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Filament Fragmentation

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Abstract. The formation and evolution processes of magnetized filamentary molecular clouds are investigated in detail by linear stability analyses and non-linear numerical calculations. A one-dimensionally compressed self-gravitating sheet-like cloud breaks up into filamentary clouds. The directions of the longitudinal axes of the resulting filaments are perpendicular to the directions of magnetic field lines unless the column density of the sheet is very small. These magnetized filaments tend to collapse radially without characteristic density, length, and mass scale for the further fragmentation during the isothermal phase. The characteristic minimum mass for the final fragmentation is obtained by the investigation of thermal processes. The essential points of the above processes are analytically explained in terms of the basic physics. A theory for the expected mass function of dense molecular cloud cores is obtained. The expected mean surface density of companions of dense cores is also discussed.

1. Introduction

At present most of theoretical works on the binary formation mechanism are focussed on the evolution of an isolated cloud whose mass is about one Jeans mass (see, e.g., the reference of Bodenheimer et al. 2000). Their results do not seem to successfully produce binaries with intermediate and large separation (Truelove et al. 1997; Tsuribe & Inutsuka 1999a,b,2000; Boss et al. 2000; see also p.184 of the poster booklet). On larger scales, star forming regions are not isolated and show remarkable filamentary structures. Although it is difficult to know the strength of the magnetic field, their projected position angles are observed extensively in star forming regions. For example in the Taurus dark cloud, the magnetic fields are perpendicular to the filamentary structures (see e.g., Heiles et al. 1993). This short article is intended to explain the formation and evolution processes of such filamentary structures in magnetized molecular clouds, and to discuss possible consequence to the binary formation.

2. The Formation of Filaments

First we describe how filamentary structures form. We adopt a scenario of "induced star formation" in which a molecular cloud is, at first, one-dimensionally compressed or swept up into sheet-like configuration. Many kinds of compressional mechanisms are expected: supernova remnants and winds from massive stars, etc. A self-gravitating sheet-like cloud is unstable and fragments into smaller clouds. The character of this gravitational instability is investigated by many authors (see the reference of Nagai, Inutsuka, & Miyama 1998). Among them, Miyama, Narita, & Hayashi (1987a,b) studied the fragmentation process of a non-magnetized sheet-like cloud and have shown that in general a sheet-like cloud fragment into filamentary clouds. Nagai et al. (1998) investigated the effect of magnetic fields on this filamentary tendency and found that the resulting configuration depends on the extent of the external pressure of the sheet-like cloud. The essence of the mechanism is explained in this section.

In a paper by Nagai et al. (1998), perturbations of a pressure-confined isothermal gas layer with uniform magnetic fields are investigated in the linear regime. The unperturbed magnetic field lines are parallel to the midplane of the layer. For the layer with a thickness much larger than the pressure scale height H ($= c_s[2\pi G\rho_c]^{-1/2}$ where c_s is the isothermal sound speed and ρ_c is the midplane density), perturbations whose wave vector is *parallel* to the magnetic fields grow faster than those perpendicular to the fields. Therefore the layer fragments into filaments, and the direction of longitudinal axis of each filament is perpendicular to the magnetic field lines. On the other hand, the layer with a thickness much smaller than H becomes more unstable for perturbations *perpendicular* to the magnetic fields. In this case it fragments into filaments, and the direction of longitudinal axis of each filament is *parallel* to the magnetic field lines. In this way, the fragmentation direction is determined only by the thickness (and hence, the surface density) of the sheet-like cloud, and not by the strength of the magnetic field.

The reason for the difference of the fragmentation processes can be explained as follows. We take the midplane of the layer as the x-y plane of the Cartesian coordinates. $\mathbf{k} = (k_x, k_y)$ is the wave vector of the normal mode. The z-axis measures the distance from the midplane. The surface density of the unperturbed sheet is $\sigma(z) = 2\int_0^z \rho(z')dz'$.

Suppose that the unperturbed magnetic field lines are in the x-direction, and let us consider two layers as the extreme cases.

A a geometrically thick layer with the boundary $z_b \gg H$ at which the external pressure is much smaller than the midplane pressure ($P_{ext} = P(z_b) \ll P_c$). In this case $\sigma(z_b) \approx \sigma(\infty) = 2\rho_c H$.

B a geometrically thin layer with the boundary $z_b \ll H$ and $P_{ext} = P(z_b) \approx P_c$. In this case $\sigma(z_b) \ll \sigma(\infty)$.

We take the same midplane density ρ_c and pressure P_c for the two layers. In both layers, the growth time t_g (the inverse of growth rates) of the most unstable linear perturbation is on the order of the free-fall time, i.e., $t_g \sim \sqrt{G\rho_c}$, because the fragmentation is due to self-gravity. The separations of fragments or the

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wavelength λ of the eigenfunction of the most unstable perturbations are a few times the effective thickness of the layers.

The fragmentation of a layer **A** is as follows: Within the growth time, the sound waves can barely propagate the effective width ($\sim 2H$) in the z-direction, and propagate the length λ inside the fragments in the x- or y- direction.

$$c_{\rm s}\, t_{\rm g} \sim \lambda \gtrsim H. \tag{1}$$

Thus, the effect of pressure is not overwhelming and the fragmentation is through straightforward compressional motion of the isothermal gas. In this case the magnetic pressure impedes the motion in the y-direction, because the y-direction motion has to compress the magnetic field lines. Therefore the growth rate of the perturbation with $k_y = 0$ is greater than that of $k_x = 0$.

The fragmentation of a layer **B** is as follows: Within the growth time, the sound wave can propagate many times in any direction inside the fragment,

$$c_{\rm s}\, t_{\rm g} \gg \lambda \gtrsim z_{\rm b}. \tag{2}$$

Thus the effect of pressure is substantial and smooth out the density perturbation so that the most unstable mode behaves as if it is of incompressible gas. This *incompressional* motion of the most unstable mode of *compressible* gas is a key to understand the physics at hand. The property itself is observed in the literature (for sheet, see e.g., Elmegreen & Elmegreen 1978; Miyama et al. 1987a; Lubow & Pringle 1993; Nagai et al. 1998; for cylinder, see Nagasawa 1987; Inutsuka & Miyama 1992). In the case of incompressible gas, however, the x-direction motions suffer from the tension of magnetic field lines and the y-direction motions do not suffer from the magnetic field. Thus, the magnetic field affects the motion in just opposite sense compared with the case **A**. In this case, the growth rate of the perturbation with $k_x = 0$ is greater than that of $k_y = 0$ (Oganesyan 1960).

In the case of the fragmentation with small column density and high external pressure, the axes of the resulting filaments are parallel to the magnetic field lines. The compressional motion is negligible and the resulting filaments are stable against radial collapse. Such quasi-equilibrium filaments may possibly break up into pieces again mainly through the motion in the axis direction of filaments, depending on the strength of the magnetic field. The resulting nearly spherical clumps, however, will not collapse into stars because the mass of each clump tends to be less than the Bonner-Ebert sphere. Thus, this case does not seem to be related to active star formation. Therefore, hereafter we will concentrate on the evolution of the filament where magnetic field lines are perpendicular to the axis of the filament

3. The Evolution of Filaments

In the previous section, the dynamical fragmentation of the sheet-like cloud is investigated in terms of the instability of the *equilibrium* sheet. This is justified because one-dimensional compression always provides quasi-equilibrium sheet-like configuration (see below).

It is important to distinguish a remarkable difference between a planar collapse and a cylindrical collapse. Consider a one-dimensional planar collapse in which the half-thickness, $z_{\rm b}$, of a sheet-like cloud is decreasing. The forces per unit mass due to pressure and gravity are as follows:

$$F_{\rm P} = -\frac{\partial P}{\partial z} \sim \rho\frac{c_{\rm s}^2}{z_{\rm b}} \propto z_{\rm b}^{-2}, \quad F_{\rm G} = -\rho\frac{\partial\varphi}{\partial z} \sim -\rho G\sigma \propto z_{\rm b}^{-1}, \tag{3}$$

Therefore a planar collapse is inevitably halted by pressure forces when the thickness becomes sufficiently small. On the other hand, during a cylindrical collapse in which the radius or the scale height, R, of the cylinder is decreasing, the two forces take the following forms:

$$F_{\rm P} = -\frac{\partial P}{\partial r} \sim \rho\frac{c_{\rm s}^2}{R} \propto R^{-3}, \quad F_{\rm G} = -\rho\frac{\partial\varphi}{\partial z} \sim -\rho\frac{GM_{\rm line}}{R} \propto R^{-3}, \tag{4}$$

where $M_{\rm line}$ is the mass per unit length (line-mass), $M_{\rm line} = \int_0^{R_b} 2\pi\rho r dr$. In the isothermal case, which is very important in the theory of star-formation, the above two forces have the same dependence to the radius. Therefore, radial collapse cannot be halted by the pressure if the gravity is sufficiently large at the beginning. The critical line-mass corresponding to an isothermal cylindrical equilibrium is $2c_{\rm s}^2/G$ (Ostriker 1964) which depends only on the temperature. Hence, a filamentary isothermal gas cloud cannot be in equilibrium for any radial density distribution and eventually collapses unless its line-mass is smaller than the above value. This collapse of isothermal filaments with various line-masses was described in terms of the one-parameter family of self-similar solutions (Inutsuka & Miyama 1992). In general, the fragmentation of a sheet-like cloud produces filaments whose line-mass is about twice the critical value (see e.g., eq.[4.1] of Miyama *et al.* 1987). Therefore within actual filaments, equilibrium is overly idealized state.

We must study massive collapsing filaments to determine the evolution of actual interstellar molecular clouds. This line of work is done by Inutsuka & Miyama (1992,1997) for non-magnetized filaments, and by Inutsuka (2000) for magnetized filaments where the magnetic fields are perpendicular to the longitudinal axes of the filaments. Such filaments with transverse magnetic fields are predicted to form as a result of the fragmentation of parental sheet-like clouds, and expected to be important in the context of star formation.

The extensive sets of linear stability analyses and non-linear numerical calculations to study the collapse and fragmentation processes of magnetized isothermal filamentary molecular clouds are done by Inutsuka (2000). While details are described elsewhere, the most important result is explained in this subsection.

Consider the evolution of a massive isothermal filament with magnetic field perpendicular to it. The numerical result shows that this massive filament with perpendicular magnetic fields continues its radial collapse unless the strength of the magnetic field is so large to stop the collapse from the beginning. This result can be explained in terms of the following simple argument. Let us consider again homologous cylindrical collapse in which the radius or the scale height, R, of the cylinder is decreasing. The magnetic flux density and the magnetic force per unit mass scales as

$$B \sim B_0\frac{R_0}{R}, \quad F_{\rm M} = -\frac{\partial B^2}{\partial r} \sim \frac{B_0^2 R_0^2}{R^3} \propto R^{-3}. \tag{5}$$

This scaling is again the same as the scaling of the gravity (see eq.[4]). Therefore, if the gravity is initially greater than the magnetic force, the gravity continues to dominate the magnetic force during the radial collapse.

The numerical calculations of the fragmentation of magnetized massive collapsing filaments show that the fragmentation is not effective during the rapid radial collapse, unless perturbations with very large amplitude are imposed initially. Therefore there is no characteristic density with which the fragmentation occurs during the isothermal phase. When the radial collapse is decelerated by some mechanism such as the change of the equation of state, the filament will fragment. Therefore the change of the equation of states is very important in the theory of fragmentation. The question of how and when the isothermal evolution is terminated is explored in the next section.

4. The Characteristic Mass of Fragmentation

In general, molecular clouds are under thermal balance described by the following relation.

$$\Gamma_{\rm g} + \Gamma_{\rm ext} = \Lambda_{\rm th}, \tag{6}$$

where $\Gamma_{\rm g}$, $\Gamma_{\rm ext}$, and $\Lambda_{\rm th}$ are the compressional heating rate of gas, the heating rate due to the external sources such as cosmic rays and visual or UV photons from surrounding stars, and the radiative cooling rate, respectively. The isothermal collapse continues during $\Gamma_{\rm g} \ll \Lambda_{\rm th}$ and thus $\Gamma_{\rm ext} \approx \Lambda_{\rm th}$, but isothermality is broken down when the compression of gas becomes so effective as to heat up the cloud sufficiently against the cooling.

Masunaga & Inutsuka (1999) discussed on the critical central density $\rho_{\rm crit}$ when the isothermal evolution is terminated in gravitational collapse. The condition with which isothermality is broken down is determined as two possibilities.

Optically Thin Case Isothermality is violated when $\Gamma_{\rm g} = \Lambda_{\rm th}$, if $\Gamma_{\rm g}$ overwhelms $\Lambda_{\rm th}$ before the optical depth of the collapsing cloud core, τ, reaches unity. This critical density is denoted by $\rho_{\rm th}$.

Optically Thick Case If $\Lambda_{\rm dif} > \Gamma_{\rm g}$ when $\tau \approx 1$, isothermality survives even after τ exceeds unity until the central density reaches a critical value $\rho_{\rm dif}$, which defines the central density when $\Gamma_{\rm g}$ becomes comparable to $\Lambda_{\rm dif}$.

The optical depth, τ, of the collapsing filamentary cloud is defined by $\tau = \int_0^\infty \kappa\rho dr = \frac{\pi}{4}\kappa\rho_{\rm c}\sqrt{\frac{2c_{\rm s}^2}{\pi G\rho_{\rm c}}}$. We also define $\rho_{\tau\sim 1}$ at which $\tau \approx 1$ is reached.

Now we evaluate $\Gamma_{\rm g}$, $\Lambda_{\rm th}$, and $\Lambda_{\rm dif}$ to derive $\rho_{\rm crit}$. Note that the heating and cooling rates are defined per unit mass. For brevity we suppose the local thermodynamical equilibrium (LTE), which admits $\Lambda_{\rm th}$ to be described simply as follows.

$$\Lambda_{\rm th} = 4\kappa(T_{\rm init})\sigma_{\rm SB}T_{\rm init}^4, \tag{7}$$

where κ is the frequency-averaged opacity per unit mass, which is independent of density due to LTE, and $\sigma_{\rm SB}$ denotes Stefan-Boltzmann constant. The temperature is kept at $T_{\rm init}$ at the initial isothermal stage. The compressional heating rate for gravitational collapse is

$$\Gamma_{\rm g} = Ac_{\rm s}^2\sqrt{4\pi G\rho}. \tag{8}$$

A numerical constant A is found to be of the order of unity and nearly constant through the evolution.

The energy transport rate due to radiative diffusion is

$$\Lambda_{\rm dif} \sim \frac{E}{\rho t_{\rm dif}} = \frac{4\kappa(T_{\rm init})\sigma_{\rm SB}T_{\rm init}^4}{\tau^2}. \tag{9}$$

The radiative diffusion time $t_{\rm dif}$ is defined by $\tau^2\lambda_p/c$, where $\lambda_p \equiv 1/\kappa\rho$ is the mean free path of a photon and c is the speed of light.

The critical densities were obtained as

$$\rho_{\rm th} = 4.7\times10^{-15}{\rm g\cdot cm^{-3}}\left(\frac{\kappa_0}{0.01{\rm cm^2\cdot g^{-1}}}\right)^2\left(\frac{T_{\rm init}}{10K}\right)^{6+2\alpha}, \tag{10}$$

$$\rho_{\tau\sim1} = 4.7\times10^{-12}{\rm g\cdot cm^{-3}}\left(\frac{\kappa_0}{0.01{\rm cm^2\cdot g^{-1}}}\right)^{-2}\left(\frac{T_{\rm init}}{10K}\right)^{-1-2\alpha}. \tag{11}$$

$$\rho_{\rm dif} = 4.7\times10^{-13}{\rm g\cdot cm^{-3}}\left(\frac{\kappa_0}{0.01{\rm cm^2\cdot g^{-1}}}\right)^{-2/3}\left(\frac{T_{\rm init}}{10{\rm K}}\right)^{(4-2\alpha)/3}. \tag{12}$$

where we assume that the opacity is approximated as $\kappa(T_{\rm init}) = \kappa_0(T_{\rm init}/10{\rm K})^\alpha$, $\alpha = 1 \sim 2$.

Equations (7) and (9) shows that $\Lambda_{\rm th}$ and $\Lambda_{\rm dif}$ are smoothly connected with each other at $\tau = 1$. In other words, $\rho_{\rm th} = \rho_{\tau\sim1}$ and $\rho_{\rm dif} = \rho_{\tau\sim1}$ degenerate into an identical line in the $T_{\rm init}$-κ plane (see Figure 1, 2 of Masunaga & Inutsuka 1998). Thus the critical density for the violation of isothermality is determined only by $\rho_{\rm th}$ or $\rho_{\rm dif}$, depending upon $T_{\rm init}$ and κ. for gravitationally collapsing clouds. The critical density of $\rho_{\tau\sim1}$ *has no significance in practice* in the case of *filamentary* collapse. This result is in contrast to the classical "opacity limited fragmentation" pictures of Low & Lynden-Bell 1976, Rees 1976, Silk 1977, and Boss 1988 where they have assumed the condition "$\tau \approx 1$ " plays the essential role.

We can evaluate $M_{\rm m}$ as the mean clump mass of a filament with the effective radius of $H_f = \sqrt{2c_{\rm s}^2/\pi G\rho_{\rm crit}}$, setting the mean separation $\lambda_{\rm m}$ between clumps to be $8\times H_f$:

$$M_{\rm m} \simeq M_{\rm line}\lambda_{\rm m} \simeq \frac{16c_{\rm s}^3}{G}\sqrt{\frac{2}{\pi G\rho_{\rm crit}}}. \tag{13}$$

Eliminating $\rho_{\rm crit}$ in equation (13) by equations (10) and (12), we have

$$M_{\rm m} = 3.7\times10^{-2}M_\odot\left(\frac{\kappa_0}{0.01{\rm cm^2\cdot g^{-1}}}\right)^{-1}\left(\frac{T_{\rm init}}{10K}\right)^{-(3+2\alpha)/2}, \tag{14}$$

for optically thin case, and

$$M_{\rm m} = 3.7\times10^{-3}M_\odot\left(\frac{\kappa_0}{0.01{\rm cm^2\cdot g^{-1}}}\right)^{1/3}\left(\frac{T_{\rm init}}{10K}\right)^{(5+2\alpha)/6}, \tag{15}$$

for optically thick case.

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5. Mass Function of Molecular Cloud Cores

As shown in Section 4.1.1 of Inutsuka & Miyama (1997), a fragment of filament becomes collapsed and isolated self-gravitating object when the amplitude of hypothetical *linearly growing* density perturbation becomes $\delta_c \approx 1.6$. This useful property is used in the following analytical calculation of the mass function of dense cores.

We consider radially and azimuthally averaged density fluctuation on the filament whose axis corresponds to the z-axis.

$$\delta(z) = \frac{\delta \rho_z}{\bar{\rho}_z} = \frac{\rho_z - \bar{\rho}_z}{\bar{\rho}_z} = \frac{\rho_z}{\bar{\rho}_z} - 1, \tag{16}$$

where

$$\frac{\rho_z}{\bar{\rho}_z} = \frac{1}{M_{\mathrm{line}}} \int_0^\infty \int_0^{2\pi} \rho(r, \phi, z)\ r d\phi\ dr, \tag{17}$$

and M_{line} is the line-mass of the unperturbed filament. The Fourier transform of $\delta(x)$ is

$$\delta_k(k) = \frac{1}{\lambda_{\mathrm{m}}} \int_{-\infty}^{\infty} \delta(z) e^{-ikz} dz, \tag{18}$$

where λ_{m} has the dimension of length.

For convenience, we measure the amplitude of overdensity by smoothing the fluctuation field as follows:

$$\delta_M(z) = \int \delta(z') W(z - z') dz', \qquad W(z) = \frac{\sin(k_M z)}{\pi z}, \tag{19}$$

where we have chosen the window function W whose Fourier transform is a sharp k-space filter for simplicity. We relate mass scale M, length scale λ_M, and wavenumber k_M by the following simple equations:

$$M = M_{\mathrm{line}} \lambda_M, \qquad k_M = \frac{2\pi}{\lambda_M} = \frac{2\pi M_{\mathrm{line}}}{M}. \tag{20}$$

If we assume density fluctuations δ_M is a Gaussian random distribution, then, the probability $P(M, \delta > \delta_c)$ of finding a region of mass scale M in which the linear density fluctuation is greater than the critical overdensity δ_c is given by an integral over the tail of a Gaussian distribution function,

$$P(M, \delta > \delta_c) = \int_{\delta_c}^{\infty} \frac{1}{\sqrt{2\pi\sigma_M^2}} \exp\left[-\frac{\delta^2}{2\sigma_M^2}\right] d\delta, \tag{21}$$

where the variance σ_M can be estimated by summing up the variance of each Fourier component in the sharp k-space filter:

$$\sigma_M^2 = \frac{1}{\lambda_{\mathrm{m}}} \int_{-\infty}^{\infty} |\delta_M(x)|^2 dx = \frac{\lambda_{\mathrm{m}}}{2\pi} \int_{-k_M}^{k_M} |\delta_k(k')|^2 dk'. \tag{22}$$

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For simplicity we assume a power law form for the fluctuation spectrum,

$$|\delta_k|^2 = A \left(\frac{k}{k_{\rm m}}\right)^n, \tag{23}$$

where $k_{\rm m} = 2\pi/\lambda_{\rm m}$ is the most unstable wavenumber and $M_{\rm m} = M_{\rm line}\lambda_{\rm m}$. The growth rate of the perturbation of filament has the following property (Inutsuka & Miyama 1992):

$$\omega \propto k \quad \text{for} \quad k \ll k_{\rm m}. \tag{24}$$

Thus, the linear growth of the variance might be approximated by

$$\sigma_M = \sqrt{\frac{2A}{n+1}} \left(\frac{k_M}{k_{\rm m}}\right)^{\frac{n+1}{2}} \exp\left[\left(\frac{t}{t_{\rm g}}\right)\left(\frac{k_M}{k_{\rm m}}\right)\right], \tag{25}$$

where $t_{\rm g}$ is on the order of free-fall time.

Finally we can calculate the mass function of collapsed dense cores by the Press-Schechter formalism (see e.g., Press & Schechter 1974; Peacock & Heavens 1990; Bond et al. 1991; Yano, Nagashima, & Gouda 1996). The resulting mass function dN/dM is given by

$$\frac{dN}{dM} = \sqrt{\frac{2}{\pi}} \frac{M_{\rm line}}{M^2} \frac{\delta_c}{\sigma_M} \left[\frac{n+1}{2} + \left(\frac{t}{t_{\rm g}}\right)\left(\frac{M_{\rm m}}{M}\right)\right] \exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right). \tag{26}$$

The power spectrum with $-1 < n < 0$ seems to be consistent with the recent observations (Motte, Andre, & Neri 1998; Testi & Sargent 1999; Onishi et al. 1999).

In the derivation of equation (26), we assume a random Gaussian field whose power spectrum is a simple power law. This assumption, however, can be directly tested by observations of actual filamentary clouds, and even determine the power law exponent n. That is, observational determination of both fluctuation spectrum and the resulting mass function is expected to be a straight forward test of the theory.

6. Mean Surface Density of Companions

According to Wiener-Khinchin's theorem, the autocorrelation function is related to the power spectrum:

$$\xi(z) = \frac{\lambda_m}{2\pi} \int_{-\infty}^{\infty} |\delta_k|^2 e^{-ikz} dk. \tag{27}$$

We again adopt a power law form for the power spectrum for simplicity (eq.[23]). Then autocorrelation function becomes

$$\xi(z) = 2\ A\ \Gamma(n+1)\ \cos[(n+1)\frac{\pi}{2}]\ (zk_{\rm m})^{-n-1}, \tag{28}$$

where $\Gamma(x)$ is Gamma Function.

Now we calculate the mean surface density of companions (Larson 1995; Simon 1997; Bate, Clarke, & McCaughrean 1998; Nakajima, Tachihara, Hanawa, & Nakano 1998; Gradwin, Kitsionas, Boffin, & Whitworth 1999). Note, however, that we are interested in the distribution of dense cores, instead of stellar distribution. The probability δP of the number of finding dense cores within the annulus whose area is $2\pi\theta\delta\theta$ corresponds to the number of dense cores within the length $2\delta z$ of the filament that is estimated as

$$\delta P = 2\pi\theta\delta\theta\Sigma(\theta) = 2\bar{n}_z\delta z(1+\xi), \tag{29}$$

where $\bar{n}_z$ is the average number of dense cores per unit length of the filament. Thus the mean surface density $\Sigma(\theta)$ is expressed as

$$\begin{aligned} \Sigma(\theta) = \frac{\bar{n}_z D}{\pi\theta}(1+\xi) \quad &\propto \quad \theta^{-n-2} \qquad \text{for } \theta \ll \theta_c \\ &\propto \quad \theta^{-1} \qquad\quad \text{for } \theta \gg \theta_c, \end{aligned} \tag{30}$$

where D is the distance from the observer to the cloud, and we defined θ_c by $\xi(\theta_c) = 1$.

The above expression should not be applied to the mean surface density of *stellar* companions, because the dynamical evolution of the positions of stars is considerable and may wipe out the information of the initial density fluctuations. Understanding the mean surface density of *stellar* companions may require understanding the collapse and fragmentation processes in each dense core including the effect of angular momentum (which might be obtained by the mutual interaction of cores), and understanding the subsequent orbital evolution of resulting protostars.

More importantly, a direct observation of the mean surface density of the *dense core* companions in actual molecular clouds can test the validity of the above argument, and hopefully determine the parameters in the model (e.g., the power spectrum exponent n).

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