MERIDIONAL COMPRESSION OF RADIO PULSAR BEAMS

JAMES D. BIGGS
Nuffield Radio Astronomy Laboratories, University of Manchester

Abstract

We have studied the radio pulsar emission beam assuming a) a magnetic dipole field geometry and b) that the beam geometry is defined by the field lines that are not contained within the light cylinder. In general, the beam is compressed in the meridional direction. When the magnetic and rotation axes are aligned the beam is circular and as, the angle between these axes increases, the ratio of meridional to longitudinal dimension decreases monotonically to the minimum value 0.62 when the axes are orthogonal. This beam shape is thus consistent with that inferred from the study of circular polarization in average pulse profiles by Radhakrishnan and Rankin. Evidence for meridional compression is also found in the extensive observational study of Lyne and Manchester (1988).The beam evolution was determined using this data set, the beamwidth being found proportional to $P^{-1/2}$, where P is the pulsar period. This relation implies that the more rapidly rotating pulsars should have larger beams, and this should aid in their detection. The more numerous, slower pulsars should have somewhat smaller beams than previously determined. This implies that the pulsar birthrate is probably close to the highest current estimates (1 in 25 yr).

Introduction

The strong magnetic fields of radio pulsars (10^8–10^{12} G) appear to be primarily dipolar in nature. Also, radio pulsar emission is usually highly polarized with a monotonic (or piece-wise monotonic) change in position angle through the main pulse and interpulse (if present). The observed emission has been interpreted as originating above the polar cap on dipolar field lines, and it has a polarization that is either parallel or perpendicular to the plane of curvature (Radhakrishnan and Cooke 1969, Manchester and Taylor 1977, Beskin, Gurevich, and Istomin 1988c). A potentially observable radiation beam originates from the unclosed field lines that intersect the neutron-star surface within the polar cap. Figure 1 illustrates the dipole field geometry assuming a static field (neglecting the effects of rotation) as well as the relationship between the polar cap, the last closed field lines, and the light cylinder.

Emission beam geometry

If one uses the definition of the polar cap as the area bounded by the last closed field lines, the shape of the polar-cap boundary (and hence the emission beam) depends on the angle α between the rotation axis and the magnetic axis (Sturrock 1971). This effect results from the difference in the size of the last closed field lines (Biggs 1990b). When aligned, the boundary is circular but as α increases the meridional dimension is compressed, while the longitudinal dimension remains unchanged, thus making the polar cap quasi-elliptical (see figure 2). The ratio $K(\alpha)$ of the meridional to longitudinal dimension is at its minimum value 0.62 when the rotation and magnetic axes are orthogonal (see figure 3). An approximate formula for $K(\alpha)$ is

$$K(\alpha) \approx 1.00 - 3.30 \times 10^{-4}\alpha - 4.40 \times 10^{-5}\alpha^2,$$

where the units of α are degrees.

Observational evidence

Recently Lyne and Manchester (1988) have studied the shape of radio pulsar beams using the polarization position-angle changes in more than 120 pulsars observed near 400 MHz. random within a circular beam that defines the overall pulse profile. Also, they found no evidence for the meridional beam elongation proposed by Jones (1980) and Narayan and Vivekanand (1983). Lyne and Manchester found that the number of pulsars with some conal emission and a small normalized impact parameter ($\beta_n = \beta / W$, where β is the angle subtended by the magnetic axis and the line of sight, and W is the angle subtended by the magnetic axis and edge of the emission beam) was higher by a factor of 2 than that for a large impact parameter. This is evidence for compression in the meridional direction.

Lyne and Manchester (1988) also determined α and β values for about 110 pulsars observed near 400 MHz which provide further evidence for meridional compression. Normalizing β by $P^{-1/2}$ to account for the beamwidth evolution with period reduces the scatter in figure 4 (upper panel), and the

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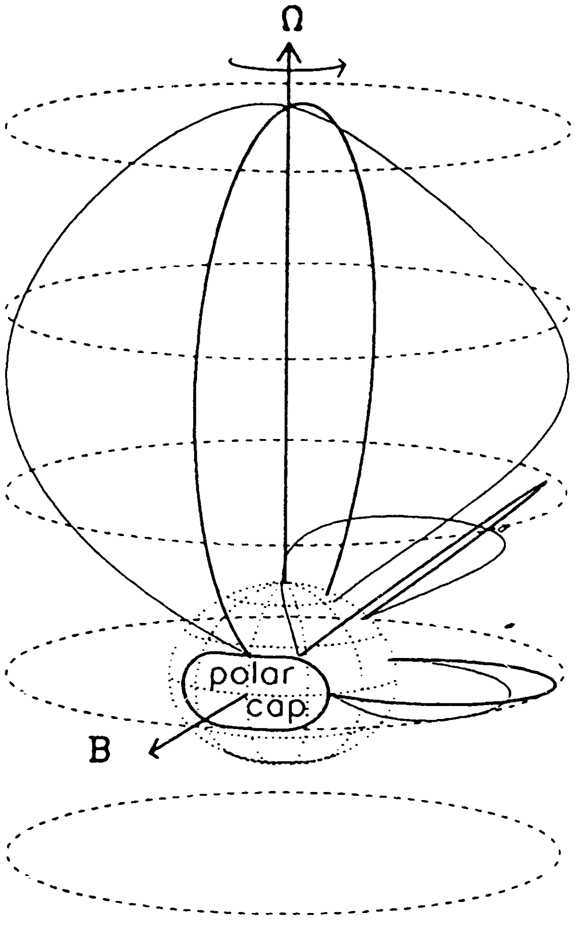

Figure 1 The dipole field geometry of a radio pulsar. Some of the last closed magnetic field lines are shown that just fit within the light cylinder, calculated for the static case (thick lines). The polar-cap boundary on the neutron star surface is also indicated. The thinner lines illustrate the effects of rotation and retardation on the magnetic field lines. The light cylinder is represented by dashed lines.

vast majority of the data can be bounded by a curve whose form is determined by the The calculation of an upper bound is appropriate for this beam geometry, since in order to observe a pulsar the impact parameter must be less than the maximum half beamwidth. The upper bound curve indicates that at 400 MHz the half beamwidth evolution (for an aligned rotator) as a function of P is given by

$$W = 6.2^\circ P^{-1/2}, \qquad (1)$$

where P is in seconds. A smaller impact parameter is required to observe core components, and this is reflected by the lower curve in figure 4, (lower panel). The form of this curve is determined by $K(\alpha)$, and it represents an upper bound to data from pulsars exhibiting both core and cone components.

Further evidence for a meridionally compressed beam geometry arises from the study of the circular polarization in average pulse profiles by Radhakrishnan and Rankin (1990). In order to explain the correlation between the sign of the rate of change

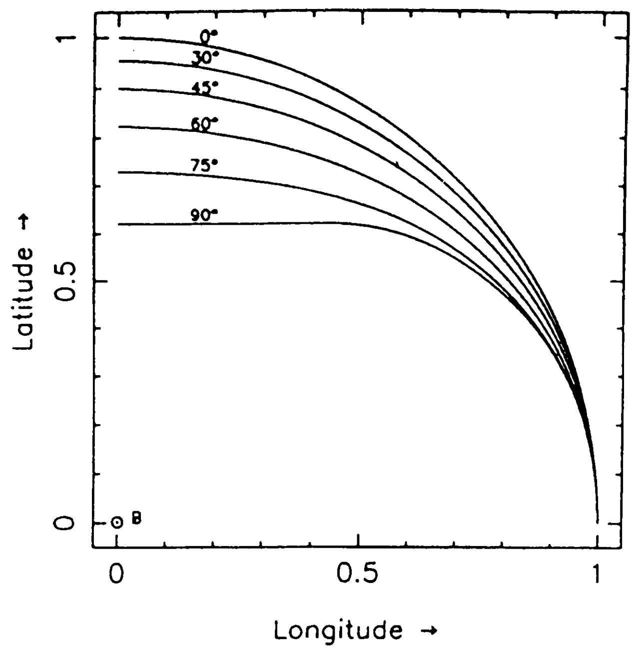

Figure 2 The approximate shape of one quadrant of the polar cap (scaled by 69 $P^{1/2}$, P in seconds) as a function of α.

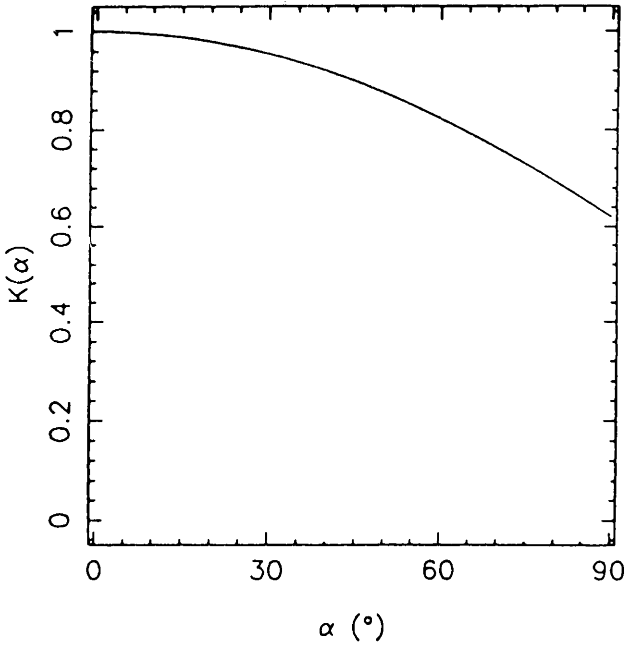

Figure 3 The meridional compression factor K(α) plotted as a function of α.

of polarization position angle with respect to longitude with the change of handedness of the circular polarization, they suggest that the emission beam must be compressed in the meridional direction.

Beaming fraction

For a circular beam with half beamwidth W, f is given by

$$f = \frac{4}{\pi} \sin W, \qquad (2)$$

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Table 1 Pulse structure of the fastest pulsars

PSR	P (ms)	pulse structure	Ref
1937+21	1.5	interpulse	1
1957+20	1.6	interpulse	2
1821−24	3.1	wide double	3
1908+00	3.6	?	4
0021−72A	4.5	5% duty cycle	5
1855+09	5.4	interpulse	6
1820−30A	5.4	≈10% duty cycle	7
1516+02A	5.5	?	8
0021−72C	5.8	15% duty cycle	9
1953+29	6.1	wide double	10
0021−72B	6.1	?	11
1516+02B	7.9	?	8
1639+36	10	15% duty cycle	12
1620−26	11	10% duty cycle	13
1745−24	11	≈5% duty cycle	14

[1] Cordes and Stinebring (1984). [2] Fruchter, Stinebring, and Taylor (1988). [3] Lyne *et al.* (1987). [4] Anderson *et al.* (1990). [5] Ables *et al.* (1989b). [6] Segelstein *et al.* (1986). [7] Biggs *et al.* (1990). [8] Wolszczan *et al.* (1989). [9] Manchester *et al.* (1990). [10] Boriakoff, Buccheri, and Fauci (1983). [11] Ables *et al.* (1988). [12] Anderson *et al.* (1989). [13] Lyne *et al.* (1988). [14] Lyne *et al.* (1990).

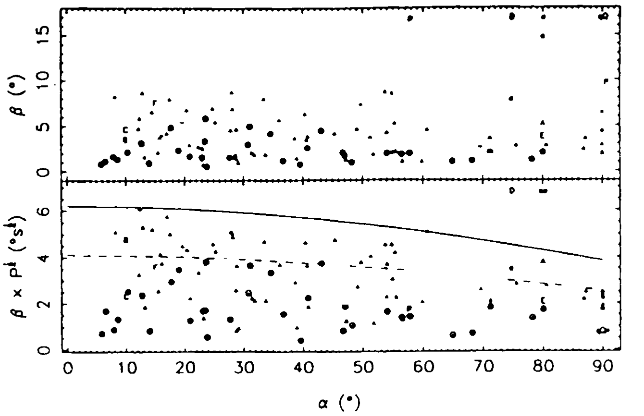

Figure 4 *Upper panel*: The impact angle β (the minimum angle between the line of sight and the magnetic axis) plotted as a function of α for pulsars with predominately conal emission (△), core and cone emission(⊙) and pulsars with interpulses or wide pulses (A, PSR 0823+26; B, PSR 0826−34; C, PSR 0950+08; D, PSR 1055−52; E, PSR 1702−19; F, PSR 1929+10; P, PSR 1855+09; Q, PSR 1937+21; uppercase main pulse, lowercase interpulse). *Lower panel*: Same as above with β normalized by $P^{-1/2}$. Also plotted are the upper bounds 6.2°K(α) (solid line) and 4.1°K(α) (dashed line). Data with large values of β are plotted at $\beta = 17°$ and 6.7° in the upper and lower panels, respectively, in order to improve the presentation of the majority of the data. Also, symbols a, e, q, P and Q have been slightly shifted in α for clarity. Note the different vertical scales.

and the probability to observe an interpulse, $P_{\rm IP}$ is given by

$$P_{\rm IP} = \frac{2}{\pi}\,(1 - \cos W), \qquad (3)$$

assuming a random distribution of α and ignoring the overlap region near the poles (Lyne and Manchester 1988). The W we need to use is that measured in the meridional direction because of the rotational symmetry. Since we have shown that W is a function of α, we must average W over α to calculate f. Eq.(2) then becomes

$$\begin{aligned} f &= \bar{K}\,\frac{4}{\pi}\,\sin W(\alpha = 0) \\ &\approx 0.818\,\frac{4}{\pi}\,\sin W(\alpha = 0), \end{aligned} \qquad (4)$$

where $\bar{K} = \int_0^{\pi/2} K(\alpha)\,\sin\alpha\,d\alpha \approx 0.818$ and $W(\alpha = 0)$ is the beamwidth of an aligned rotator. An average over α is not appropriate to calculate the correction to $P_{\rm IP}$ since to observe interpulses α cannot vary greatly from $\approx 90°$ for the majority of pulsars, and so the compression factor is best left in the expression for W. Thus, eq.(3) becomes

$$P_{\rm IP} = \frac{2}{\pi}\,\{1 - \cos[W(\alpha = 0)\;K(90° - W(\alpha = 0))]\}. \qquad (5)$$

Using the beamwidth formula of eq.(1) enables the calculation of the evolution of f and $P_{\rm IP}$ with P. These are plotted in figure 5 along with those

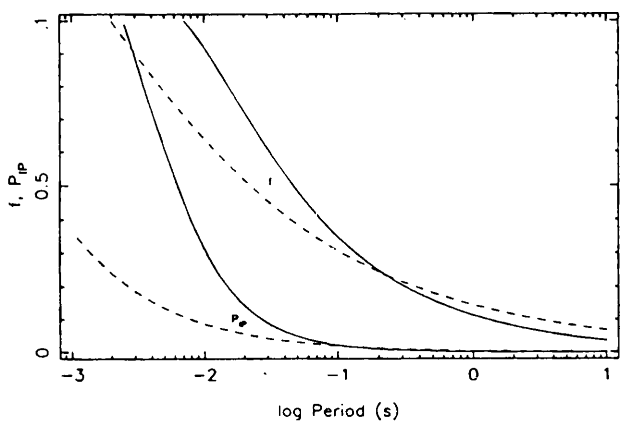

Figure 5 The beaming fraction f and the probability to observe an interpulse $P_{\rm IP}$ as a function of P, calculated in the present work assuming $W = 6.2°\,P^{-1/2}$ (solid lines) and by Lyne and Manchester (1988) (dashed lines).

calculated by Lyne and increases f and $P_{\rm IP}$ for $P \lesssim 0.2$ s and decreases them for $P \gtrsim 0.2$ s as compared with those of the earlier work. Figure 5 indicates that if the effect of beam patchiness was inoperative, then it should be possible to detect all pulsars with cone components whose $P < 7$ ms. This figure also indicates that all pulsars with $P \lesssim 2.5$ ms should exhibit interpulses. The observations (see table 1)—wherein the two fastest radio pulsars ($P < 1.6$ ms) exhibit interpulses and

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the majority of the other millisecond pulsars have interpulses or wide profiles—appear to support this premise.

The vast majority of the radio pulsars have $P \approx 0.5$ s and a small beaming fraction. An estimate of the average beaming fraction can be derived by multiplying eq.(4) by the sensitivity-corrected P distribution of Lyne *et al.* (1985) and The actual beaming fraction may be even smaller since it depends on the details of the geometry of the pulse emission mechanism within the cone defined by the last closed field lines. Following Lyne and Manchester we calculated the mean fraction of pulsars with interpulses by dividing P_{IP} by f and then multiplying by the sensitivity corrected P distribution. The value so derived is 0.0252 which is smaller than the 0.0344 obtained by Lyne and Manchester and consistent with the observed ratio of 9 interpulsars in 440 pulsars.

Population and birthrate

The pulsar birthrate BR_{PSR} has an inverse dependence on f, and the low average f calculated above indicates that the higher estimates of this quantity are preferred. Lyne *et al.* (1985) have calculated BR_{PSR} to lie within 1 in 30 yrs and 1 in 120 yrs, whereas Narayan (1987) using a different model for the pulsar luminosity function calculates 1 in 20 yrs $> BR_{PSR} >$ 1 in 60 yrs. The smaller birthrates were calculated assuming that f was influenced by beam elongation and, since no evidence for this phenomenon persists, the higher values are probably the better estimates. Thus 1 in 20 yrs $> BR_{PSR} >$ 1 in 30 yrs is probably the best estimate for the currently known pulsar population. This value is in good agreement with the death rate of massive stars that can become supernovae (the possible progenitors of pulsars) and is not inconsistent with the birthrate of supernova remnants (Lyne, Manchester, and Taylor 1985).

For a more detailed discussion of the matters raised here, the reader is referred to the recent work by Biggs (1990).