SOLAR CONSTRAINTS

Janine PROVOST
Observatoire de Nice, B.P. 139, 06003 Nice-Cedex, France

ABSTRACT.

Accurate tests of the theory of stellar structure and evolution are available from the Sun's observations. The solar constraints are reviewed, with a special attention to the recent progress in observing global solar oscillations. Each constraint is sensitive to a given region of the Sun. The present solar models (standard, low Z, mixed) are discussed with respect to neutrino flux, low and high degree five-minute oscillations and low degree internal gravity modes. It appears that actually there do not exist solar models able to fully account for all the observed quantities.

1. INTRODUCTION.

As the nearest star, the Sun provides the opportunity to test very accurately the theory of stellar structure and evolution. Particularly in the last years solar oscillation observations have considerably progressed and their unusually high accuracy has even more constrained the solar model, leading to still unresolved difficulties.

Models of the Sun must in the first place satisfy the major usual constraints, i.e. they must lead at the actual age of the Sun ($t_\odot = 4{,}6\ 10^9$ years) to the present values of luminosity ($L_\odot = 3{,}86\ 10^{33}$ erg/sec from Bahcall et al., 1982) and of radius ($R_\odot = 6.9599\ 10^{10}$cm) for the given mass of the Sun ($M_\odot = 1{,}989\ 10^{33}$g). Models of the Sun are more specifically constrained firstly by the present chemical composition of the solar surface, which is assumed to reflect the initial composition of the heavy elements, secondly by the observed abundances of nuclearly processed elements, such a lithium and ^{3}He. This point will be considered subsequently in more detail by Schatzman. The most famous constraint on solar models has been provided by the measure of the solar neutrino flux by Davis and his collaborators. The measured value corresponds to (2.1 + 0.3) SNU (Cleveland ,Davis and Rowley, 1981) (1 SNU corresponds to 10^{-36} capture per target atom per second). It is only about 25 to 40

A. Maeder and A. Renzini (eds.), Observational Tests of the Stellar Evolution Theory, 47–65.

https://doi.org/10.1017/S0074180900030539 Published online by Cambridge University Press

percents of the present theoretical prediction of the standard solar model as recently reviewed by Bahcall et al. (1982).

The accurate measurements of the solar oscillation frequencies made within the last ten years provide an additional independent constraint on the solar model. The properties of these oscillations are closely related to the internal structure of the Sun, the different observed components of oscillations permitting to infer the properties of different zones of the Sun. Since the discovery by Leighton, Noyes and Simon (1962) of 5 mn solar oscillations, by measuring time variations of the Dopplershift of a Fraunhofer line, quite interesting progress has been made. The high degree five-minute acoustic modes with small horizontal wavelengths, typically of 10 000 km, have been first observed by Deubner (1975) and later by many observers (e.g. Deubner et al., 1979; Harvey et al., 1983; Hill et al., 1983). The low degree five-minute acoustic modes, i.e. with large horizontal wavelengths of the order of the solar radius, have been recognised by Claverie et al. (1979, 1981), Grec et al. (1980, 1983), Scherrer et al. (1982), Woodard and Hudson (1983). This year, two important observational results have been obtained : acoustic modes with intermediate degree have been isolated by Duvall and Harvey (1983) and oscillations have been found in the low frequency solar spectrum (160-360 mn) by different groups of observers (Delache and Scherrer, 1983; Fröhlich, 1983; Van der Raay, 1983; Severny et al., 1983) which have been interpreted as gravity modes. Other oscillations with periods longer than 5 mn, detected by solar diameter measurements or limb brightness fluctuations (e.g. Bos and Hill, 1983; Rösch and Yerle, 1983) are not yet fully identified and understood. Just as the well known 160 mn oscillation (e.g. Kotov et al., 1983; Scherrer and Wilcox, 1983) which might be a gravity mode if it is of solar origin.

An important fact is that all these considered solar constraints concern different parts of the Sun, so that the whole solar model can be tested. This is shown in figure 1 which represents the contribution of the different layers of the Sun to different observed quantities. The neutrino flux from bore decay, measured in the Davis experiment, originates from the inner five percents of the solar mass. The mean separation which characterizes the observed frequencies of low degree acoustic modes originates from the outer ten percents of the solar mass. Nevertheless, it will be shown in section 5 that other features of these oscillations are closely related to the deep interior of the Sun. The solar luminosity and the neutrino flux of the proton-proton reaction (which could be detected by the ^{71}Ga experiment as discussed by Bahcall et al. 1978) come mostly from an intermediate region. The"spacing" period which characterizes the periods of low degree gravity modes (see section 6) is mainly sensitive to the inner five percents of the mass which determine also the neutrino flux.

The main purpose of this review is to point out the importance of solar oscillations as a probe of the Sun's interior, without attempt to consider this subject extensively. There exist many reviews on solar oscillations : for their observations e.g. Deubner 1980, Stenflo 1980, for their interpretations e.g. Christensen-Dalsgaard (1980, 1982), Gough (1980, 1983b). More information can be found in the proceedings of two recent meetings, the 66th IAU Colloquium, held in Crimea in 1981 and the Europhysics Study Conference held at Catania in June 1983. In a first part it is recalled how the neutrino problem has led to question the solar standard model, and some ways of modifying this model are briefly presented. Then the constraints on the solar structure, due to observed global solar oscillations, are reviewed. The final discussion shows that presently no model is able to account for all the solar constraints.

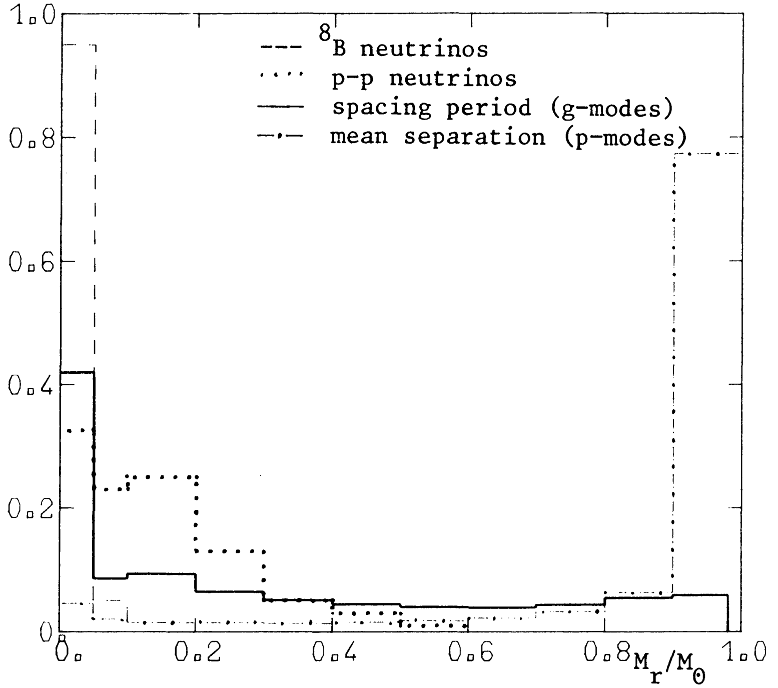

Figure 1 : Histogram of the fractional contributions to low degree p-mode mean separation ν_o, the low degree g-mode spacing period P_o (computed for the standard solar model of Schatzman and Maeder, 1981) the ^{8}B neutrino flux and the neutrino flux from the p-p reaction (taken from Bahcall, 1981), as a function of $M_r/M_\odot = \int_o^r 4\pi \rho r^2 dr/M_\odot$

2. STANDARD SOLAR MODELS AND THE NEUTRINO PROBLEM.

Solar models are computed with usual simplifying assumptions, i.e. spherical symmetry, thermal and hydrostatic equilibrium at each time of the evolution, and a uniform chemical composition at the zero age. The

https://doi.org/10.1017/S0074180900030539 Published online by Cambridge University Press

convection zone which extends on 30% of the solar radius is crudely described with the mixing length theory. Other macroscopic motions are ignored, as well as the possibility of chemical mixing in the solar core. The initial solar model is specified by the chemical composition parameters Y and Z, i.e. helium and heavy elements contents, and the mixing length parameter λ, which characterizes the efficiency of convective transport. No direct information on Y and λ is available. A discussion of protosolar helium abundance has been given recently by Gough (1983a). A usual calibration of a solar model consists in adopting either the value of Z or the value of Z/X most directly determined from spectroscopic measurements ($Z \cong 0.02$ or $Z \cong 0.023$ from Ross Aller 1976). Y and λ are then fitted to adjust present luminosity and radius at the actual age of the Sun. So calibrated standard models (e.g. Bahcall et al., 1982; Christensen-Dalsgaard, 1982; Ulrich, 1982...) correspond to $Y \cong 0.25$. This helium abundance is not in conflict with the content of the galaxy and with the prediction of cosmological theories. These models have an extended convective zone (about 30% of the radius). But they predict a too large neutrino flux compared to the measured one. According to Bahcall et al. (1982) who have estimated all the uncertainties that could affect the neutrino flux, the predicted value is (7.6 + 3.3) SNU, which has to be compared to the observed one (2.1 + 0.3) SNU (Cleveland et al., 1981). The so-called neutrino problem has led to a careful examination of the uncertainties of standard models. Many improvements of the physical inputs have been made, which concern the equation of state, the opacity and the nuclear cross sections, without reducing noticeably the neutrino flux.

The production rate of neutrinos detected by the Davis experiment, which come essentially of ^8B decay, depends very strongly on the central temperature of the Sun ($F \cong T_c^{20}$ according to Iben, 1969). Thus, the main idea is to bring down the central temperature in order to put the predicted flux in agreement with the observed one. This can be obtained in particular by increasing the proportion of hydrogen available for nuclear burning. So, the solar luminosity can be generated in the core at a lower temperature. As shown by Iben (1969), a possibility to lower the neutrino flux is to lower the helium content. But the adjusting of the neutrino flux requires extremely low Z (Abraham, Iben, 1971; Bahcall and Ulrich, 1971), uncompatible with the observed surface chemical abundances of the Sun. To avoid this difficulty "dirty" solar models have been computed by Christensen-Dalsgaard et al. (1979) with low Z in the interior, hence a low neutrino flux, but assuming that the outer layers have accreted heavy elements during the evolution of the Sun and thus have $Z \cong 0.02$ at the surface. Low helium and low heavy elements contents are associated with a shallow convection zone : this is a consequence of a lower opacity which requires reducing the efficiency of the convection to adjust the computed radius to the solar radius. Another way to lower the predicted neutrino flux is provided by some sort of mixing of the solar core, which brings more hydrogen into the central region. Several attempts have been made (as for example by Ezer and Cameron, 1968; Shaviv and Beaudet, 1968; Bahcall et al., 1968), but without a discussion of the nature of the mixing. The physical mechanisms of mixing in stellar evolution are discussed by Schatzman (1984). The basic assumption is the existence of some sort of marginal instability, driven for example by differential rotation, which generates turbulence . The turbulent transport can be described by a turbulent diffusion coefficient, proportional to microscopic viscosity, and to a pseudo-Reynolds number which measures the efficiency of the mixing, as shown by Schatzman (1977) and Genova and Schatzman (1979). Diffusion mixed solar models with a normal chemical composition ($Y = 0.25$, $Z = 0.02$) have been computed by Schatzman and Maeder (1981). The neutrino flux is highly reduced by turbulent diffusion mixing, by a factor 2.6 to 8, when the pseudo-Reynolds number R_e^* varies from 50 to 200. For these models, the convection zone depth is almost unchanged.

3. DESCRIPTION OF SOLAR OSCILLATIONS

In this section general properties of solar oscillations are rapidly described. An extensive presentation of stellar oscillations is given in Ledoux and Walraven (1951), Cox (1980) and Unno et al. (1979). Oscillations of a star are usually studied assuming spherical symmetry of the equilibrium model; rotation, magnetic field and all horizontal inhomogeneities are neglected. The observed small amplitude oscillations can be studied in a linear approximation. The radiative dissipative time being much larger than the characteristic time of the motion, the adiabatic approximation can be used, at least for eigenfrequencies studies. Global solar oscillations are standing waves, whose motion can be decomposed into components varying sinusoïdally in time with a frequency ν. A physical quantity related to the wave can be described by : $X = f_n(r)\ Y_\ell^m(\theta,\phi) \cos 2\pi\nu t$, where $Y_\ell^m(\theta,\phi)$ is the spherical harmonic of azimutal order m and degree ℓ. Each oscillation can thus be described by the radial order n, the degree ℓ and the azimutal order m, which represent respectively the number of zeros along a solar radius, on a sphere of radius r and along a circle parallel to the equator. At each position r, it is possible to define a local radial wavenumber $k_r \cong n/r$ and a local horizontal wavenumber

$k_h \cong \ell/r$ ($k_h = \frac{\sqrt{\ell(\ell+1)}}{r}$ due to the properties of the spherical harmonics). As long as rotation is neglected, the frequencies are independent of m.

Two sorts of waves exist in a compressible self gravitating medium: the acoustic waves (p-modes) are driven by the pressure fluctuation produced by compression, and they depend on the structure of the sound speed c ; the gravity waves (g-modes) are driven by the buoyancy force and they depend on the structure of the Brunt-Väissälä frequency

N ($N^2 = g(\frac{1}{\Gamma_1}\frac{1}{p}\frac{dp}{dr} - \frac{1}{\rho}\frac{d\rho}{dr})$). These different modes are trapped in

different regions of the Sun and their frequencies depend evidently on the structure of the trapping zones. Acoustic modes are reflected at the surface where the density scale height becomes shorter than the

https://doi.org/10.1017/S0074180900030539 Published online by Cambridge University Press

wavelength. Due to the increase of the sound speed from the surface to the centre, they are reflected at the level where their frequencies become equal to the acoustic local frequency (or Lamb frequency) $S_\ell = k_h c = \sqrt{\ell(\ell+1)}\, c/r$. Thus five-minute acoustic modes are trapped in an outer part of the solar envelope if they are of a high degree (the outer 10% of radius if $\ell > 200$); they penetrate to the bottom of the convective zone if the degree ℓ is about 40 ; low degree oscillations are reflected near the centre of the Sun and their frequencies depend on the structure of the sound speed across the whole Sun. Gravity modes are trapped in convectively stable regions ($N^2 > 0$) and they are evanescent in convection zones. Thus, solar gravity modes can be trapped either in the radiative interior or in the solar atmosphere. Their frequencies must be smaller than the maximum of the Väissälä frequency in the trapping region, i.e. their period must be larger than 1/2 hour, for the modes trapped in the interior, and larger than 4 mn for modes trapped in the upper atmosphere. Dziembowski and Pamjatnyk (1978) have shown that the observation of large degree gravity modes trapped in the interior is unlikely, because of the rapid decay of their amplitude through the convection zone. It is not the case for low degree gravity modes which have been recently observed (see section 6). Despite not yet identified atmospheric modes, some observed photospheric motions could be interpreted in terms of gravity waves (Hill et al., 1982). In the next sections the different observed oscillations, high and low degree acoustic modes and low degree gravity modes will be analysed in more details in relation with the solar structure.

4. FIVE MINUTE OSCILLATION OF HIGH DEGREE : A CONSTRAINT ON THE CONVECTION ZONE.

Five minute solar oscillation has been spatially resolved into different components of horizontal wavelengths of 5 000 to 20 000 km by Deubner (1975). The two dimensional power spectrum obtained in a horizontal wavenumber-frequency plane shows that the power is maximum along well defined ridges. Each ridge corresponds to oscillations with the same radial order n ($n \lesssim 10$) and varying ℓ from 140 to 700. The interpretation of these oscillations as acoustic modes trapped below the solar surface, first suggested by Ulrich (1970), has been confirmed by an approximate agreement between observations (Deubner, 1975; Deubner, Ulrich and Rhodes, 1979) and theoretical p modes calculations (Ando and Osaki, 1975, 1977; Ulrich and Rhodes, 1977). As these oscillations are trapped in the outer 10 percents of the radius, their frequencies depend essentially on the structure of the sound speed in the convective zone. The roughly parabolic shape of the ridge in the ($k - \omega$) plane is predicted with a simple model of the convection zone, i.e. an adiabatically stratified polytrope (Gough 1978). This model shows also that the frequencies are proportional to the adiabatic temperature gradient. Studies of the sensitivity of the theoretical five-minute frequencies to the structure of the Sun have shown that the frequencies depend essentially on the mixing length parameter λ, hence on the depth of the convection zone (Berthomieu et al., 1980; Lubow et al., 1980). The

https://doi.org/10.1017/S0074180900030539 Published online by Cambridge University Press

frequencies decrease when λ increases with a saturation at large λ. The best fit with the observations is obtained for models with $\lambda \cong 2$ and a convection zone depth about 200 000 km (see for example Fig. 1 in Berthomieu et al., 1980). Such large convection zones are related to a normal helium abundance for standard models as well as for diffusive mixed models. Thus high degree five-minute oscillations constrain the solar model to have a large convective zone.

Improvements in the accuracy of the observations allow to infer more precisely the structure of the convection zone. From Harvey, Duvall and Rhodes observations, Duvall (1982) has found that most of the information about the solar interior contained in high degree five-minute modes can be reduced to a dispersion relation of the form $(n + 3/2)\pi/\omega = f(k/\omega)$, where n is the radial order, k the horizontal wave number and $\omega = 2\pi\ \nu$. Such a relation is obtained for a polytrope of index 3, only if it is adiabatically stratified (Gough, 1978; Christensen-Dalsgaard, 1980). This led Gough (1983a) to the conclusion that Duvall's result is an observational evidence of the nearly adiabatic stratification of most of the convection zone. This confirms some features of the structure given by the mixing length theory, despite its crudeness. Information on the usually ignored macroscopic motions in the convection zone can be obtained from observations like those of Hill, Toomre and November (1983), which have revealed significant change in the position of the ridges in the $(k - \omega)$ diagram for different days. The position of the ridges are sensitive to large scale subphotospheric velocity and temperature fields (Gough and Toomre,1983). Variations of about 100 m/sec are obtained from one day to the other and may be due to a giant cell convective pattern according to Gough et al. (1983).

5. FIVE MINUTE OSCILLATIONS OF LOW DEGREE : AN INFORMATION ON THE WHOLE SUN.

Low degree five-minute oscillations will provide a more important constraint on the solar interior, because propagating nearly vertically ($\ell \ll n$) they penetrate deeper towards the centre of the Sun. Such oscillations have been observed by the Birmingham group (Claverie et al., 1979, 1981) measuring variations in the Dopplershift of a Fraunhofer line in light integrated over the entire solar disk. The temporal spectrum shows a sequence of almost uniformly spaced peaks with a mean separation of 68 μ Hz and amplitudes of about 20 cm/sec. Subsequent observations have improved the frequency resolution, which is about 2 μ Hz for the observations of Grec, Fossat and Pomerantz (1980, 1983) at the South Pole; the Birmingham group has reached a resolution of about 0.1 μ Hz by combining 3 months of data from two different stations of observations at Tenerife and Hawaī. Similar spectra have been obtained by Woodard and Hudson (1983) for the total irradiance fluctuations. Due to averaging over regions where the oscillations have opposite signs, the sensitivity of such observations decreases rapidly when the degree ℓ increases (Dziembowski,1977; Christensen-Dalsgaard and Gough 1982), and only modes with $\ell \leqslant 3$ can be observed in light integrated on the solar disk. With differential observations made at Stanford (mean on a central region minus concentric annulus), modes with ℓ = 3, 4, 5 have been observed (Scherrer et al., 1982).

Identification of observations requires a comparison with accurately numerically computed theoretical frequencies. Nevertheless some main features of their properties are rather well reproduced by asymptotic results. The eigenfrequencies satisfy the following asymptotic relation, derived by Tassoul (1980), neglecting the perturbation of the gravitational potential :

$$\nu = \nu_o (n+\ell/2+\varepsilon) - A\, \nu_o^2 (\ell(\ell+1)+\delta)/\nu \tag{1}$$

with $\nu_o = 1/\left(2\int_o^{R_\odot} (dx/c)\right)$

$$A = \left\{ c(R_\odot) / R_\odot - \int_o^{R_\odot} \frac{dc}{dr}\, \frac{dr}{r} \right\} / 2\pi^2 \nu_o$$

In this formula, n and ℓ are the radial order and the degree of the mode. ν_o is the inverse of the sound travel time across a solar diameter. The main contribution to ν_o comes from the regions where the sound speed is small, i.e. from the envelope and from the atmospheric layers (see Figure 1). The atmosphere must be taken into account in numerical calculations, its contribution to ν_o being about 5%. ε is a constant related to the effective polytropic index of the solar surface. A is a positive constant related to the variation of the sound speed across the solar radius and is very sensitive to the gradient of sound speed in the central region. δ is a more complex integral on the model (δ is negative in the particular isothermal problem as shown by Gough 1983b). As n is much greater than ℓ, the second term in the asymptotic formula is a correction to the quantity $\nu_o(n+\ell/2+\varepsilon)$. If ℓ increases by 2 and n decreases by 1, the frequency is almost unchanged. This property explains that the observed power spectrum has been resolved into almost equidistant peaks, corresponding to even and odd ℓ values, the mean separation $\nu_o/2$ corresponding to 68 μ Hz. In fact, due to the corrective term, there is a small splitting between frequencies of modes with a given value of n + ℓ/2, which permits to identify the degree ℓ of the mode. Grec et al. (1980) have been able to detect this small splitting, with a continuous 120 hours record of observations obtained at the South Pole. They found that $\nu_{n,o} - \nu_{n-1,2}$ = 9,4 μ Hz and $\nu_{n,1} - \nu_{n-1,3}$ = 15.3 μ Hz. More than 100 modes ($0 \leqslant \ell \leqslant 5$, $12 \leqslant n \leqslant 35$, $1.88 \leqslant \nu_{m\,Hz} \leqslant 5.08$) have been identified by different groups of observers with an average accuracy of 2 μ Hz. Now the solar model is constrained not only to reproduce the mean separation ν_o, but its theoretical frequencies must be equal to the observed frequencies within 2 μ Hz.

The observed frequencies sequence is in rather good agreement with

https://doi.org/10.1017/S0074180900030539 Published online by Cambridge University Press

that predicted by standard solar models with a normal chemical composition ($Z = \cong 0.02$, $0.21 \lesssim Y \lesssim 0.28$) as computed by many groups (Christensen-Dalsgaard and Gough, 1980, 1981; Shibahashi and Osaki, 1981; Scuflaire et al., 1981, 1982; Gabriel et al., 1982; Noels et al., 1983; Shibahashi et al., 1983; Ulrich and Rhodes, 1983). Attempts to improve this agreement by variation of chemical composition (Christensen-Dalsgaard and Gough, 1981; Shibahashi et al., 1983) or by making various adjustments of the solar model (different sort of mixing, high strength internal magnetic fields as done by Ulrich and Rhodes, 1983), have not succeeded. Between observed and computed frequencies there remain significant discrepancies, up to 15 μ Hz, larger than the observational errors (2 μ Hz). This can be seen for instance in Figure 2 of Shibahashi et al. (1983) where their theoretical frequencies and the observational results are compared. The systematic differences reach up to 10 μ Hz, most of the theoretical frequencies being too small, which is a general result obtained in other works. Thus, present standard solar models have too small frequencies compared to the observed ones, and a little too large mean separations. This result has led to a careful examination of some possible source of uncertainties : numerical errors by computing the frequencies are discussed by Christensen-Dalsgaard (1982) and Noels et al. (1983). Uncertainties in the input physics, as equation of state, nuclear cross sections, opacities,can change the frequencies of about 1 μ Hz, as estimated by Ulrich and Rhodes (1983). The most important modification of frequencies, about 10 μ Hz, is obtained varying the position of the outer boundary condition, but it does not improve the fit between observed and computed frequency sequences. So, despite the remaining uncertainties in the computation of equilibrium models and of the oscillation frequencies (Christensen-Dalsgaard, 1982), the conclusion is that something is missing, or wrong, in the standard theory of solar models (Noels et al., 1983; Scuflaire et al., 1981; Ulrich and Rhodes, 1983).

According to the asymptotic formula, a way to increase the frequencies could be to decrease the value of A, hence the splitting value between the frequencies of modes with given $n + \ell/2$. Table I indicates the splitting values for the observations, for standard models, for low Z models and for some mixed models. It appears that the fit with the observations is not improved for low Z models and mixed models. A better agreement is obtained with standard models which give generally the smallest theoretical splitting. The splitting is increased in low Z models or in mixed models. This is explained by the fact that for these models the sound speed in the central region varies more rapidly than in the standard model, due to the more gradual variation of the mean molecular weight relatively to the temperature variation.

The continuity between low and high degree 5 mn modes has been obtained recently by observations of Duvall and Harvey (1983). They have succeeded in connecting the low degree range to the high degree one, by projecting longitudinally averaged Doppler measurements onto zonal harmonics. The power spectrum they obtained looks like Deubner's spectrum, but corresponds to degree $\ell < 140$. An important consequence is

https://doi.org/10.1017/S0074180900030539 Published online by Cambridge University Press

that the order n of the low degree modes becomes unambiguously identified (Gough, 1983c). A comparison between observed frequencies and those computed by Ulrich and Rhodes (1983) for degree ℓ up to 40 shows the greatest discrepancy of order 15 μ Hz for degree ℓ about 20, the theoretical frequencies being too small. The higher degree modes that sample the convection zone agree relatively well. That leads Duvall and Harvey to the conclusion that there exists a substantial difference between the model and the Sun below the convection zone.

To summarize, five-minute oscillations trapped in the convection zone are relatively well described by current standard models, whereas oscillations which penetrate deeper into the radiative inner zone cannot be accounted for by these models : it seems to be a problem of the present description of the radiative interior of the Sun. An independent constraint on this radiative interior has been provided recently by gravity modes observations.

6. LOW DEGREE GRAVITY MODES : A CONSTRAINT ON THE RADIATIVE INTERIOR

The recent analysis of the low frequency solar spectrum obtained from velocity measurements (Delache and Scherrer, 1983; Severny et al., 1983; Van der Raay, 1983) or from the data of the ACRIM radiometer on board of the SMM (Fröhlich 1983) has shown the existence of low frequency global solar oscillations ranging in periods of 160-360 mn, which have been interpreted as low degree solar gravity modes. The low frequency spectrum is very complex; it presents a number of sets of lines with separations about 11.57 μ Hz corresponding to day sidebands, which come from the nightly gaps in the data. To eliminate these sidebands, Delache and Scherrer have used an iterative peak removal technique to find and remove the peaks one at a time; they obtained 14 peaks with frequencies in the range 45-105 μ Hz.

The identification of these oscillations has been made using the theoretical properties of low degree gravity modes. In fact, the period P of high order low degree gravity modes can be approximated by an asymptotic relation. For a solar model with a radiative inner zone, P is given by the formula, derived by Vandakurov (1967) and Tassoul and Tassoul (1968), without perturbing the gravitational potential :

$$P = \frac{P_o}{\sqrt{\ell(\ell+1)}} (n + \ell/2 - 1/4) \qquad (2)$$

with $P_o = 2\pi^2 / \int_o^{r_c} dr \ \frac{N}{r}$

n and ℓ are the order and degree of the mode, P_O is a characteristic frequency which depends only on the Brunt-Väissälä frequency N, i.e. on the density stratification, in the radiative interior (r_c is the position of the base of the convection zone). Numerical calculations in complete solar models show that neglecting the perturbations of the

	Y	Z	$\overline{\nu_{n,0} - \nu_{n-1,2}}$	$\overline{\nu_{n,1} - \nu_{n-1,3}}$	
CLAVERIE et al. 1981			8.3		Observations
GREC et al. 1983			9.4	15.3	
CHRISTENSEN-DALSGAARD	0.25	0.02	10	16	
and GOUGH 1980	0.19	0.04	13	17	
NOELS, SCUFLAIRE and	0.28	0.02	9.3		
GABRIEL 1983	0.27	0.018	9.4		
SHIBAHASHI, NOELS	0.23	0.02	10.5		
and GABRIEL 1983	0.229	0.018	6.6		
SCHATZMAN, MAEDER 1981	0.25	0.02	8.8	15.6	$R_e^* = 0$ n = 22
ULRICH, RHODES 1983	-	0.021	8.3	16.3	
	0.27	0.018	9.2	16.0	n = 22
		0.005	9.7	17.5	
CHRISTENSEN-DALSGAARD 1982	0.27	0.02	15	22	Homogeneous
SCHATZMAN,MAEDER 1981	0.25	0.02	12.4	20.5	$R_e^* = 100$ n = 22
ULRICH, RHODES 1983a	-	-	15.4	20.3	0.05MO mixed,n=22
	-	-	17.3	19.7	0.05MO mixed,n=17
ULRICH, RHODES 1983b	-	-	13.4	-	diffusive mixed n = 17

Table I : Splitting values of low degree five-minute oscillations for the observations and for some standard, low Z, and mixed models. The observation values from Grec et al. are power weighted mean values; other values are arithmetic mean values, corresponding to the observed frequency range, except when the radial order is quoted.

gravitational potential has little influence on the periods of low degree gravity modes, at this order of approximation, and that the asymptotic formula (2) is a good approximation for orders larger than about 10. It can be seen on Figure 2, where are plotted,as a function of $n + \ell/2 - 1/4$,the periods of gravity modes multiplied by the square root of $\ell(\ell+1)$, numerically computed by Berthomieu et al.(1983) for 2 different models of Schatzman and Maeder (1981). The points corresponding to a given model and to different ℓ values are on a straight line with a slope P_o. Asymptotic expressions for the periods show that for a given degree ℓ, the periods are equidistant, the difference between the periods of modes with two consecutive radial orders being $P_o/\sqrt{\ell(\ell+1)}$. In what follows, P_o will be referred to as "spacing" period.

https://doi.org/10.1017/S0074180900030539 Published online by Cambridge University Press

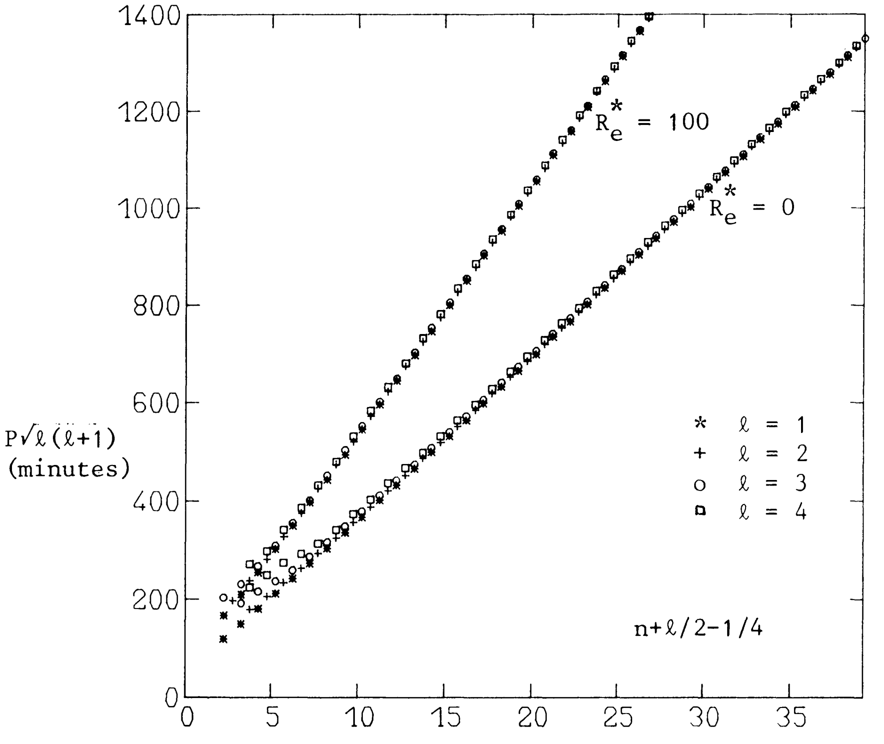

Figure 2 : Periods (multiplied by $\sqrt{\ell(\ell+1)}$) of the solar gravity modes for the models of Schatzman and Maeder (1981) versus $n+\ell/2 - 1/4$. (From Berthomieu et al. (1983)).

It is this property that has led to the identification of gravity modes. The value of the spacing period P_0 derived from observations is of the order of 38.6 mn (Delache and Scherrer 1983; Fröhlich 1983). It is suggested that it could be higher, of the order of 41 mn (Van der Raay, 1983). The amplitude of modes identified by Delache and Scherrer are compatible with an equipartition of energy between the different modes (Delache, 1983).

How this result constrains the solar interior has been discussed by Berthomieu, Provost and Schatzman (1983). In table II the chemical composition, convection zone depth, neutrino flux and estimated spacing period of gravity modes are given for different models. It appears that standard models which produce a too large neutrino flux give a too low value of P_o, of the order of 35 to 36 mn. Low Z models, which produce a lower neutrino flux (but possess a shallower convection zone) correspond to larger values of P_o of the order of 37 to 38 mn, as can be estimated from published gravity modes periods. Berthomieu et al. have shown that turbulent diffusion mixing, a physical mechanism very efficient to lower the neutrino flux, increases significantly the periods of gravity modes as can be seen in table II. It is also seen in Figure 2, where the slope of the straight line corresponding to the Schatzman-Maeder model with pseudo Reynolds number $R_e^* = 100$, is much larger than for the model without mixing. Thus, turbulent diffusion enhances greatly the spacing period of gravity modes. This effect is much greater than the effect of chemical composition change. The variation of the spacing period as a function of R_e^* can be obtained by an interpolation formula. It is almost linear for $R_e^* \lesssim 100$, with $\partial P_o/\partial R_e^* \cong 0.2$ mn. The main result is, that a rather moderate turbulent diffusion mixing ($R_e^* \cong 25$) is required to account for the observed spacing period of low degree gravity modes. Thus, the observation of solar gravity modes severely constrains the degree of mixing in the core of the Sun.

Y	Z	H_c 10^5km	N SNU	P_o min	
					SOME STANDARD MODELS
0.236	0.02	1.6	10.3	35	Iben and Mahaffy 1979
0.25	0.02	1.7	5.2	36	Christensen-Dalsgaard et al. 1979
0.25	0.02	1.6	11.96	34.3	Schatzman and Maeder $R_e = 0$ 1981
					HELIUM DEFICIENT MODELS
0.173	0.01	1.	4.4	37	Iben and Mahaffy 1979
0.186	0.004	1.1	2.3	38	Christensen-Dalsgaard et al. 1979
0.159	0.001	0.62	1.7	37.7	" " " " "
					TURBULENT DIFFUSION MIXED MODELS
0.25	0.02	1.77	2.38	52	Schatzman,Maeder $R_e^* \doteq 100$ 1981
0.25	0.02	1.7	1.43	58.7	Schatzman,Maeder $R_e^* = 200$ 1981

Table II : Initial abundances, depth of the convective zone, neutrino flux and estimated spacing period P_o for some solar models (from Berthomieu et al., 1983).

7. DISCUSSION

Now the compatibility of all the considered solar constraints and the limits they impose on different solar models will be discussed. Concerning the chemical surface abundances, observations of lithium depletion yields a constraint on the convection zone depth. This depth must not be too large, since the temperature at the bottom of the convection zone must be smaller than the lithium burning temperature by an amount which depends on the physical mechanisms carrying lithium from the bottom of the convection zone to the lithium burning zone, i.e. convective overshooting or turbulent diffusion,or both. Lithium observation puts limits on the efficiency of mixing as it is shown by Schatzman (1984) and by Baglin and Morel (1984). Their results imply for the Sun a convection zone depth of the order of 200 000 km and a pseudo-Reynolds number about 60,just below the convection zone. Observation of the ^{3}He at the solar surface cannot be explained without some transport of this isotope from ^{3}He peak, located deep in the interior, towards the bottom of the convection zone. The observed ^{3}He abundance limits also the efficiency of the mixing to a value of the pseudo-Reynolds number of about 40 (Schatzman, 1984). The measured neutrino flux requires a sufficiently low central temperature, which can be obtained by providing more nuclear fuel to the central region compared to the standard model, i.e. for models with extremely low heavy elements content, or for mixed models. To account for the neutrino flux, the pseudo-Reynolds number must be about 100, or smaller (Schatzman, 1984).

To satisfy the constraint imposed by high degree five-minute oscillations, a solar model must have a large convection zone, about 200 000 km, which excludes low Z models. There is a problem with low degree 5 mn oscillations, since their frequency spectrum is not reproduced in all details by any existing models. The observed fine structure, i.e. the splitting of the modes, constrains the mixing to be sufficiently small. The recent observations of intermediate degree 5 mn oscillations seems to localize the origin of the problem below the convection zone, according to Duvall and Harvey (1983), but this remains to be confirmed by further theoretical work. Finally, solar gravity modes provide a probe of the radiative interior of the Sun. It is difficult to account for their spacing period within the context of standard models or even low Z models, except for rather unrealistic models. Their observation limits the mixing to a rather moderate value ($R_e^* \cong 25$),too small to fit the observed neutrino flux. It is clear that the simple model with a constant pseudo-Reynolds number in the whole Sun seems to present a contradiction between the values of R_e^* derived from neutrino flux and from gravity modes. More elaborate models, taking into account recent results on rotational instabilities, as given by Spruit et al. (1983) and Zahn (1983), and with different chemical compositions, are necessary to discuss both the neutrino flux and the oscillations, particularly the spacing period of gravity modes. This is partly discussed by Law, Knobloch and Spruit (1984).

8. FUTURE PROSPECTS.

The above discussion shows that no present solar models are able to account for all the observed solar constraints. In fact, solar oscillations contain a great amount of information on the solar interior. In principle it should be possible to use the inverse method developed in terrestrial seismology by Backus and Gilbert (1968), to infer from the observation of solar oscillations the physical conditions inside the Sun. Helioseismology requiresto know a sufficiently large spectrum of oscillations, representing all parts of the Sun. At the present time, well identified modes with periods ranging from 5 mn to 160 mn are missing. They probably have a very low surface amplitude, as indicated by present 5 mn observations. Thus their detection will require very long sequences of observation to pick up the oscillation frequencies out of the noise.

Another important question concerns the dynamics of the stellar interior, specially the distribution of angular velocity, which is of fundamental importance for the understanding of stellar structure and evolution (Tassoul, 1984). Moreover, the knowledge of the internal solar rotation will permit to compute the resulting solar oblateness and consequently the modification of the solar gravity field, which is needed to test the general relativity. Solar oscillations will provide, in a near future, the possibility to measure the solar rotation. In fact, their frequencies are splitted by the rotation by a very small amount, which can be detected only with a long time sequence of observations. The rotational splitting provides an average value of the rotation on the trapping region of the oscillation. Two groups have reported detection of 5 mn splitting (Claverie et al., 1981 and Hill et al., 1982) and they have found that the Sun rotates 2-6 times faster in the centre than at the surface. However these observations and their interpretation are still not clearly understood and the existence of an intense rotating magnetic field in the solar core has been proposed and discussed by Dicke (1982), Gough (1982) and Isaak (1982). Very recently, splitting of low degree gravity modes has been reported by Delache and Scherrer from Stanford observations and by Fröhlich from SMM data. It is about 1.22 μ Hz and corresponds also to an indication of fast rotation in the centre of the Sun, about 3 times the surface rotation.

In conclusion,more observations on a long time continuous sequence are needed, both to complete the observed solar spectrum of oscillations and to resolve rotational splitting. This will be possible with projects of combining data from many stations at the Earth surface,as proposed independently by Fossat and Isaak, a project of observations at the South Pole within a balloon as,proposed by Fossat,and maybe,in a long range, by spatial observations. Finally, the Sun is a normal main sequence star and the promising preliminary results of helioseismology make way for stellar seismology, which would yield new tests for the theory of stellar evolution and greatly improve our knowledge on stellar structure and evolution.

I am grateful to Gabrielle Berthomieu for constructive comments and to Philippe Delache and Evry Shatzman for useful discussions. I would like to thank all colleagues who sent information prior to publication.

REFERENCES.

Abraham Z. and Iben I.,Jr : 1971, Astrophys. J. 170, 157.

Ando H. and Osaki Y. : 1975, Publ. Astron. Soc. Japan 27, 581.

Ando H. and Osaki Y. : 1977, Publ. Astron. Soc. Japan 29, 221.

Backus G.E. and Gilbert J.F. : 1968, Geophys. J.R. Astron. Soc. 16, 169.

Baglin A. and Morel P. : 1984, this volume, p. 529.

Bahcall J.N., Bahcall N.A. and Ulrich R.K. : 1968, Astrophys. Letters 2, 91.

Bahcall J.N. and Ulrich R.K. : 1971, Astrophys. J. 170, 593.

Bahcall J.N., Cleveland B.T., Davis R. Jr, Dostrovsky I., Evans J.C. Jr, Frati W., Friedlander G., Lande K., Rowley J.K., Stoenner R.W. and Weneser J. : 1978, Phys. Rev. Letters 40, 1351.

Bahcall J.N. : 1981, Proceedings of Neutrino-81 (Maui,Hawaï), R.J.Cence editor.

Bahcall J.N., Huebner W.F., Lubow S.H., Parker P.D. and Ulrich R.K. : 1982, Rev. of Modern Physics, 54, 767.

Berthomieu G., Cooper A.J., Gough D.O., Osaki Y., Provost J. and Rocca A. : 1980, Non Radial and Non Linear Stellar Pulsation, eds. H.A. Hill and W.A. Dziembowski, Springer Verlag Berlin, p. 307.

Berthomieu G., Provost J. and Schatzman E. : 1983, submitted to Nature.

Bos R.J. and Hill H.A. : 1983, Solar Phys. 82, 89.

Christensen-Dalsgaard J. : 1980a, Proc. Vth European Regional Meeting in Astronomy, Institut d'Astrophysique, Liège.

Christensen-Dalsgaard J. : 1980b, Mon. Not. R. Astron. Soc. 190, 765.

Christensen-Dalsgaard J. : 1982a, COSPAR Meeting, Ottawa, to be published.

Christensen-Dalsgaard J. : 1982b, Mon. Not. R. Astron. Soc. 199, 735.

Christensen-Dalsgaard J., Gough D.O. and Morgan J.G. : 1979, Astron. Astrophys. 73, 121.

Christensen-Dalsgaard J. and Gough D.O. : 1980, Nature 228, 544.

Christensen-Dalsgaard J. and Gough D.O. : 1981, Astron. Astrophys. 104, 173.

Christensen-Dalsgaard J. and Gough D.O. : 1982, Mon. Not. R. Astron. Soc. 198, 141.

Claverie A., Isaak G.R., McLeod C.P. and Van der Raay H.B. : 1979, Nature 282, 591.

Claverie A.,Isaak G.R., McLeod C.P., Van der Raay H.B. and Roca Cortes T. : 1981, Nature 293, 443.

Cleveland B.T., Davis R., Jr, and Rowley T.K. : 1981, in "Weak Interactions as Probes of Unification", edited by G.B. Collins, L.N. Chang and J.R. Fience (AIP Conference Proceedings n° 72),p. 322.

Cox J.P. : 1980, Theory of Stellar Pulsation, Princeton University Press.

Delache Ph. : 1983, Proceedings of the Study Conference "Oscillations as a Probe of the Sun's Interior". Catania. Ed. G. Belvedere and L. Paterno. To be published.

Delache Ph. and Scherrer Ph. : 1983, Nature, in press.

Deubner F.L. : 1975, Astron. Astrophys. 44, 371.

Deubner F.L., Ulrich R.K. and Rhodes E.J. Jr : 1979, Astron. Astrophys. 72, 177.

Dicke R.: 1982, Nature 300, 693.

Duvall T.L. : 1982, Nature 300, 242.
Duvall T.L. and Harvey J.W. : 1983, Nature 302, 24.
Dziembowski W.A. : 1977, Acta Astr. 27, 203.
Dziembowski W.A. and Pamjatnykh A.A. : 1978, "Deuxième Assemblée Européenne de Physique Solaire. Pleins feux sur la Physique solaire", J. Rösch ed. (Paris CNRS) p. 135.
Ezer D. and Cameron A.G.W. : 1968, Astrophys. Lett. 1, 177.
Fröhlich C. : 1983, Proceedings of the Study Conference "Oscillations as a probe of the Sun's Interior". Catania. Ed. G. Belvedere and L. Paterno. To be published.
Gabriel M., Scuflaire R. and Noels A. : 1982, Astron. Astrophys.110, 50
Genova F. and Schatzman E. : 1979, Astron. Astrophys. 78, 323.
Gough D.O. : 1978, Proceedings of the "Workshop on Solar Rotation". Catania. Ed. G. Belvedere and L. Paterno.
Gough D.O. : 1980, Lectures Notes in Physics, Springer Verlag Heidelberg, 125,273.
Gough D.O. : 1982, Nature 298, 350.
Gough D.O. : 1983a, To appear in Proceedings Primordial Helium Workshop, ed. P. Shaver, European Southern Observatory, Garching 1983.
Gough D.O. : 1983b, To appear in "Physics Bulletin"
Gough D.O. : 1983c, Nature 302, 18.
Gough D.O., Hill F. and Toomre J. : 1983, Proceedings of the Study Conference "Oscillations as a Probe of the Sun's interior". Catena. Ed. G. Belvedere and L. Paterno. To be published.
Gough D.O. and Toomre J. : 1983, Solar Physics 82, 41.
Grec G., Fossat E. and Pomerantz M. : 1980, Nature 288, 541
Grec G., Fossat E. and Pomerantz M. : 1983, Solar Phys. 82, 55.
Harvey J.W., Rhodes E.J. and Duvall T.L. : 1983, in preparation.
Hill F., Toomre J. and November L.J. : 1983, Solar Phys. 82, 411.
Hill H.A., Goode P.R. and Stebbins R.T. : 1982, Astrophys. J. 256, L17.
Hill H.A., Bos R.J. and Goode P.R. : 1982, Phys. Rev. Letters 49, 1794.
Iben I. Jr : 1969, Ann. Physics 54, 164.
Iben I. Jr and Mahaffy J. : 1976, Astrophys. J. 209, L39.
Isaak G.R. : 1982, Nature 296, 130.
Kotov V.A., Severny A.B., Tsap T.T., Moiseev I.G., Efanov V.A. and Nesterov N.S. : 1983, Solar Phys. 82, 9.
Law, W.Y.,Knobloch E. and Spruit H.C. : 1984, this volume, p. 523.
Ledoux P. and Walraven T. : 1958, Lecture Notes in Physics, Springer Verlag Berlin 51, Chap. 4.
Leighton R.B., Noyes R.W. and Simon G.W. : 1962, Astrophys. J. 155, 474.
Lubow S.H., Rhodes E.J. and Ulrich R.K. : 1980, Lecture Notes in Physics, Springer Verlag Heidelberg, 125 , 300.
Noels A., Scuflaire R. and Gabriel M. : 1983, preprint.
Rösch J. and Yerle R. : 1983, Solar Phys. 82, 139.
Ross J.E. and Aller L.H. : 1976, Science 191, 1223.
Schatzman E. : 1977, Astron. Astrophys. 56, 211.
Schatzman E. : 1984, this volume, p. 491.
Schatzman E. and Maeder A. : 1981, Astron. Astrophys. 96, 1.
Scherrer Ph. and Wilcox J.M. : 1983, Solar Phys. 82, 37.
Scherrer Ph., Wilcox J.M., Christensen-Dalsgaard J. and Gough D.O. : 1982 Nature 297, 312.

https://doi.org/10.1017/S0074180900030539 Published online by Cambridge University Press

Scuflaire R., Gabriel M. and Noels A. : 1981, Astron. Astrophys. 99, 39.
Scuflaire R., Gabriel M. and Noels A. : 1982, Astron. Astrophys. 113, 219.
Severny A.B., Kotov V.A. and Tsap T.T. : 1983, Proceedings of the Study Conference "Oscillations as a Probe of the Sun's Interior" - Catania. Ed. G. Belvedere and L. Paterno. To be published.
Shaviv G. and Beaudet G. : 1968, Astrophys. Lett. 2, 17.
Shibahashi H., Noels A. and Gabriel M. : 1983, Astron. Astrophys. 123, 283.
Shibahashi H. and Osaki Y. : 1981, Publ. Astron. Soc. Japan 33, 713.
Spruit H.C., Knobloch E. and Roxburgh I.W. : 1983, to appear in Nature.
Stenflo J.O. : 1980, Proc. Vth European Regional Meeting in Astronomy, Institut d'Astrophysique , Liège.
Tassoul J.L. and Tassoul M. : 1968, Ann. Astrophys. 31, 251.
Tassoul M. : 1980, Astrophys. J. Suppl. 43, 469.
Tassoul J.L. : 1984, this volume, p. 475.
Ulrich R.K. : 1970, Astrophys. J. 162, 993.
Ulrich R.K. : 1982, Astrophys. J. 258, 404.
Ulrich R.K. and Rhodes E.J. : 1977, Astrophys. J. 218, 521.
Ulrich R.K. and Rhodes E.J. : 1983, Astrophys. J. 265, 551
Ulrich R.K. Rhodes E.J., Tomczyk S., Dumont Ph. J. and Brunish W.M. : 1983b, AIP Conference Proceedings, n° 96, Ed. H.C. Wolfe.
Unno W., Osaki Y., Ando H. and Shibahashi H. : 1979, Non Radial Oscillations of Stars, University of Tokyo Press.
Vandakurov Yu.V. : 1967, Astr. Zh. 44, 786.
Van der Raay H.B. : 1983, Proceedings of the Study Conference "Oscillations as a Probe of the Sun's Interior" - Catania. Ed. G. Belvedere and L. Paterno. To be published.
Woodard M. and Hudson H. : 1983, Solar Phys. 82, 67.
Zahn J.P. : 1983, Saas Fee Lectures, 13th Advanced Course of the Swiss Society of Astronomy and Astrophysics. Ed. B. Hauck and A. Maeder.

DISCUSSION

Cox: What is the effect of the non-adiabaticity on the periods? They may be important for the longer periods.

Provost: The effect of the non-adiabaticity is to decrease the frequencies of low degree 5 mn oscillations of about 5μHz, according to Christensen-Dalsgaard and Frandsen (1983). For gravity modes, the modification of periods could be larger since they have longer periods and non-adiabatic exchanges have time to operate. However, the maximum amplitude of g-modes occurs in the radiative inner region where the non-adiabaticity due to radiation is quite negligible. An exact non-adiabatic computation is necessary to conclude. As far as I know, there do not exist published results, except some stability studies; these studies do not indicate both adiabatic and non-adiabatic periods.

https://doi.org/10.1017/S0074180900030539 Published online by Cambridge University Press

Hesser: Could you please repeat how much faster the interior is thought to rotate, as a result of the study of the rotational splitting of the g-modes?

Provost: The rotational splitting of g-modes reported by Delache and Scherrer (1983) and Fröhlich (1983) is of the order of 1.22 μHz. This indicates that the interior of the Sun rotates about three times faster than the solar surface.

Zahn: In your discussion you consider the parameter R_e^* characterizing the turbulent diffusion as a constant throughout the Sun. But it could well depend on depth. Is it possible to refine the determination of R_e^* to make it depth-dependent? For instance, to which part of the Sun are the g-modes most sensitive (since they yield the smallest value of R_e^*)?

Provost: Models with constant R_e^* are a first approach in the construction of mixed models and more elaborated models are needed. Depth-dependent values of R_e^* can be theoretically obtained assuming marginal instability (Spruit et al, 1983, Zahn, 1983). In fact it is possible to estimate different values of R_e^* at different depths of the Sun from the observations. For instance, a value of R_e^* for the region below the convection zone and above the lithium-burning zone can be determined from the observation of lithium depletion, as discussed by Baglin and Morel (1983). In the same way, an estimate of R_e^* for the inner five percents of the mass could be obtained from the observed low degree gravity modes periods.

https://doi.org/10.1017/S0074180900030539 Published online by Cambridge University Press