CHAPTER 11

SUMMARY OF THE CONFERENCE

OVERVIEW OF THE CONFERENCE: IMPLICATIONS OF SEISMOLOGICAL DATA FOR ASTROPHYSICS

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1. Introduction

This review of the conference will necessarily consider the seismological data implications for only stellar astrophysics. While there are some aspects of this conference that interface with subjects like relativity, gravity, stellar systems, studies of chaos, etc., these will not be discussed here. What we are doing here is discussing the interiors of stars. We want to learn about their masses and composition structures. Pulsation periods can be used to measure stellar mean densities. Further details that seem accessible are the solar rotation speed versus depth and latitude and the structure of both solar and stellar atmospheres.

Most of the contributions at this conference dealt with the hard problems of our understanding oscillations of the sun. As we shall see in many cases, the problems in understanding the stars by observing their pulsation periods are even more difficult. Similarities and differences between helioseismology and asteroseismology will be a principal theme of this review.

2. Solar p Modes

The most mature aspect of solar oscillation research is the measurement of solar p (acoustic) modes in the period range from about 3 to 8 minutes. These are the strongest modes, probably because they are most closely coupled to their driving source, the turbulent convection. They are seen both in radial velocity variations as well as in the observed variations of total light. In this latter case, however, they are not seen for high degree modes with rapid spatial variations across the solar surface, even viewed through small diaphragms, because these alternatively bright and dark areas cancel each other to give little net variation in luminosity.

The assumption has always been that these global modes, with lifetimes on the sun of typically ten days, are all of the shape given by the spherical harmonic functions with the integer parameters l and m. For any l, the number of

J. Christensen-Dalsgaard and S. Frandsen (eds.), Advances in Helio- and Asteroseismology, 561–575.

node lines on the surface, there are m node lines that go through a suitably defined pole. These functions (and a few others also) satisfy the hydrodynamic equations of motion in a gravitational field when the motions are small. Thus there are (linear theory) reasons for expecting these spheroidal modes, and interpretation of observations in terms of them is most often done. It is relatively easy on the resolved disc of the sun to actually see these spherical harmonic modes, but for stars with no resolved disc , mode identification is difficult and frequently controversial.

The m pole-to-pole crests in the spherical harmonic functions move relative to the solar surface in either prograde or retrograde directions. For a nonrotating star, each period both prograde and retrograde crests move $1/m$ of the way around the equator, and each superperiod, mP, the crests have moved completely around the sun.

If the star is rotating, these crests move differently because of coriolis forces that come into play in both radial and latitudinal directions. They are small and the velocity change is small for high order p modes with mostly radial motions. Matter moves toward the equator and upward to form the crests of the longitude moving modes, and, due to angular momentum conservation, they lag in the rotation compared to matter already at the equator. Thus a prograde moving crest moves more slowly than it does for a nonrotating star. A retrograde crest would appear to move faster due to this effect. Note that the crests are either slowed or speeded up by this effect, depending on whether it is prograde or retrograde, but the troughs are affected in the opposite direction. Therefore, a rotating star has sectorial modes with unequal spacings between the crests and troughs.

Note also that if the motion is prograde, that is, in the direction of the slow solar rotation, an external observer sees the arrival of the crests that go with the mode with m lines passing through the rotation poles a bit early compared to a case with less or no rotation. This is a doppler effect that decreases the intrinsic mode period if prograde, or increases the period if retrograde.

One then sees that for a given l value it is possible to have twice that value of modes plus one. Modes for m between 1 and l are retrograde, and those between -1 and $-l$ are prograde; $m = 0$ has all l node lines as lines of constant latitude. For odd $l - |m|$ integers, there is a node line at the equator. There is always symmetry of the constant latitude node lines around the equator.

3. Internal Rotation

The current frontier in the measurement of the p mode periods is in their use for the estimation of the internal rotation structure. It seems to be established that the sun rotates with the surface equatorial velocity (or just slightly less!) down to about a point 20 percent of the radius from the center. There it increases to maybe 3 to 5 times faster, but the p modes do not probe this part of the sun very effectively. Actually for $l = 1$ in high order p modes that are just not high enough to leak into the chromosphere and be damped, the eigenfunction penetrates to just less than 0.05 of the solar radius. However, this mode has only two m values that can sense, respectively, the prograde and retrograde motions relative to the deep solar rotation. For the 300 second modes the penetration is down to 0.06 of the solar radius. For larger l at the same frequency, the turning point of the acoustic wave is further out by the factor $\sqrt{l(l+1)}$, but then there are more m values to reveal the rotation.

The dependence of the rotation on latitude is even more difficult to measure. Large l modes with m approximately equal to l are concentrated toward the equator. Thus for each l the latitude dependence of the rotation can be measured by comparing the rotational mode period splitting for different m values. Large positive or negative m (near l) values sense only the equatorial rotational splitting of periods whereas lower m frequency splittings feel both the equatorial rotation and the slower, higher latitude rotation.

The rotation results to date, which are not in good agreement between the several teams making the measurements and analyses, indicate that through the entire convection zone, the internal rotation is not appreciably different than that for the surface, with faster rotation at the equator. Rotation seems to be constant on cylinders, with those nearer the rotation axis having slower rotation as seen actually at the surface. Deeper than the convection zone (using l values less than 20), there seems to be no latitude effect, just solid body rotation.

I should mention briefly that Hill and students at Arizona get large rotation velocities (like 2.9 microhertz) through almost all the sun using their frequency splittings from low order g modes seen as solar limb fluctuations. They propose that the difficult-to-measure p mode splittings may be contaminated by surface brightness fluctuations due to causes other than oscillations. Sunspot activity can make the solar surface not conform to the assumed spherical harmonic shapes to produce small errors in mode periods. The light and velocity fluctuations observers of the p modes on the solar surface doubt, in return, that the g modes and their rotational splittings are reliably detected by the limb data.

Details for stars are much harder to obtain because only much lower l values can be detected in the integrated light of the stellar disc. Yet for the nonradial pulsators near the main sequence, such as the δ Scuti, 53 Persei, β Cephei, and ζ Ophiuchi variables some average rotation speeds have been suggested. The $v \sin i$ values from the rotationally broadened shapes of all lines in the spectrum can then be combined with the rotation v value to give an inclination of the rotation axis to the line of sight. At best only a few reliable rotation velocities are available, and the much smaller differential m splittings giving latitudinal effects are not yet available.

4. Solar Structure

The solar p modes have been earlier used to measure the solar structure at and below the convection zone. Deeper convection to over two million kelvin is indicated. It now seems that the p modes are increasing their periods as the solar sunspot cycle goes through a minimum according to van der Raay (1984), Woodard and Noyes (both teams at $l = 0$ and 1), and Hill and students, for $l = 7$ to 12 at Arizona. For l values between 15 and 22 the Arizona workers get period decreases. This may indicate changes in the structure of the convection zone, because the low l modes penetrate deep into and beyond the convection zone, the intermediate l values probe all the convection zone, while the larger l modes see only into the upper half of this zone. The reality of these frequency changes needs to be confirmed further around the solar cycle if for no other reason than the fact that they are only 0.4 microhertz for the low l and 0.04 microhertz for the largest l values.

The project by Dappen and Gough to use the p mode periods to determine an accurate helium abundance has still not progressed to a definitive answer. The fitting of the p mode ridges in the k–ω diagram gives a Y like 0.25. The current best value (Lebreton and Maeder 1986) is just at $Y = 0.28$ to match things like the solar mass, luminosity, radius, surface composition, and age, but the consistency with the details of the p mode periods is not satisfactorily in hand.

Actually different theoretical teams calculate slightly different p mode periods, and these differences are much larger than the uncertainties in the observed periods. The causes of these differences are due to both the physical property data used by different teams, but also due to different methods of solution for the linear theory eigensolutions. Christensen-Dalsgaard, Gabriel, and Ulrich are coordinating an effort to cross check theoretical calculations.

The typically half percent difference between the measured and observed p mode frequencies is disturbing because it means that there is an error in eigensolution predictions from all theoretical teams. The observed frequencies are greater than predictions at low frequencies and smaller at the highest p mode frequencies. Further, for l values above 20, where all the action is located in the convection zone or just below, the errors are systematically larger.

5. Solar Composition

Demarque and collaborators suggest that maybe there has been some sinking of the surface helium and heavier elements from the homogeneous convection zone, and that the deep primordial composition is richer in helium and especially the heavier elements. This composition gradient helps to decrease (but not eliminate) the discord between observations and theory mostly because the opacity at a few million kelvin is increased by the increased heavy element abundances, giving higher temperatures and sound speeds there.

Of course, higher helium and higher heavy elements give higher opacities at the solar center also. Usually that composition is not considered because the predicted solar neutrino flux then becomes even higher and disagrees even more with the chlorine detection results.

There has been in the last eight years, a feeling that there may be something incorrect about the equation of state of the solar material. That is a parallel worry about the much older question as to whether the opacities are correct. My feeling is that both the equation of state and opacity are correct for the assumed compositions, but there is a real possibility that somehow the original composition and its evolved structure is still not well known. Discrepancies between observation and theory for the p mode periods are probably due to composition (not necessarily the helium) problems.

If there is any mixing in the sun induced by rotation effects, it seems that it would be just below the convection zone where the temperature gradient is not very subadiabatic. Models for this case have been considered by Lebreton, Berthomieu and Provost, and they discuss both the effects of the ^{3}He and ^{7}Li abundances as well as the p mode period changes. Unfortunately, the mixing of these fragile elements to deeper depths does not explain at all the major solar neutrino discrepancy.

6. Mode Excitation

Recent observations by Duvall, Harvey and Pomerantz give mode line widths increasing with both frequency and l value. These line widths indicate mode lifetimes, because the sharpness of the line depends on how long the line is present to allow and perfect its detection. They use p modes that sample either all (l = 20–39), part (l = 40–59), the upper layers (l = 60–79), or just the surface regions (l = 80–98) of the convection zone. Lifetimes longer than a million seconds are difficult to measure, but many modes decay more rapidly than this lifetime of about 10 days.

My concept for the excitation of the p modes in the sun and solar like stars is that they are induced by the turbulent elements of the convection which have similar spatial shapes and periods as the global oscillation patterns. The turnover time for a convective element, taken as the pressure scale height divided by the mixing length convective velocity, is perhaps over a million seconds at the bottom of the zone, but is only a few hundred seconds at the top. The reason for the selection of the 300 second modes in the sun rather than longer period ones, that could also couple with deeper convection, is that the 300 second modes match the convection time scale in the highly superadiabatic, pulsation driving, surface regions of the convection zone.

The generation of acoustic noise according to Goldreich and Kumar in the first approximation should give equal energy in all five minute modes, regardless of the l value. This is because in the convection zone the mode oscillation pattern is very closely the same from $l = 0$, $k = 23$ to $l = 60$, $k = 11$. This can be seen from plots of these mode structures given by Christensen-Dalsgaard, Gough, and Toomre in a review in *Science* (1985). Our calculations for $l = 90$ and $k = 8$ give again very similar modal structure, and there should also be good convection driving for these high degree modes.

The nonadiabatic calculations by Kidman and Cox (1984) give the result that the 300 second modes up to at least degree $l = 5$ decay by radiative damping in the subphotosphere layers in time scales of only half a day, much shorter than observed lifetimes. More recent calculations with turbulent pressure included, but important only between the photosphere and the depth where the temperature is 12,000K, give even larger damping and shorter lifetimes. This indicates that there must be a source of considerable continuous pulsation driving which, however, is just not large enough to keep the pulsations going all the time in the presence of the strong radiative damping.

In these calculations there is the usual κ and γ effects driving at temperatures up to about 9500 K, and radiative damping exterior to 8000 K. The maximum damping is only a few optical thicknesses deep at about 7000 K. If there was not such strong convection, carrying more than 99 percent of the luminosity, there would be considerably more of the radiation blocking driving peaking at the usual temperature of about 11,000 K as it occurs in the lower density giant star pulsators which also have convection zones.

For the $l = 90$ case, the radiative damping is about twice as large as for lower l. This is probably due to the easier horizontal heat flow in the optically thin damping regions. It seems that the increased damping is the reason for the shorter lifetimes for the highest degree modes considered by Duvall, Harvey, and Pomerantz.

These linear, nonadiabatic, nonradial calculations, using a new Lagrangian formulation for the nonradial pulsation eigensolutions give different results from those of Ando and Osaki (1975) and Goldreich and Keeley (1977a). These authors get strong κ and γ mechanism driving that overcomes the photosphere damping. We get the same driving and damping regions, but damping dominates. The cause of the different results is not known, but it may be an important function of the material opacity. Our calculations have used the Stellingwerf (1975ab) fit for the opacities, which we have found to match the actually calculated opacity points very well. The photosphere structure and the details of the radiation transport there may also be of crucial importance. Christensen-Dalsgaard and Frandsen (1983) discuss this problem and suggest also that the Ando and Osaki results may be incorrect. Just as in pulsating stars, the net balance of the driving and damping is extremely important in making a stars stable or variable.

It seems that the line width observations agree reasonably with theory, if convection is strongly driving in the surface part of the oscillation modes and the damping is due to subphotosphere and photosphere radiation losses.

It also could be that the convection driving is reduced for high l, and then the radiative damping could dominate even more and decrease the mode lifetime. That does not seem to be the case, however, according the Antia et al. calculations.

These convection driving calculations of Antia, Chitre and Narasimha (1982) actually give a net pulsation instability. They find that there is good coupling between the convective eddies and the pulsation modes, especially if l and k are large enough. Apparently, the physical mechanism is that during the compression phase of the pulsation, the amount that the temperature gradient is superadiabatic is decreased, reducing the convective luminosity. During the subsequent expansion phase, the gradient becomes more superadiabatic to give a larger luminosity. This cycling of the luminosity during pulsation is exactly like that for radiation blocking by the κ and γ effects.

Coupling of the convection with the pulsation must occur exterior to the numerous oscillations in the variation amplitude of the structure variables, in which there are about k nodes deeper. Thus the spatial structure for both the convection and the pulsation modes is similar. It is in the last mixing length before the surface, and the region of growing amplitude of the pulsation solution toward the surface, that the driving occurs. The variation of the temperature is greater at the photosphere than deeper for this part of pulsation mode, and therefore at compression, the temperature increase there is larger than for deeper levels. Then it is not unreasonable that the eigensolutions give a reduction of the superadiabaticity at maximum compression phase, blocking the flow of luminosity until a later time when it can be used to drive pulsations.

This coupling of the convective motions to the normal mode motions by buoyancy is what is called by Goldreich and Keeley (1977b) a dipole process. These earlier authors actually thought that the turbulent pressure coupling was more important, and they focussed on this other (so-called quadrupole) process. Apparently the new Goldreich and Kumar work and the Antia et al. ideas center on coupling by a buoyancy effect that I also feel is appropriate.

While the reduction in the temperature gradient at maximum compression has an important effect for the convective luminosity, because for that luminosity the temperature gradient is relative to the adiabatic gradient, it apparently does not cause significant pulsation driving in purely radiative regions. There must, however, be a small radiation blocking effect that has always been included previously in the radial and nonradial nonadiabatic calculations for many other classes of variable stars.

It is not clear to me what the name of this driving process is. It is frequently called the Cowling mechanism, but it seems that for that effect, there needs to be a restoring force by a magnetic field. Then the convective eddies are restrained from going too far from their starting point, and as they return, they have gained (lost) heat on their downward (upward) excursions. There is a similar process called the Kato mechanism where, however, the restoring force is a composition gradient in the nuclear burning regions deep in the star. The former case has been suggested for the excitation of high order p modes in the convection zone of Ap stars, while the latter may help excite pulsations in the β Cephei variables. This "gradient" mechanism, that can occur without any restoring force on the convective elements, seems to have been discussed first by Spiegel (1964). Similar considerations were discussed for gravity modes by Souffrin and Spiegel (1967).

I feel that the Ando and Osaki, the Goldreich and Keeley, and the Antia, Chitre, and Narasimha results that give self excited oscillations must somehow overestimate the driving. This is simply because the modes are actually observed to be decaying through their significant line widths. If the radiative damping that we and Christensen-Dalsgaard and Frandsen get is too much, as some authors might insist, there is always the possibility that turbulent viscosity can become important and cause further damping.

7. Solar g Modes

The gravity modes for the sun have been reported many times by the workers in Crimea, Stanford, and at SCLERA (University of Arizona) as well as possibly in the ACRIM data from the solar maximum mission satellite. The strongest mode reported, the 160 minute (1/9 day!) g mode, has not been strongly confirmed by the Stanford and ACRIM data. Their sensitivity is not so great for this mode though. There is a feeling that this mode may not be detected at all, though the SCLERA results can even identify the mode as g_9 with $l = 2$ and $m = 2$ (retrograde). Theoretical predictions that this mode may decay over perhaps one hundred thousand years, make it reasonable that the Crimea and other observations give the same pulsation phases over many years. However, amplitude changes as recently reported from Stanford, make the identification very suspect. It may be that surface magnetic field activity such as plages and sunspots, contaminate these Stanford interpretations, and the observed amplitude changes are not actually due to a variable g mode amplitude. Scherrer reports that problems with removing the data drifts may also be a cause of the apparent amplitude changes.

For upper main sequence and the white dwarf and pre-white dwarf stars, the longer period g modes are most easily detected. Only in the δ Scuti and possibly the β Cephei variables are the p modes easily seen. There is an important difference, however, between the sun and these stars. These stars have a mechanism that produces net driving, and they are self excited to a limiting amplitude set by a balance between driving and damping.

There is another important difference between the sun and the giant stars. The deep and strong convection, giving a large superadiabatic region at the top of the convection zone, blocks the appearance of the g modes because they cannot exist in this region. It is only by tunneling through the convection zone that we can see them at all. The penetration is less for the higher order g modes that have shorter radial spacing between nodes. The evanescent layer is both apparently and actually thicker for them. The decay of the oscillation in this evanescent layer is exponential on a spatial scale that is shorter the more the temperature scale is superadiabatic. Nevertheless, g modes up to about g_{20} for very low l may be observable. l values above 10 or more will be difficult to detect because their oscillations in amplitude are confined to the deep layers, and also the period spacing gets very small.

According to theory, the horizontal motions exceed the radial (vertical) motions at a period longer than about 167 minutes for the solar case. The scaling for this ratio goes as the square of the period times the mean density, so for giant B stars, the horizontal motions can equal the radial ones for periods like one day. Detection of these horizontal motions has not been made for the B stars, and the observers predict that they may be much smaller than expected from the theory with the assumption of spheroidal modes. Since the shortest period (lowest order) g mode in the sun is expected to be about 35 minutes, and there are several modes at $l = 1, 2$, or 3 with periods shorter than the 167 minute value, vertical motions dominate. These should be detectable with either the radial velocity or light variations. Apparently these modes are seen readily in the limb variation measurements at Arizona, but higher g modes at low l may be hard to detect. Periods over several hundred minutes may not be reliably reported.

An important feature for the g modes is that for high order (and therefore at least moderate degree to keep the radial motions large relative to the horizontal ones for easier observations), it is theoretically expected that their period spacing should be a constant that I call P_0. Solar models that are conventionally evolved to the solar age have P_0 equal to about 37 minutes. Analysis of data from several sources, most recently ACRIM, have indicated that the observations give P_0 about the same value. Mixing hydrogen down into the central regions, to give hydrogen burning for the observed luminosity without the high temperature that gives large neutrino flux predictions, makes P_0 considerably larger. Thus it has been often stated that deep mixing, that is hard to do anyway in the presence of the steep hydrogen μ gradient, has not occurred in the sun.

8. WIMPs

At this conference, it has been proposed that the ACRIM data can also support a P_0 of 29.85 minutes. The problem with the g modes is that their frequency spacing of a few microhertz is just slightly larger than the $2l + 1$ component frequency splitting due to rotation in the solar core region. Note that velocity and light variations integrated over the entire solar surface cannot detect modes where $l + m$ is odd, so that there are really only $l + 1$ modes observable. Noise free data can then determine both the P_0 and the central rotation, but such high quality data do not exist. We do not reliably know either P_0 or the central rotation speed.

The reason for looking at the ACRIM time series data for 29 minute period spacing is the Faulkner and Gilliland (1985) proposal that there are weakly interacting massive particles (WIMPs) orbiting the solar center which conduct energy from the center much more effectively than photons. In that case the central temperature is reduced to perhaps less that 13 million kelvin, and the central density is increased (to maintain the required central pressure) to maybe 200 g/cm^3.

Predictions for P_0 in this model range from 29 minutes (Faulkner, Gough, and Vahia, 1986) to 32 minutes (Dappen, Gilliland, and Christensen-Dalsgaard, 1986) instead of the larger values of 37 minutes because these central regions are even more subadiabatic with the shallower temperature gradient. The Brunt-Väisälä frequency is therefor increased significantly. Note that mixing hydrogen into the central regions gives a lower temperature gradient also, but this higher hydrogen at the center gives a higher opacity, and the temperature gradient is then just barely subadiabatic. Mixing in hydrogen results in a very low Brunt-Väisälä frequency and a very large P_0.

I must point out that it is not unreasonable for the observed P_0 to be lower than the standard model asymptotic value of, say, 37 minutes. This value is not reached until order 30 or more. Such high order g modes are not likely to be observable in the ACRIM or any other data because of their evanescent behavior in the deep convection zone and because the high order modes have very small radial motions and very small luminosity variations. Actually, the P_0 increases from just over 20 minutes to about 34 minutes at k=10, giving an average of about 29 minutes over this range even for a standard model. Averaged over the first 20 orders, the value is still just 32 minutes. If the ACRIM data can see only these modes, in spite of the fact that higher order modes were expected in the data, the observed 29.85 minutes for P_0 actually indicates the standard model with no WIMPs.

9. Solar g Mode Excitation

Pulsation excitation for the deep-seated g modes have been discussed in a number of papers, and the possibility that the ^{3}He periodic burning can cause pulsations and even mixing is dealt with here again by Gavryusev and Gavryuseva. At Los Alamos, we can calculate some pulsation driving all through the deep regions at temperatures above about 10 million kelvin, but is is probably not enough to make g modes self excite. This problem was most recently considered by Saio (1980), and it seems that damping in the convection zone may be important. Low order and degree g modes may be linearly unstable as suggested by Saio, but certainly not the 160 minute mode that has too much radiative damping in the same deep layers that also give the ϵ mechanism driving.

10. Solar Like Variables

The question of solar like stars pulsating as the sun does has been an interesting one to pursue. ϵ Eridani has observed p mode periods and separations (170 microhertz) that seem to accord well with theoretical expectations for a mass of perhaps 0.8 $M_{\odot}$. At this conference there were two papers that state that ϵ Eridani could be a young star with an age of less than two billion years and give the observed p mode period spacings. This is in better agreement with the observed large chromospheric activity, and changes the arguments that Guenther and Demarque (1986) have made earlier for an age over 10 billion years. They now suggest that the metal abundance is more than double that assumed before, giving a larger luminosity at a young age. Such large Z may be reasonable, since the measured abundances may have been affected by the strong stellar activity that filled in many absorption lines.

The case for Procyon is also reasonably good. It seems that a mass of 1.5 $M_\odot$ can give a stellar radius that is larger than that for the sun and that is consistent with the rather low frequency p mode spacing of 80 microhertz.

The real problem with the solar like oscillations in other stars is met with α Centauri A. Its p mode period separations seem to be observed by Fossat at 165 microhertz, but this high frequency spacing is almost the same as for ϵ Eridani. That indicates a very low radius, and yet the accurately observed mass and luminosity point to a radius at least 20 percent larger than the sun. There has been discussion in the literature that perhaps periods may depend on the amplitude of the pulsations. From calculations of radial modes in yellow giants, both linear and nonlinear, the expected period changes are at most a few percent, with all modes experiencing essentially the same period shift. There seems no possibility that the high order modes in these solar like stars are affected at all in their frequency separations.

There is another problem, not very much addressed at this conference, but of great interest here. The amplitudes of the observed oscillations in these three bright stars are much larger than seen in the sun. In ϵ Eridani, they are over 100 times larger. My opinion is to not worry about that problem because many self excited yellow giants also have amplitudes that are not very well understood. The detailed calculations of the balance between the driving and damping of pulsations are difficult to make accurately whether the amplitudes are small or large. For these solar like oscillations, my prejudice is that they are excited by convection and damped by radiation losses. These losses may be greatly reduced if the composition is more metal rich, extending the lifetime of modes and allowing excitation to larger amplitudes.

11. Oscillation H-R Diagram

A very useful concept has been advanced at this conference. A plot of δ (the average spacing between the radial, $l = 0$, and $l = 2$ p mode frequencies ranging over k as small as 10 to as large as 30) versus Δ (the average spacing between mode orders at fixed l) was constructed by Christensen-Dalsgaard for masses between 0.7 and 1.5 $M_\odot$ at ages up to hydrogen exhaustion in the center. The ordinate δ measures mostly the age through the probing of the central hydrogen abundance. The abscissa generally measures the total mass by indicating the mean sound speed travel time across the star. This diagram can be used easily by future observers to give a first guess about the structure of a star using precise (space based?) periods.

These spacings have been discussed in connection with the WIMP problem, because the δ has some sensitivity to the central solar structure. Observations give a value of δ of 9.2 microhertz with perhaps 0.6 microhertz uncertainty. WIMP models predict δ ranging from 8.8 to 9.2, whereas standard unmixed models predict δ as 10.0 microhertz. Mixed models with larger central hydrogen give δ even larger. Thus proponents of WIMPs suggest that even the p modes support the existence of WIMPs, though I am not all that certain about the accuracy of the p mode separations.

12. Ap Variables

The Ap stars with strong magnetic fields and peculiar surface compositions (enhanced helium at the magnetic poles and depleted helium at the magnetic equator) exhibit high order p mode oscillations similar to those seen on the sun. These Ap variables fit in well with the theoretical expectations for the Δ period spacing. Even though these stars are probably excited by magnetic convection (Cowling mechanism), their periods are fixed by their general stellar structure. A frequency spacing near 60 microhertz is appropriate for an evolved star somewhat more massive than the sun. There are eleven known stars in this class. Their few minute periods can be determined to great precision with a long, gap-free, series of data taking. The various campaigns for observing these stars, and maybe even space based observations, will pay good dividends.

For most of these stars there are only a few periods observed. In HD 60435 periods between 12 and 20 minutes are seen according to the results of an observing campaign from CTIO, Las Campanas and SAAO. In this case the spacing in frequency is about 70 microhertz.

13. White Dwarfs

White dwarf variables actually come in three classes: the PG 1159–035 pre-white dwarfs (GW Vir variables) that have temperatures above 80,000 K; the DB variables, with temperatures about 25,000 K; and the classical white dwarfs with temperatures about 11,000 K, the ZZ Zeti variables. All these stars are observed to pulsate only in g modes, with the order being very high. Much progress is being made in understanding the white dwarf structure that is being probed by these pulsation modes as reviewed by Winget.

Of particular interest is Kepler's work on the white dwarf G117B15A which has a persistent period of 215.2 seconds that has remained constant for 11 years now. This means that the evolution time is longer than 6.9×10^8 years which agrees with the expected evolution period change time of about 5×10^9 years. The expected period change should become observable in 5 to 10 more years, if the expected theoretical 1 part in 10^{14} period change is correct. It is fortunate that this star is a very hot ZZ Ceti variable, right at the blue edge of the pulsation instability strip, because the evolution rate becomes slower, the cooler the white dwarf gets.

An exciting result for the pre-white dwarf PG 1159-035 by Kawaler is that the spacing between the 8 periods seen is either 8.8 or 21.1 seconds for the g modes of order between 20 and 40. This indicates that the l value is either 3 or 1, respectively. Models give this P_0 of 30 seconds if the mass is very close to 0.60 $M_{\odot}$. Rotation spinup seen in observed period decreases had previously indicated $l = 3$ for the 516 second period.

In my view, the periods of the white dwarfs are not going to be able to probe their unknown composition structure. There seems to be no dispute that the mass of all these stars is right at 0.6 $M_\odot$,and the general white dwarf theory (using the degenerate equation of state), yields a definite radius. However, the settling of the helium and the upward diffusive migration of helium and heavier elements over the long life of the degenerate configuration, probably needs further study. That together with the controversial amount of surface hydrogen left on the star after its production from a red giant makes period identifications and predictions extremely difficult. Our work at Los Alamos at least shows that thick hydrogen surface layers, predicted by many evolution calculations, can allow unstable nonradial pulsations, previously thought forbidden for layers thicker than 10^{-8} $M_\odot$.

The cause of the white dwarf nonradial pulsations has also been a recent problem. The very strong convection, due mostly to the very high density in the surface layers of these high gravity objects, very effectively blocks the deeper radiation luminosity. The normal approximation for this convection is to assume that it varies too slowly to be affected by the pulsating configuration. For the white dwarfs, this is very incorrect. The adaptation time is short. Recent calculations at Los Alamos show that the frozen-in approximation produces a periodic blocking of the total emergent luminosity, just as the κ and γ effects for radiation luminosity. This gives pulsation driving, but when the luminosity is allowed to adapt readily (and in a very approximate way), the driving disappears, and the models are predicted to be stable. Thus it appears that some convection blocking is necessary to make the models pulsationally unstable as observed. The complicated facts seem to be that there is the normal κ (and γ) effect at the blue edge, but in the middle of the instability strip, convection blocking is the main driver of nonradial pulsation instability.

There is a further problem. The exact degree of interaction of the convection zone with the normal mode seems to be an important aspect of mode selection. Many more modes are predicted to be unstable than observed, even at very low amplitude. Also the period of a selected mode seems to depend importantly on the structure at the bottom of the convection zone. Thus mode identification may be extremely difficult to do.

14. Other Variable Stars

I have a few comments about other variable stars that were mentioned during the conference. Massive stars have been thought to be unstable against radial modes due to the ϵ effect of hydrogen burning at the center of the star. Klapp, Langer, and Fricke surprised everyone by stating that stars up to 440 $M_\odot$ were stable, because the limiting mass is thought to be less than 100 $M_\odot$. Work by Odell and colleagues, Baker and colleagues, and now by Cahn, Cox and Ostlie at Los Alamos has shown that somehow, Klapp et al. must be wrong. The earlier work by many others such as Ziebarth (1970), Aizenman, Hansen, and Ross (1975) seems to be confirmed.

I would like to support the very sensitive velocity measurements for the red giants being made at Arizona. These stars are not usually thought to be pulsationally unstable, but if a period can be found for them, excited by any random process, such as stochastic convection driving, details of the internal structure might be revealed. A single period gives only the mean stellar density, but there are possible surprises. A second period (giving a reasonably accurate period ratio) can indicate details such as the convection zone structure or a composition effect. There may not be any nonradial modes in the yellow and red giants due deep radiative damping, and therefor all observed modes are probably radial.

The report by Jerzykiewicz, Sterken, and Manfroid about the radius of Spica reinforces the problems for this star. Transient pulsations were seen by many observers for many years with up to four periods seen. Yet no theoretical predictions for any unstable mode have ever been made. The period coming closest to being pulsationally unstable is the radial fundamental mode, but the best fit to the observed mode is the radial overtone or even the second overtone. The cause for all the B star pulsations remains to be discovered.

The cause of pulsation is known for three other classes of variable stars, but the occurrence of two modes is unexpected. The double mode classical Cepheids, the double mode RR Lyrae variables, and the double or multiple mode δ Scuti variables are not understood. Nonlinear hydrodynamic calculations give only one mode predicted at a time. For the δ Scuti variables, it may be that most of the several modes may be nonradial with the usual κ and γ effects driving, and they may not interfere or be interfered with by the radial modes. However, that prediction is not possible with our lack of fast 3D hydrocodes today.

In conclusion, I would like to remark that the problems of solar oscillations are many and difficult. They will take even more precise observations, especially for the g modes. However, the problems for the other types of stellar variability are even more difficult. For a stellar stability theoretician, solar oscillations are easy!

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