2. Solving for x means finding one or more parallels to the y-axis through the points of intersection of the given curves.
3. When only the first power of y is present in both given curves, each parallel to the y-axis cuts each given curve in only one point.
4. When one equation contains the second power of y and the other equation only the first power, each parallel cuts the first curve in two points and the second in one point.
5. The solutions required are given only by the points where *both* given curves meet a parallel; wrong results mean points where *only one* given curve meets a parallel.

It would not be feasible, of course, to introduce to a beginner the hyperbola cited in Mr Ridley's note, but a grasp of the geometrical facts for the simpler curves should help to eliminate the beginner's idea that as every equation is a true statement, "true" results must follow from combining any sets of equations.

(*Cf.* Godfrey and Siddon's "Algebra," § 177).

G. D. C. Stokes.

An Exact Geometrical Construction for the Exponential Curve.—Consider the curve $y = a\epsilon^{-t/t_0}$. (Fig. 1.)

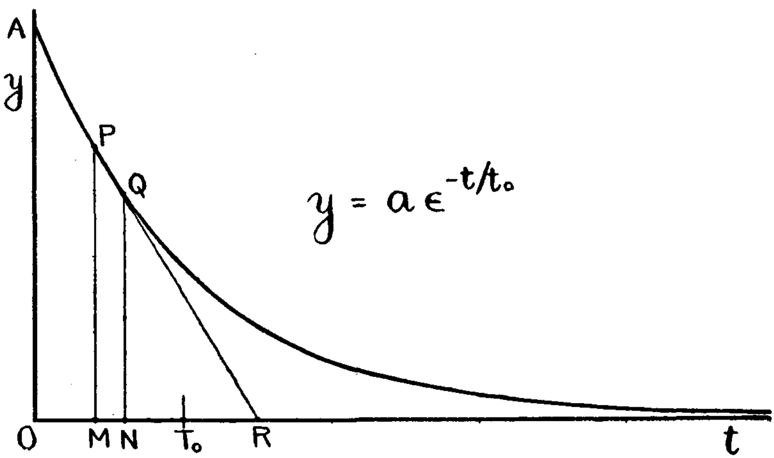

Fig. 1.

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Mark off along the t-axis a distance OT_0 equal to the time constant t_0.

Divide OT_0 into n equal parts and erect ordinates at each point of division.

M, N, are the m^{th} and $(m+1)^{\text{th}}$ of these, counting from O.

MP, NQ, are the corresponding ordinates; P, Q, the points on the curve; and R the intersection of the chord, produced, with the t-axis.

Then, $MP = a\epsilon^{-m/n}$ and $NQ = a\epsilon^{-(m+1)/n}$.

Hence the slope of the chord is

$$-\frac{MP - NQ}{MN} = -\frac{a\epsilon^{-m/n}(1-\epsilon^{-1/n})}{t_0/n}.$$

Hence,

$$MR = \frac{MR}{MP} \times MP = \frac{t_0/n}{a\epsilon^{-m/n}(1-\epsilon^{-1/n})} \times a\epsilon^{-m/n} = \frac{t_0}{n(1-\epsilon^{-1/n})}$$

= constant, and so we have the following construction:—

Set off along the y-axis the length $OA = a$. (Fig. 2.)

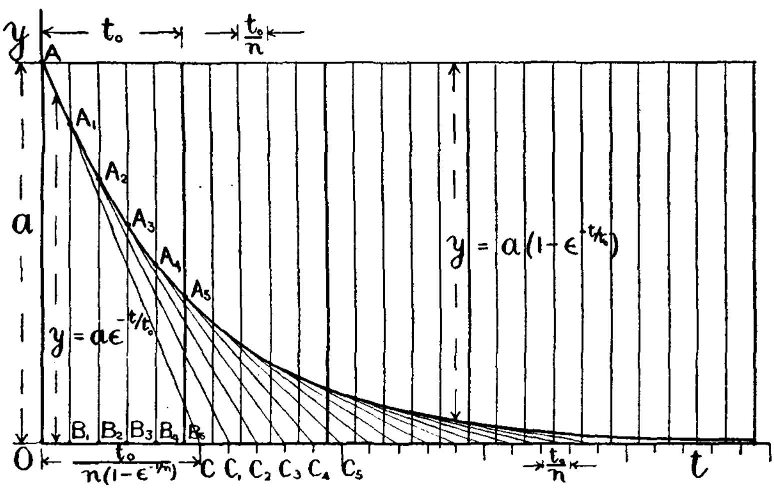

Fig. 2.

Beginning with O erect ordinates B_1, B_2, B_3, etc., at intervals of t_0/n.

Take OC along the t-axis $= \dfrac{t_0}{n(1-\epsilon^{-1/n})}$.

Beginning with C, mark off a number of points C_1, C_2, C_3, etc., at intervals of t/n.

AC cuts the ordinate B_1 in A_1.

A_1C_1 cuts the ordinate B_2 in A_2.

A_2C_2 cuts the ordinate B_3 in A_3, and so on.

A_1, A_2, A_3, etc., are points on the curve.

The following are values of the function OC/t_0

n	3	5	10	∞
$\{n(1-\epsilon^{-1/n})\}^{-1}$	1·176	1·105	1·051	1.

A convenient number to take for n is 5, for which OC may be taken with sufficient accuracy for most purposes as 1·1. This fits in very easily with the use of squared paper. The case $n=\infty$ gives the well-known property of the subtangent which has often been applied to a geometrical construction, but fails because of the necessity of working with a finite number of ordinates.

Obvious modifications make the construction applicable to the curve $y=a(1-\epsilon^{-t/t_0})$.

DAVID ROBERTSON.

Integration by Parts. A Failing Case.—Integrate $\tan x$ by parts as follows:—

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\frac{\cos x}{\cos x} + \int \tan x dx,$$

i.e. $0=-1$, which is absurd.

It might be conjectured at first sight that the above result is due to a disregarded constant of integration, but this is not so, because we can imagine all the integrals taken between the same limits, in which case there is no constant. The true explanation is to be found by considering the proof of the theorem of integration by parts.

Let u and v be functions of x, then the following is an identity

$$uv = \frac{du}{dx}\int v dx + uv - \frac{du}{dx}\int v dx.$$

Re-write as,

$$uv = \frac{d}{dx}\left(u\int v dx\right) - \frac{du}{dx}\int v dx \qquad (1)$$

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