{\Omega}}$. However, we assume that in the integration of the equations of motion an algorithm is used which handles in a stable way perturbations with a timescale of about one day (this is anyway needed in most cases). Then precession and lunisolar nutation are reproduced in the integrated orbit with about the same accuracy as that of the available data.

A similar treatment applies to length-of-day variations by deducing $\Delta\Omega_z$ and $\Delta\dot{\Omega}_z$ from UT measurements, then calculating the apparent forces. Integration of the equations of motion with this term added will give - on the same assumptions as before - the correct longitude drift and acceleration.

3. DETERMINATION OF POLAR MOTION

The method currently used to determine polar motion by satellite tracking is kinematical, i.e., the orbit is integrated in an inertial frame with polar motion affecting only the station positions.

In a body-fixed frame, polar motion can be determined by a different method. It can be modelled by a suitable fitting depending on a set of

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unknown parameters, while precession and nutation are modelled from observational data. Then the unknown parameters will appear in the equations of motion via the apparent forces. Therefore they can be determined by a differential corrections iterative process, with the same method used to determine any other set of solve-for parameters appearing in the force model (e.g.,geopotential coefficients). We remark that for polar motion determinations a high satellite orbit is better, while in general for geophysical studies low orbits are more useful.

We are studying the possibility of using this approach to analyse laser range data from the LASSO system which will be carried by the geosynchronous satellite SIRIO 2, to be launched by ESA in 1981 (Serene and Albertinoli, 1980; Bertotti et al., 1980). In this case the acceleration due to polar motion is of the order of 2×10^{-5} cm s^{-2}, mainly in the z direction. To estimate the attainable accuracy, this value must be compared with the "true" dynamical perturbations. Among these, some are well known (e.g.,Earth's oblateness, lunisolar gravitational forces) and others produce effects with a different signature (e.g. resonant harmonics of the geopotential, which cause a semimajor axis libration - Kamel et al., 1973). The most critical perturbation is due to solar radiation pressure because (a) it cannot be accurately modelled neither in direction, nor in absolute value (b) its z-component has an annual period, hence it masks the Fourier components of polar motion with similar periods.
For a satellite of an area-to-mass ratio of 0.05 cm^2 g^{-1}, the solar radiation pressure produces an acceleration of the order of about 3×10^{-6} cm s^{-2}. The z-component will have a maximum of about 10^{-6} cm s^{-2}. If we assume for this component a 20% uncertainty, apparent forces due to polar motion cannot be determined better than 1%.
As a matter of fact, the accuracy limits depend mainly on the precision and geometry of the available range data. At present the laser method (as applied in the LASSO mission) seems capable of about the same accuracy in polar motion determinations as that of the traditional methods However, more advanced laser systems are planned and the attainable accuracy will be improved.

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