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LEAST-SQUARES FITTING A SMOOTH CURVE TO RADIOCARBON CALIBRATION DATA

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ABSTRACT. We Fourier transformed and filtered calibration curve data to compensate for the averaging effect of radiocarbon-dating sets of adjacent tree rings. A Wiener Filter was also applied to minimize the effects of the counting errors of the dates on the resulting calibration curve and to produce a least-squares curve through the data. The method is illustrated using a short ^{14}C-dated tree-ring sequence from New Zealand to produce a calibration curve at yearly intervals for New Zealand matai (*Prumnopitys taxifolia*). The resulting curve has a nominal standard error of 10 ± 3 yr, which is *ca.* half the average standard error of the original raw data.

INTRODUCTION

We previously showed (McFadgen, Knox and Cole 1994) that conversion of radiocarbon dates to calendar dates using currently accepted methods results in an artificial spreading and clumping of the calendar dates. The spreading and clumping, referred to as calibration stochastic distortion (CSD), is brought about by the interaction of the standard errors of the dates with the change in slope of the calibration curve. The distortion increases both the overall spread of dates and the possibility of date reversals. We suggested that the CSD effect could be overcome by deconvolving counting statistics from the ^{14}C dates to obtain the true distribution of ^{14}C dates, and then mapping the deconvolved set through the calibration curve onto the calendar axis in the usual way. The efficacy of the whole procedure depends on minimizing those changes of slope of the calibration curve caused by counting statistics.

There are calibration curves for terrestrial samples and for marine samples (Stuiver and Reimer 1993). Marine calibration data are derived from terrestrial data (Stuiver and Braziunas 1993) and are not considered further here. Terrestrial calibration curves are based on ^{14}C dates of tree-ring dated wood (*e.g.*, Stuiver and Pearson 1993; Pearson and Stuiver 1993; Stuiver and Becker 1993). Each dated sample comprises a group of adjacent rings, and the dates have statistical errors associated with them that introduce spurious wiggles into the calibration curves and contribute to changes in the slopes of the curves.

Terrestrial calibration data span some 8000 yr and are derived from measurements of several tree-ring chronologies. The longest chronologies are from the Northern Hemisphere. Southern Hemisphere chronologies include a ^{14}C-dated tree-ring sequence from New Zealand spanning the period from AD 1335 to 1745 (Sparks *et al.* 1995).

In addition to their use in calibration, smoothed, accurate and precise versions of these curves are a prerequisite for comparison of the Northern and Southern Hemisphere data to test the assumption that ^{14}C variations in the Southern Hemisphere match those of the Northern Hemisphere. They are also necessary to shed light on the relevant geophysical processes that produce the major changes in slope of the calibration curve.

We here describe a method of deriving a smoothed, more accurate and precise calibration curve by removing the spurious wiggles introduced by counting errors inherent in ^{14}C measurements of tree rings, and by compensating for the averaging effect brought about by dating sets comprised of a number of adjacent tree rings. Our analysis uses the short ^{14}C-dated tree-ring sequence from New Zealand in order to establish the method. The longer published Northern Hemisphere sequences will be considered in a subsequent paper.

Finally, the method described here has wider application than just to ^{14}C calibration curves. It is generally applicable to producing least-squares smoothed curves through any regularly spaced set of discrete data points with known error estimates.

METHODS

The raw ^{14}C calibration data set of ^{14}C *vs.* tree-ring age is Fourier transformed from the time to the frequency domain, where we design and apply filters to the transformed data, based on 1) the standard deviation of the measured tree rings, and 2) the fact that each sample measured contained wood spanning ten rings. We transform back to the time domain to obtain a smoothed calibration curve with substantially reduced errors attributable to counting statistics, and with some compensation for the averaging effect of using wood spanning ten adjacent tree rings in each measurement.

We develop the method using ^{14}C dates of tree rings for New Zealand matai (*Prumnopitys taxifolia*) measured at the Rafter Radiocarbon Laboratory, New Zealand Institute of Geological and Nuclear Sciences (Sparks *et al.* 1995: Table 2). Before Fourier transforming the data set, we remove the ideal (straight line) ^{14}C age *vs.* tree-ring age. The difference between raw data points and corresponding points on the ideal line is the detrended data listed in Table 1, columns 4 and 8. The end points of the data set differ from the ideal by only 1–2 yr. Since ^{14}C dates are normally reported to the nearest year, we use 1 yr (y) as our unit of time, and correspondingly 1 cycle per year (y^{-1}) as the unit of frequency.

Table 1 contains 42 points at 10-yr intervals, extending over 420 annual tree rings. Computer programs in the field of Fourier analysis often require the number of data points to be a power of 2, so we extend our data period to 512 yr by padding it with an approximately equal number of zeros at the beginning and end.

We use the discrete Fourier transform (*e.g.,* Press *et al.* 1994: §12.1)

$$T_n \equiv \sum_{k=0}^{N-1} \tau_k e^{2\pi i k \frac{n}{N}} \tag{1}$$

and its inverse

$$\tau_k = \frac{1}{N} \sum_{n=0}^{N-1} T_n e^{-2\pi i k \frac{n}{N}}, \tag{2}$$

where $i = \sqrt{-1}$, N = 512, n and k are integers in the range 0 to N–1 inclusive, and τ_k takes the data values given in Table 1 or else is zero. n/N is the frequency in the chosen units (y^{-1}).

To check how zero padding and choice of N affect accuracy, the set of τ_k was Fourier transformed, and immediately inverse Fourier transformed to recover the set of τ_k (including the zeros between and outside the values of τ_k given in Table 1). The recovered values agree with the original values to better than 0.003 yr, confirming the padding and the choice of N = 512 as adequate for calculating to the nearest year.

TABLE 1. ^{14}C age and calendar age of tree-ring dated wood of New Zealand matai (*Prumnopitys taxifolia*) from Sparks *et al.* (1995: Table 2). Detrended data = 1950 – ^{14}C age – calendar age. k = the index number of the data point in the extended data set after zero padding.

Calendar age (AD) T_k	Conventional ^{14}C age (yr BP)	k	Detrended data τ_k(yr)	Calendar age (AD) T_k	Conventional ^{14}C age (yr BP)	k	Detrended data τ_k(yr)
1335	617 ± 22	50	−2	1545	324 ± 14	260	81
1345	635 ± 19	60	−30	1555	322 ± 18	270	73
1355	639 ± 20	70	−44	1565	307 ± 22	280	78
1365	683 ± 20	80	−98	1575	377 ± 25	290	−2
1375	637 ± 22	90	−62	1585	385 ± 26	300	−20
1385	618 ± 17	100	−53	1595	396 ± 21	310	−41
1395	593 ± 19	110	−38	1605	361 ± 12	320	−16
1405	599 ± 19	120	−54	1615	367 ± 17	330	−32
1415	530 ± 20	130	5	1625	360 ± 21	340	−35
1425	471 ± 21	140	54	1635	286 ± 18	350	29
1435	484 ± 21	150	31	1645	288 ± 20	360	17
1445	422 ± 21	160	83	1655	290 ± 16	370	5
1455	453 ± 17	170	42	1665	220 ± 20	380	65
1465	450 ± 19	180	35	1675	163 ± 22	390	112
1475	420 ± 22	190	55	1685	163 ± 20	400	102
1485	417 ± 17	200	48	1695	182 ± 23	410	73
1495	380 ± 23	210	75	1705	167 ± 21	420	78
1505	380 ± 21	220	65	1715	157 ± 17	430	78
1515	372 ± 15	230	63	1725	167 ± 20	440	58
1525	334 ± 21	240	91	1735	176 ± 21	450	39
1535	323 ± 15	250	92	1745	206 ± 17	460	−1

In the actual calibration procedure, taking successive sets of D(=10) tree rings at a time to supply the carbon for dating is mathematically equivalent to taking a running mean over D yr of the true calibration curve, and then sampling the running mean once every D yr. Because the width of rings varies from year to year, the mean over D yr is not well defined, so we simply assume the ring widths within any D yr set to be constant. This assumption must introduce some error into the correction for averaging, but as the correction itself, given below, is found to make a difference of somewhat less than 1 yr, the overall error introduced should not be significant.

For constant ring width, then, the running mean in the time domain amounts to convolving a response function, of amplitude 1/D y^{-1}, constant from −D/2 to D/2 y and zero elsewhere, with the true calibration curve (Press *et al.* 1994: §13.1). In the frequency domain this is equivalent to multiplying together the Fourier transforms of the response function and the calibration curve (Press *et al.* 1994: §12.0). The Fourier transform of the response function can be shown to be

$$R_n = \frac{\sin\left(\pi D \frac{n}{N}\right)}{\pi D \frac{n}{N}} \qquad (3)$$

(Press *et al.* 1994: §12.0, 12.1); thus, to correct for the running mean in the time domain by deconvolving it from the true calibration curve, the frequency domain representation of the data set must be divided by R_n.

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A possible problem arises with the above deconvolving procedure due to R_n becoming zero for n/N = 1/D; *i.e.,* we would be dividing the Fourier component at frequency 1/D(=0.1)y^{-1} (and higher harmonics) by zero. We avoid this problem, however, because we filter out by multiplying by zero all Fourier components at frequencies ≥0.5/Dy^{-1}, in order to remove the discrete character of the raw data set (*i.e.,* finite values at intervals of D yr and zero values every year in between). In general, to remove the discreteness a specifically designed filter would be required, but it will be seen below that with the data we are using here, the discreteness is removed incidentally by a further filter that is required in order to reduce variation due to counting statistics. This further filter multiplies by zero all Fourier components at frequencies greater than a cut-off value which, in this case, is considerably less than 0.5/Dy^{-1}.

Variation due to counting statistics amounts to adding a component of noise to the quantity being measured. A filter that minimizes such added noise, in the sense that when applied to the noisy data it produces a least-squares curve passing through the data, is the Wiener filter (Press *et al.* 1994: §13.3). If T_n and Y_n are, respectively, the Fourier transforms of the noisy data and of the noise alone, the Wiener filter is

$$\Phi_n = 1 - \frac{|Y_n|^2}{|T_n|^2}, \tag{4}$$

where $|Y_n|^2$ and $|T_n|^2$ can be shown to be power spectra (Press *et al.* 1994: §13.4). The expression for calculating T_n has already been given; for the noise alone it is

$$Y_n = \sum_{k=0}^{N-1} \upsilon_k e^{2\pi i k \frac{n}{N}}, \tag{5}$$

where each υ_k noise is obtained as a number of years selected randomly according to a normal distribution having a standard deviation equal to that given for the corresponding k in Table 1.

A difficulty arises because only one randomly chosen value of υ_k is used at each value of k. Different runs of randomly chosen values were in general found to give a very irregular power spectrum $|Y_n|^2$ (Press *et al.* 1994: §13.4), varying appreciably from one run to the next. However, an average of 500 runs of $|Y_n|^2$ was found to produce an acceptably constant and smooth set of values, denoted here by $\langle |Y_n|^2 \rangle$.

The same difficulty must appear in the power spectrum $|T_n|^2$ because each τ_k is measured only once and only one set of data is available, but overcoming the difficulty requires a more elaborate procedure than that given above for $|Y_n|^2$. To make a first estimate of a least-squares smoothed curve through the data points τ_k, take the inverse Fourier transform of T_n multiplied by the filter

$$\Phi'_n = 1 - \frac{\langle |Y_n|^2 \rangle}{|T_n|^2}, \tag{6}$$

i.e., take the least-squares curve as

$$\theta'_k = \frac{D}{N} \sum_{n=0}^{N-1} \Phi'_n T_n e^{-2\pi i k \frac{n}{N}}, \tag{7}$$

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where the need for the normalizing factor D will be discussed later. Now, as in deriving Y_n above, for each value of k in Table 1 add to θ'_k a number (of years) selected randomly according to a normal distribution having a standard deviation equal to that given for the corresponding k, and represented as τ'_k. The Fourier transform of a set of τ'_k is

$$T'_n = \sum_{k=0}^{N-1} \tau'_k e^{2\pi i k \frac{n}{N}}, \tag{8}$$

and we may average as many runs of $|T'_n|^2$ as necessary to obtain an acceptably constant and smooth spectrum. As with $|Y_n|^2$, an average of 500 runs was found to be sufficient, and we represent the average as $\langle |T'_n|^2 \rangle$.

The above procedure finally allows us to give our best estimate of the Wiener filter as

$$\langle \Phi'_n \rangle = 1 - \frac{\langle |Y_n|^2 \rangle}{\langle |T'_n|^2 \rangle}, \tag{9}$$

and the least-squares smoothed curve through the data points τ_k, also corrected for the running mean over D tree rings, as

$$\theta_k = \frac{D}{N} \sum_{n=0}^{N-1} \frac{\langle \Phi'_n \rangle}{R_n} T_n e^{-2\pi i k \frac{n}{N}}. \tag{10}$$

The normalizing factor D is required because the power in the spectrum $|T_n|^2$ derives only from the finite data values τ_k separated by D yr with zero values assumed for all years in between, whereas the finite values of θ_k are for *every* year in the range of interest. $\langle \Phi'_n \rangle$ is listed in Table 2.

TABLE 2. Wiener Filter ($\langle \Phi'_n \rangle$) *vs.* Frequency ($(n/N)y^{-1}$)

n	Frequency: $(n/N)y^{-1}$	$\langle \Phi'_n \rangle$
0	0	1
1	0.001953125	0.973
2	0.003906250	0.987
3	0.005859375	0.952
4	0.007812500	0.969
5	0.009765625	0.819
6	0.011718750	0.378
7	0.013671875	0.059
8	0	0
.	.	and zero for all higher frequencies

The ideal ^{14}C *vs.* tree-ring curve, initially subtracted to produce the data in the fourth and eighth columns of Table 1, is now added to θ_k to yield the smoothed, error-reduced and running mean corrected ^{14}C *vs.* tree-ring calibration. This calibration is listed in the Appendix and plotted as a graph in Figure 1.

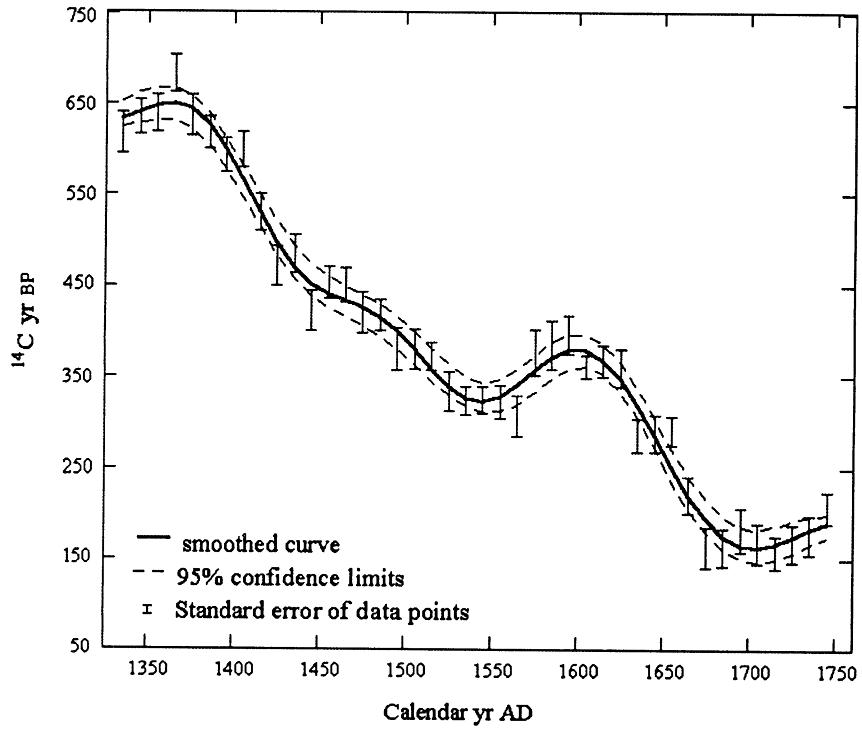

Fig. 1. Least-squares smoothed calibration curve for New Zealand matai (*Prumnopitys taxifolia*) corrected for a running mean over 10 tree rings compared with ±1 standard error range at each of the measured 42 data points. Mean nominal standard error of the curve is 10 ± 3 yr.

Statistical tests and standard deviation of curve

We now test to see if the deviation between 10-yr averages of the above calibration curve and the data is Gaussian. A χ^2 test (Snedecor and Cochran 1967: 84) of the differences between the 42 raw data points in Table 1 (= ^{14}C ages) and a mean over 10 yr centered on the corresponding points of the smoothed calibration curve (column 2, Appendix), using the corresponding standard deviations listed in Table 1, yields $\chi^2 = 5.0$ (df = 5), which is not significant at the 0.95 level ($\chi^2_{0.95,\, df=5} = 11.1$). This indicates that the set of data points constitutes a Gaussian distribution about the averaged calibration curve with the appropriate standard deviations, as it should.

We determine the likely error in the calibration curve itself by constructing from it a set of simulated raw data and then recovering a curve from these data by the procedure described in this paper. Repeating this 500 times allows us to obtain 95% confidence limits and 68% confidence limits. In a Gaussian distribution these confidence limits would correspond respectively to ±2 and ±1 standard deviations, but here this correspondence is only nominal, as there is no guarantee that errors in the estimation of the calibration curve have a Gaussian distribution. The distribution of the data points about the averaged calibration curve, however, is still Gaussian.

In implementing the procedures described in the preceding paragraph we took a normal distribution centered at each point of the 10-yr running mean of the calibration curve corresponding to a value of τ_k in Table 1, and with the corresponding standard deviation of the measured date. A raw data set is simulated by randomly selecting one value from each of these normal distributions, and this simulated data set is processed as described to give a simulated calibration curve. Inspection of the 500 simulated points for each τ_k allowed an estimate of the 95% confidence limits for the calibration curve that are plotted as the dashed lines in Figure 1.

The 68% confidence limits were derived in the same way and averaged to yield an effective overall nominal standard deviation of 10 ± 3 yr. This standard deviation is approximately half the average standard deviation of each raw data point, indicating that the curve has been smoothed in a running fashion over *ca.* 4 consecutive data points.

Comment on the Use of Fourier Analysis

The method presented here, of estimating the true ^{14}C calibration curve from discrete measured data regularly spaced in calendar time, assumes that the ^{14}C age is a continuous, single-valued function of calendar age. It further assumes that after subtracting out the ideal straight line representing equality of ^{14}C and calendar ages, the amplitude of the curve representing deviation from the ideal is everywhere finite. From inspection of the calibration data and consideration of the physics involved, we consider that both assumptions are valid.

Under the above two assumptions, any finite length of curve may be as closely approximated as desired by a weighted sum of sinusoids, *i.e.*, the Fourier sum. Once expressed in this form, the well-developed techniques of Fourier analysis readily allow the weights, and therefore the curve, to be derived from the data while eliminating much of the variation due to counting (or any other known) statistics. Furthermore, distortions of the curve by known processes, such as averaging the ^{14}C age over a number of tree rings, may be corrected by the technique of deconvolving.

Other techniques are available for deriving a continuous curve from the calibration data, but have disadvantages. For example, simple cubic splines create a continuous line through the data points, but in so doing cannot eliminate any of the statistical variation: many of the smaller wiggles in the curve are merely artifacts due to counting statistics. Straight lines joining the data points suffer the same disadvantage, while also introducing artificial discontinuities of slope at the data points.

Running means can eliminate some statistical noise, but in general do not do so in an optimal fashion related to the signal-to-noise ratio in the data. Also, running means require additional criteria for deciding the type of mean, and how many data points the mean is to be taken over.

The method described here can be considered a particular case of least-squares fitting a regression, the Fourier sum, to the data. Least-squares fitting other regressions to the data are potentially capable of equalling or surpassing the performance of the present method, but since we do not know all the physical processes responsible for the deviations of the calibration curve away from the straight line ideal, we cannot choose the correct mathematical form for the regression. We should therefore use a general-purpose function, such as a Fourier sum, which is capable of approximating as closely as desired any mathematical function satisfying the assumptions made earlier concerning the calibration curve. Finally, Fourier analysis has the advantage of being a mathematically well-developed, and widely used and understood technique.

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Appendix

Least-squares smoothed calibration curve at yearly intervals for New Zealand matai (*Prumnopitys taxifolia*) corrected for a running mean over 10 tree rings. Mean nominal standard error of the curve is 10 ± 3 yr.

Tree-ring date (AD)	^{14}C age (yr BP)	^{14}C age (AD)
1330	629	1321
1331	630	1320
1332	630	1320
1333	631	1319
1334	631	1319
1335	632	1318
1336	633	1317
1337	633	1317
1338	634	1316
1339	635	1315
1340	635	1315
1341	636	1314
1342	637	1313
1343	638	1312
1344	639	1311

Tree-ring date (AD)	^{14}C age (yr BP)	^{14}C age (AD)
1345	639	1311
1346	640	1310
1347	641	1309
1348	642	1308
1349	643	1307
1350	643	1307
1351	644	1306
1352	645	1305
1353	646	1304
1354	646	1304
1355	647	1303
1356	647	1303
1357	648	1302
1358	648	1302
1359	649	1301

Tree-ring date (AD)	^{14}C age (yr BP)	^{14}C age (AD)
1360	649	1301
1361	649	1301
1362	650	1300
1363	650	1300
1364	650	1300
1365	650	1300
1366	649	1301
1367	649	1301
1368	649	1301
1369	648	1302
1370	648	1302
1371	647	1303
1372	647	1303
1373	646	1304
1374	645	1305
1375	644	1306
1376	643	1307
1377	641	1309
1378	640	1310
1379	638	1312
1380	637	1313
1381	635	1315
1382	633	1317
1383	631	1319
1384	629	1321
1385	627	1323
1386	625	1325
1387	623	1327
1388	620	1330
1389	618	1332
1390	615	1335
1391	612	1338
1392	609	1341
1393	606	1344
1394	603	1347
1395	600	1350
1396	597	1353
1397	594	1356
1398	591	1359
1399	587	1363
1400	584	1366
1401	580	1370
1402	577	1373
1403	573	1377
1404	570	1380
1405	566	1384
1406	563	1387
1407	559	1391
1408	555	1395

Tree-ring date (AD)	^{14}C age (yr BP)	^{14}C age (AD)
1409	552	1398
1410	548	1402
1411	544	1406
1412	541	1409
1413	537	1413
1414	533	1417
1415	530	1420
1416	526	1424
1417	523	1427
1418	519	1431
1419	516	1434
1420	512	1438
1421	509	1441
1422	505	1445
1423	502	1448
1424	499	1451
1425	496	1454
1426	493	1457
1427	490	1460
1428	487	1463
1429	484	1466
1430	481	1469
1431	479	1471
1432	476	1474
1433	474	1476
1434	471	1479
1435	469	1481
1436	467	1483
1437	464	1486
1438	462	1488
1439	460	1490
1440	458	1492
1441	457	1493
1442	455	1495
1443	453	1497
1444	452	1498
1445	450	1500
1446	449	1501
1447	447	1503
1448	446	1504
1449	445	1505
1450	444	1506
1451	443	1507
1452	442	1508
1453	441	1509
1454	440	1510
1455	439	1511
1456	438	1512
1457	437	1513

Tree-ring date (AD)	^{14}C age (yr BP)	^{14}C age (AD)
1458	436	1514
1459	436	1514
1460	435	1515
1461	434	1516
1462	434	1516
1463	433	1517
1464	432	1518
1465	432	1518
1466	431	1519
1467	431	1519
1468	430	1520
1469	429	1521
1470	429	1521
1471	428	1522
1472	427	1523
1473	427	1523
1474	426	1524
1475	425	1525
1476	424	1526
1477	423	1527
1478	422	1528
1479	422	1528
1480	421	1529
1481	420	1530
1482	418	1532
1483	417	1533
1484	416	1534
1485	415	1535
1486	414	1536
1487	412	1538
1488	411	1539
1489	409	1541
1490	408	1542
1491	406	1544
1492	405	1545
1493	403	1547
1494	401	1549
1495	400	1550
1496	398	1552
1497	396	1554
1498	394	1556
1499	392	1558
1500	390	1560
1501	388	1562
1502	386	1564
1503	384	1566
1504	382	1568
1505	380	1570
1506	378	1572

Tree-ring date (AD)	^{14}C age (yr BP)	^{14}C age (AD)
1507	375	1575
1508	373	1577
1509	371	1579
1510	369	1581
1511	367	1583
1512	365	1585
1513	362	1588
1514	360	1590
1515	358	1592
1516	356	1594
1517	354	1596
1518	352	1598
1519	350	1600
1520	348	1602
1521	346	1604
1522	344	1606
1523	342	1608
1524	340	1610
1525	339	1611
1526	337	1613
1527	335	1615
1528	334	1616
1529	332	1618
1530	331	1619
1531	330	1620
1532	329	1621
1533	328	1622
1534	326	1624
1535	326	1624
1536	325	1625
1537	324	1626
1538	323	1627
1539	323	1627
1540	322	1628
1541	322	1628
1542	322	1628
1543	321	1629
1544	321	1629
1545	321	1629
1546	322	1628
1547	322	1628
1548	322	1628
1549	322	1628
1550	323	1627
1551	324	1626
1552	324	1626
1553	325	1625
1554	326	1624
1555	327	1623

Tree-ring date (AD)	^{14}C age (yr BP)	^{14}C age (AD)
1556	328	1622
1557	329	1621
1558	330	1620
1559	331	1619
1560	332	1618
1561	334	1616
1562	335	1615
1563	337	1613
1564	338	1612
1565	340	1610
1566	341	1609
1567	343	1607
1568	344	1606
1569	346	1604
1570	348	1602
1571	349	1601
1572	351	1599
1573	353	1597
1574	354	1596
1575	356	1594
1576	358	1592
1577	359	1591
1578	361	1589
1579	362	1588
1580	364	1586
1581	365	1585
1582	367	1583
1583	368	1582
1584	370	1580
1585	371	1579
1586	372	1578
1587	373	1577
1588	374	1576
1589	375	1575
1590	376	1574
1591	377	1573
1592	378	1572
1593	379	1571
1594	379	1571
1595	380	1570
1596	380	1570
1597	381	1569
1598	381	1569
1599	381	1569
1600	381	1569
1601	381	1569
1602	381	1569
1603	381	1569
1604	380	1570

Tree-ring date (AD)	^{14}C age (yr BP)	^{14}C age (AD)
1605	380	1570
1606	379	1571
1607	378	1572
1608	378	1572
1609	377	1573
1610	376	1574
1611	374	1576
1612	373	1577
1613	372	1578
1614	370	1580
1615	369	1581
1616	367	1583
1617	365	1585
1618	364	1586
1619	362	1588
1620	360	1590
1621	357	1593
1622	355	1595
1623	353	1597
1624	350	1600
1625	348	1602
1626	345	1605
1627	343	1607
1628	340	1610
1629	337	1613
1630	334	1616
1631	331	1619
1632	328	1622
1633	325	1625
1634	322	1628
1635	319	1631
1636	316	1634
1637	313	1637
1638	310	1640
1639	306	1644
1640	303	1647
1641	300	1650
1642	296	1654
1643	293	1657
1644	289	1661
1645	286	1664
1646	282	1668
1647	279	1671
1648	275	1675
1649	272	1678
1650	269	1681
1651	265	1685
1652	262	1688
1653	258	1692

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Tree-ring date (AD)	^{14}C age (yr BP)	^{14}C age (AD)
1654	255	1695
1655	252	1698
1656	248	1702
1657	245	1705
1658	242	1708
1659	238	1712
1660	235	1715
1661	232	1718
1662	229	1721
1663	226	1724
1664	223	1727
1665	220	1730
1666	217	1733
1667	214	1736
1668	211	1739
1669	208	1742
1670	206	1744
1671	203	1747
1672	200	1750
1673	198	1752
1674	196	1754
1675	193	1757
1676	191	1759
1677	189	1761
1678	187	1763
1679	185	1765
1680	183	1767
1681	181	1769
1682	179	1771
1683	178	1772
1684	176	1774
1685	174	1776
1686	173	1777
1687	172	1778
1688	170	1780
1689	169	1781
1690	168	1782
1691	167	1783
1692	166	1784
1693	165	1785
1694	165	1785
1695	164	1786
1696	163	1787
1697	163	1787
1698	163	1787
1699	162	1788
1700	162	1788
1701	162	1788
1702	162	1788
1703	162	1788
1704	162	1788
1705	162	1788
1706	162	1788
1707	162	1788
1708	162	1788
1709	163	1787
1710	163	1787
1711	163	1787
1712	164	1786
1713	164	1786
1714	165	1785
1715	166	1784
1716	166	1784
1717	167	1783
1718	168	1782
1719	168	1782
1720	169	1781
1721	170	1780
1722	171	1779
1723	172	1778
1724	172	1778
1725	173	1777
1726	174	1776
1727	175	1775
1728	176	1774
1729	177	1773
1730	178	1772
1731	178	1772
1732	179	1771
1733	180	1770
1734	181	1769
1735	182	1768
1736	183	1767
1737	183	1767
1738	184	1766
1739	185	1765
1740	185	1765
1741	186	1764
1742	187	1763
1743	187	1763
1744	188	1762
1745	188	1762
1746	189	1761
1747	189	1761
1748	190	1760
1749	190	1760
1750	190	1760