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AN INVESTIGATION INTO RECENT THEORIES OF RELAXATION TOWARDS EQUILIBRIUM

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There are many open questions surrounding the statistical mechanical characterisation of systems out of equilibrium, particularly regarding their relaxation towards equilibrium or nonequilibrium steady states, as well as the fluctuations which occur throughout their dynamics. The dissipation function of Evans and Searles [3] has found successful applications in both of these areas. In this work, we investigate the use of the dissipation function to characterise these behaviours in a number of dynamical systems.

In a system which allows us to tune the chaoticity of its dynamics, we use the dissipation function to study the necessary conditions for fluctuation relations to be observed. This section of work teases apart the conditions of chaotic dynamics and transient mixing (T-mixing) [4, 6], showing that fluctuation relations hold in nonchaotic systems, counter to the contemporary theories of the necessary conditions. Our results suggest that the Evans–Searles fluctuation theorem [1, 2] may provide a more fruitful path forward for the study of fluctuations in dynamical systems.

We have found that the dissipation function captures apparent relaxation in the absence of particle–particle interactions. Without interactions between particles, this relaxation is akin to relaxation in a single-particle system. It is not clear whether there is any way to give a physical interpretation of this relaxation or whether this should be interpreted as dissipation in any sense. The absence of particle–particle interactions also makes it impossible for the system to relax in the momentum distribution. Despite this, the system appears to undergo the same degree of dissipation in the absence of particle–particle interactions, suggesting that the dissipation function may not capture all aspects of relaxation within a system.

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Finally, we have applied the dissipation function to the study of relaxation in a microcanonical system. To the best of our knowledge, this represents the first application of the dissipation function to a microcanonical system. One of the few conditions which can be placed on the study of relaxation with the dissipation function is that the equilibrium state of the system will be dissipationless. However, the system we have studied appears to undergo a large number of apparently dissipationless states beyond which relaxation continues. These dissipationless states occur both instantaneously ($\Omega_t^f = 0$) and when measured over long time averages ($d\Omega_{0,t}^f/dt = 0$). We also find that the state of maximal dissipation does not necessarily indicate the maximally relaxed state of a system. These results raise deep questions about the use of the dissipation function in the study of equilibration.

Some of this research has been published in [5].

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