DISK STABILITY

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What has prevented the formation of a strong bar in the majority of disc galaxies? No truly satisfactory answer has yet been given to this question and the difficulty remains a major obstacle to our understanding of the dynamics of these systems. In this review, I will discuss the implications of recent studies of this problem.

Should The Results Be Believed?

In order to determine whether a stellar dynamical model is globally stable one can either perform a linear stability analysis or run a computer simulation. Unfortunately, analytic studies are extremely difficult and have been completed in only a few simple cases (Kalnajs 1972, Zang 1976, Toomre 1981). Large n-body simulations are therefore necessary for most models of interest.

Since the conclusions are far reaching, it is important to demonstrate that the results can be trusted. One of the best tests is to show that identical results are obtained by the two methods for the same model; Sellwood (1982) finds discrepancies of only a few percent between the predicted eigen-frequencies and those observed in the simulations. Tests of the non-linear behaviour are more difficult to devise. Nishida et al (in preparation) are working with a completely different numerical technique in which they integrate the Vlasov equation. Comparison between their results and those from n-body codes in the non-linear regime will again provide a useful test of both methods.

Stability Criteria

The unavoidable conclusion from all studies of disc stability to date is that some fraction of the mass of an unbarred galaxy must reside in a spherically distributed component. The interesting question is what fraction? Despite some systematic searching, no criterion for stability has been found that is a reliable guide for any arbitrary model.

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Ostriker and Peebles (1973) suggested that a single parameter, $t = KE_{rot}/|PE|$, was sufficient to discriminate stable from unstable stellar systems. They asserted that wherever t exceeded ∿0.14 the system should want to form a bar, and argued that only a small fraction of the mass of a galaxy could reside in a cool, rotationally supported disc. Their argument has often been cited as further evidence of the existence of massive halos around galaxies, yet it should be clear that this is evidence only for halo mass interior to the outer radius of the disc; any mass spherically distributed beyond that exerts zero gravitational force on the disc and therefore cannot affect its stability. Yet the t parameter can be made arbitrarily small by adding shells of matter at large radii. A revised parameter, t*, which avoids this nonsense was proposed by Lake and Ostriker (unpublished) and tested by Efstathiou et al (1982). They found that even the revised parameter was not an infallible guide to the stability of realistic models. Other counter-examples to the t* criterion can be found in Kalnajs' (1972) study of Maclaurin discs and in Zang's (1976) thesis work. Both parameters, therefore, are of little use.

Both Hohl (1976) and Sellwood (1980) concluded from n-body experiments of differentially rotating discs that the total spheroidal mass (well within the outer radius of the disc) had to be roughly twice the disc mass for the models to be stable. But, Hohl found less halo mass was needed if the disc rotates rigidly.

Efstathiou et al (1982) found that $Y = V_m(\alpha M_D G)^{-\frac{1}{2}}$ should be larger than ∿1.1 for a class of reasonably realistic model galaxies having exponential discs and a halo component chosen to give a predominantly flat rotation curve (Fall and Efstathiou 1980). V_m is the rotational speed of material on the flat part of the rotation curve, α^{-1} is the scale length of the exponential disc having a total mass M_D. This criterion can be applied only to galaxies with the assumed form of rotation curve.

A purely local criterion has been found by Toomre (1981). He finds that a parameter, X, should be larger than 3 to prevent swing amplification. X is defined as the ratio of the circumferential wavelength of the instability to the Jeans length in a cold rotating disc (Toomre 1964).

$$X = \frac{r\ \kappa^2}{2\pi m G \Sigma(r)}$$

where κ is the local epicyclic frequency, Σ is the surface density of matter in the disc and m is the number of arms: i.e. 2 for a bar instability. Unfortunately, X varies rapidly with radius for most galaxies, so this local criterion cannot easily be used to assess global stability.

While these parameters are rather diverse they all make similar demands on the distribution of mass, when applied to the Fall and Efstathiou models, for example. Figure 1(a) shows the variation with radius of the ratio of bulge+halo mass to the disc mass, both interior

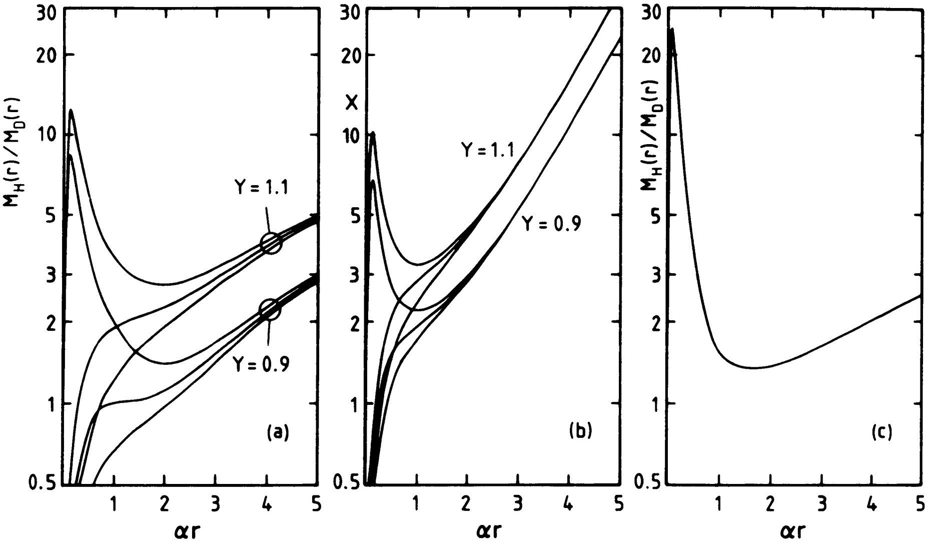

Figure 1. (a) Halo to disc ratio as a function of radius for some Fall & Efstathiou models. Y has the indicated value for each group of three curves which correspond to αr_m = .1, .8 & 1.3. (b) Toomre's X parameter for the same models as in (a). (c) Halo to disc ratio for the model of our Galaxy by Bahcall *et al*.

to r, for their models. Different values of Y determine the overall halo to disc ratio while the parameter αr_m determines the prominence of the bulge component. At two disc scale lengths from the centre, just beyond the half-mass radius, the structure of the models is nearly independent of αr_m and the condition $Y>1.1$ is precisely that required to ensure that the halo to disc ratio exceeds two at this radius. Figure 1(b) shows Toomre's X parameter for the same models. Again it is clear that X reaches its critical value at two disc scale lengths for the same value of Y. Thus for these models at least, three stability criteria agree that the bulge+halo mass interior to the half-mass radius of the disc should be more than twice the disc mass in the same region.

Confrontation with Observations

The most serious attempt made to check stability criteria against observational evidence is due to Efstathiou *et al* (1982). Taking all existing rotation curve data that could be fitted to their models, they concluded that the mass to light ratio of the discs of Sc galaxies could not greatly exceed unity without violating their stability criterion. This is rather uncomfortably low, but they claimed it to be consistent with the Larson and Tinsley (1978) population models.

Mass models of our Galaxy are inconclusive: both the Bahcall et al (1982) model, shown in Figure 1(c), and the Caldwell and Ostriker (1981) model fail by a substantial margin to place sufficient mass in the bulge+halo interior to the sun for the disc to be stable. However, it should be recognised that the disc mass in both these models is very dependent on Oort's measurement of the local surface density, a quantity which is rather badly determined by the existing observations.

We have also heard, presented at this conference, mass models of two edge-on disc galaxies. Van der Kruit, drawing together a number of strands of evidence, concluded that the bulge+halo mass interior to the outer edge of NGC 891 was roughly three times that of the disc. This galaxy should be well fitted by a model of the type shown in Figure 1 with $\alpha r = 4.5$ at the outer edge, and $Y = 1$, placing it marginally on the unstable side of the criterion. Cassertano's model of NGC 5907 falls well short of the stability criteria since he finds $M_H \sim 1.5\ M_D$ at $\alpha r = 3.5$.

Fall and Efstathiou (1980, in their Figure 9) noticed that Y did not differentiate barred from unbarred galaxies when M was estimated from the disc luminosity, which led Efstathiou et al to conjecture that most galaxies lie on the borderline of stability. I feel that the hotch-potch of evidence reviewed here does not support their hypothesis, although it is too uncertain to rule it out: in every case it has to be stretched to find sufficient mass in the bulge+halo components for even marginal stability.

The Frequency of Barred Types Along the Hubble Sequence

One characteristic defining the Hubble sequence is the bulge to disc ratio; which decreases from type S0 to Sc. If the fraction of luminous bulge is the principal determinant of disc stability one would expect the frequency of bars to increase towards later types. In fact, the opposite is the case.

Figure 2 shows the frequency of barred types taken from the Revised Shapley-Ames catalog of galaxies (Sandage and Tammann 1981). Amongst early type systems (S0 to Sb) Sandage finds more than twice as many unbarred as barred galaxies, whereas slightly fewer bars occur in Sc types. The paucity of bars amongst bright Sc galaxies is made more significant when one takes into account a selection effect notices by Van den Bergh (1982) who showed that the barred Sc galaxies in the catalog are systematically less luminous than the unbarred. Leaving aside the smaller galaxies of types Sc and later, bars are actually much rarer amongst systems having little visible bulge.

Thus, if bulge+halo mass provides stability, one must conclude that on average Sc galaxies have relatively more mass in a dark halo than earlier type galaxies have in visible bulge and dark halo combined.

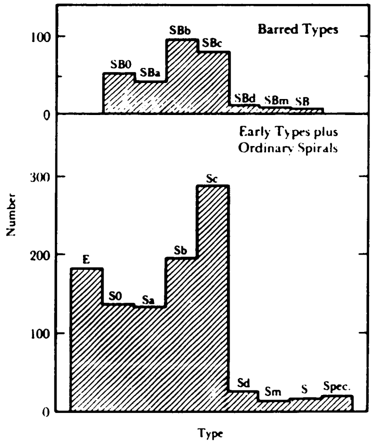

Figure 2. Histogram showing the frequency of Hubble types in the RSA catalogue. Intermediate types are combined into the main classes. Reproduced from Sandage and Tammann (1981).

Hot Discs?

All three stability criteria discussed above were determined on the basis that stellar discs are very largely rotationally supported, having sufficient random motion only to suppress local Jeans instabilities. However, global stability should be very substantially affected by a fair degree of pressure support among the stars of the disc near the centre. If we neglect this contribution we may overestimate the bulge+halo mass required.

Disc stars in the solar neighbourhood are believed to move on nearly circular orbits and to have a mean rotational velocity some five times the dispersion of the radial velocity components. But the sun lies far from the rotation axis and it is natural to expect that velocity dispersion will rise towards the centre. Observational evidence that this rise may be substantial is beginning to be found, although the measurements are still rather close to the limits of technical feasibility. (See reviews by Kormendy and Illingworth in this volume). In one case only is the observed random motion less than one third the mean rotation rate. Here, Illingworth and Schechter (1982) give an upper limit of 50 to 80 km/sec for the velocity dispersion of the disc stars in NGC 3115. However, as they clearly measured the tangential component, I would increase their limit by 50% since the radial component should be larger by roughly this amount.

Apart from reducing the bulge+halo mass required for global stability, hot discs have other advantages. For example, all globally unstable cool disc simulations give narrow, very strong bars, regardless of how marginally unstable they were. Unstable hot discs form fatter, weaker bars more consistent with intermediate SAB galaxies. Thus we could envisage a picture in which most galaxies have only moderate fractions of mass in the bulge+halo and the presence and strength of a bar reflects the extent of pressure support amongst the disc stars.

Much work needs to be done to put this hypothesis on a firmer basis. We need to determine the extent to which pressure support trades with bulge+halo mass at marginal stability. We also need to understand how galaxies can acquire large degrees of random motion in the plane when we expect disc stars to have formed on nearly circular orbits. A suggestion as to how this could be achieved is contained in the work of Carlberg and Sellwood (presented at this meeting). In their picture, the bar morphology could be related to the rate at which the disc is built up during galaxy formation.

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